# Glossary of Z notation

## Names

| | |
|---|---|
| $a$,$b$ | identifiers |
| $d$,$e$ | declarations (e.g., $a : A; b, ... : B ...$) |
| $f$,$g$ | functions |
| $m$,$n$ | numbers |
| $p$,$q$ | predicates |
| $s$,$t$ | sequences |
| $x$,$y$ | expressions |
| $A$,$B$ | sets |
| $C$,$D$ | bags |
| $Q$,$R$ | relations |
| $S$,$T$ | schemas |
| $X$ | schema text (e.g., $d$, $d \mid p$ or $S$) |

## Definitions

| | |
|---|---|
| $a == x$ | Abbreviation definition |
| $a ::= b \mid ...$ | Free type definition (or $a ::= b\langle\!\langle x \rangle\!\rangle \mid ...$) |
| $[a]$ | Introduction of a given set (or $[a, ...]$) |
| $a\_$ | Prefix operator |
| $\_a$ | Postfix operator |
| $\_a\_$ | Infix operator |

## Logic

| | |
|---|---|
| $true$ | Logical true constant |
| $false$ | Logical false constant |
| $\neg\ p$ | Logical negation |
| $p \land q$ | Logical conjunction |
| $p \lor q$ | Logical disjunction |
| $p \Rightarrow q$ | Logical implication ($\neg\ p \lor q$) |
| $p \Leftrightarrow q$ | Logical equivalence ($p \Rightarrow q \ \land\ q \Rightarrow p$) |
| $\forall X \bullet q$ | Universal quantification |
| $\exists X \bullet q$ | Existential quantification |
| $\exists_1 X \bullet q$ | Unique existential quantification |
| **let** $a == x; ... \bullet p$ | Local definition |

## Sets and expressions

| | |
|---|---|
| $x = y$ | Equality of expressions |
| $x \neq y$ | Inequality ($\neg\ (x = y)$) |
| $x \in A$ | Set membership |
| $x \notin A$ | Non-membership ($\neg\ (x \in A)$) |
| $\varnothing$ | Empty set |
| $A \subseteq B$ | Set inclusion |
| $A \subset B$ | Strict set inclusion ($A \subseteq B \land A \neq B$) |
| $\{x, y, ...\}$ | Set of elements |
| $\{X \bullet x\}$ | Set comprehension |
| $\lambda X \bullet x$ | Lambda-expression – function |
| $\mu X \bullet x$ | Mu-expression – unique value |
| **let** $a == x; ... \bullet y$ | Local definition |
| **if** $p$ **then** $x$ **else** $y$ | Conditional expression |
| $(x, y, ...)$ | Ordered tuple |
| $A \times B \times ...$ | Cartesian product |
| $\mathbb{P}\ A$ | Power set (set of subsets) |
| $\mathbb{P}_1\ A$ | Non-empty power set |
| $\mathbb{F}\ A$ | Set of finite subsets |
| $\mathbb{F}_1\ A$ | Non-empty set of finite subsets |
| $A \cap B$ | Set intersection |
| $A \cup B$ | Set union |
| $A \setminus B$ | Set difference |
| $\bigcup A$ | Generalized union of a set of sets |
| $\bigcap A$ | Generalized intersection of a set of sets |
| $first\ x$ | First element of an ordered pair |
| $second\ x$ | Second element of an ordered pair |
| $\# A$ | Size of a finite set |

## Relations

| | |
|---|---|
| $A \leftrightarrow B$ | Relation ( $\mathbb{P}(A \times B)$ ) |
| $a \mapsto b$ | Maplet ( $(a, b)$ ) |
| $\mathrm{dom}\ R$ | Domain of a relation |
| $\mathrm{ran}\ R$ | Range of a relation |
| $\mathrm{id}\ A$ | Identity relation |
| $Q \mathbin{;} R$ | Forward relational composition |
| $Q \circ R$ | Backward relational composition ($R \mathbin{;} Q$) |
| $A \lhd R$ | Domain restriction |
| $A \unlhd R$ | Domain anti-restriction |
| $R \rhd A$ | Range restriction |
| $R \unrhd A$ | Range anti-restriction |
| $R(\!\vert A \vert\!)$ | Relational image |
| $iter\ n\ R$ | Relation composed $n$ times |
| $R^n$ | Same as $iter\ n\ R$ |
| $R^\sim$ | Inverse of relation ($R^{-1}$) |
| $R^*$ | Reflexive-transitive closure |
| $R^+$ | Irreflexive-transitive closure |
| $Q \oplus R$ | Relational overriding ( $(\mathrm{dom}\ R \unlhd Q) \cup R$ ) |
| $a\ \underline{R}\ b$ | Infix relation |

## Functions

| | |
|---|---|
| $A \pfun B$ | Partial functions |
| $A \longrightarrow B$ | Total functions |
| $A \pinj B$ | Partial injections |
| $A \rightarrowtail B$ | Total injections |
| $A \psurj B$ | Partial surjections |
| $A \twoheadrightarrow B$ | Total surjections |
| $A \bij B$ | Bijective functions |
| $A \ffun B$ | Finite partial functions |
| $A \finj B$ | Finite partial injections |
| $f\ x$ | Function application (or $f(x)$ ) |

