Glossary of Z notation

Names

- \(a, b\) identifiers
- \(d, e\) declarations (e.g., \(a : A; b : \ldots : B\))
- \(f, g\) functions
- \(m, n\) numbers
- \(p, q\) predicates
- \(s, t\) sequences
- \(x, y\) expressions
- \(A, B\) sets
- \(C, D\) bags
- \(Q, R\) relations
- \(S, T\) schemas
- \(X\) schema text (e.g., \(d, d\mid p\) or \(S\))

Definitions

- \(a \equiv x\) Abbreviation definition
- \(a := b\mid\ldots\) Free type definition (or \(a := b\langle x\rangle\mid\ldots\))
- \([a]\) Introduction of a given set (or \([a_1, \ldots]\))
- \(\underline{a}\) Prefix operator
- \(-\underline{a}\) Postfix operator
- \(-\cdot\) Infix operator

Logic

- \(\text{true}\) Logical true constant
- \(\text{false}\) Logical false constant
- \(\neg p\) Logical negation
- \(p \land q\) Logical conjunction
- \(p \lor q\) Logical disjunction
- \(p \Rightarrow q\) Logical implication (\(\neg p \lor q\))
- \(p \Leftrightarrow q\) Logical equivalence (\(p \Rightarrow q \land q \Rightarrow p\))
- \(\forall X \bullet q\) Universal quantification
- \(\exists X \bullet q\) Existential quantification
- \(\exists_1 X \