## Numbers

| | |
|---|---|
| $\mathbb{Z}$ | Set of integers |
| $\mathbb{N}$ | Set of natural numbers $\{0, 1, 2, ...\}$ |
| $\mathbb{N}_1$ | Set of non-zero natural numbers ($\mathbb{N} \setminus \{0\}$) |
| $m + n$ | Addition |
| $m - n$ | Subtraction |
| $m * n$ | Multiplication |
| $m \mathsf{~div~} n$ | Division |
| $m \mathsf{~mod~} n$ | Modulo arithmetic |
| $m \leq n$ | Less than or equal |
| $m < n$ | Less than |
| $m \geq n$ | Greater than or equal |
| $m > n$ | Greater than |
| $succ\ n$ | Successor function $\{0 \mapsto 1, 1 \mapsto 2, ...\}$ |
| $m \mathinner{\ldotp\ldotp} n$ | Number range |
| $min\ A$ | Minimum of a set of numbers |
| $max\ A$ | Maximum of a set of numbers |

## Sequences

| | |
|---|---|
| $\mathrm{seq}\ A$ | Set of finite sequences |
| $\mathrm{seq}_1\ A$ | Set of non-empty finite sequences |
| $\mathrm{iseq}\ A$ | Set of finite injective sequences |
| $\langle\rangle$ | Empty sequence |
| $\langle x, y, ...\rangle$ | Sequence $\{1 \mapsto x, 2 \mapsto y, ...\}$ |
| $s \frown t$ | Sequence concatenation |
| $\frown / s$ | Distributed sequence concatenation |
| $head\ s$ | First element of sequence ( $s(1)$ ) |
| $tail\ s$ | All but the head element of a sequence |
| $last\ s$ | Last element of sequence ( $s(\#s)$ ) |
| $front\ s$ | All but the last element of a sequence |
| $rev\ s$ | Reverse a sequence |
| $squash\ f$ | Compact a function to a sequence |
| $A \upharpoonleft s$ | Sequence extraction ( $squash(A \lhd s)$ ) |
| $s \upharpoonright A$ | Sequence filtering ( $squash(s \rhd A)$ ) |
| $s \mathsf{~prefix~} t$ | Sequence prefix relation ($s \frown v = t$) |
| $s \mathsf{~suffix~} t$ | Sequence suffix relation ($u \frown s = t$) |
| $s \mathsf{~in~} t$ | Sequence segment relation ($u \frown s \frown v = t$) |
| $\mathsf{disjoint}\ A$ | Disjointness of an indexed family of sets |
| $A \mathsf{~partition~} B$ | Partition an indexed family of sets |

## Bags

| | |
|---|---|
| $\mathrm{bag}\ A$ | Set of bags or multisets ($A \nrightarrow \mathbb{N}_1$) |
| $[\![\,]\!]$ | Empty bag |
| $[\![x, y, ...]\!]$ | Bag $\{x \mapsto 1, y \mapsto 1, ...\}$ |
| $count\ C\ x$ | Multiplicity of an element in a bag |
| $C \sharp x$ | Same as $count\ C\ x$ |
| $n \otimes C$ | Bag scaling of multiplicity |
| $x \sqsubset C$ | Bag membership |
| $C \sqsubseteq D$ | Sub-bag relation |
| $C \uplus D$ | Bag union |
| $C \uminus D$ | Bag difference |
| $items\ s$ | Bag of elements in a sequence |

## Schema notation

```
┌─ S ────────
│ d
├──────
│ p
└────────────
```

**Vertical schema.** New lines denote ' ; ' and '$\wedge$'. The schema name and predicate part are optional. The schema may subsequently be referenced by name in the document.

```
│ d
├──────
│ p
```

**Axiomatic definition.** The definitions may be non-unique. The predicate part is optional. The definitions apply globally in the document.

```
┌═[a, ...]══
│ d
├──────
│ p
└────────────
```

**Generic definition.** The generic parameters are optional. The definitions must be unique. The definitions apply globally in the document.

| | |
|---|---|
| $S \mathrel{\widehat=} [X]$ | Horizontal schema |
| $[T; ... \mid ...]$ | Schema inclusion |
| $z.a$ | Component selection (given $z : S$) |
| $\theta S$ | Tuple of components |
| $\neg\ S$ | Schema negation |
| $\mathrm{pre}\ S$ | Schema precondition |
| $S \wedge T$ | Schema conjunction |
| $S \vee T$ | Schema disjunction |
| $S \Rightarrow T$ | Schema implication |
| $S \Leftrightarrow T$ | Schema equivalence |
| $S \setminus (a, ...)$ | Hiding of component(s) |
| $S \upharpoonright T$ | Projection of components |
| $S \mathbin{;} T$ | Schema composition ($S$ then $T$) |
| $S \gg T$ | Schema piping ($S$ outputs to $T$ inputs) |
| $S[a/b, ...]$ | Schema component renaming ($b$ becomes $a$, etc.) |
| $\forall X \bullet S$ | Schema universal quantification |
| $\exists X \bullet S$ | Schema existential quantification |
| $\exists_1 X \bullet S$ | Schema unique existential quantification |

## Conventions

| | |
|---|---|
| $a?$ | Input to an operation |
| $a!$ | Output from an operation |
| $a$ | State component before an operation |
| $a'$ | State component after an operation |
| $S$ | State schema before an operation |
| $S'$ | State schema after an operation |
| $\Delta S$ | Change of state (normally $S \wedge S'$ ) |
| $\Xi S$ | No change of state (normally $[S \wedge S' \mid \theta S = \theta S']$ ) |

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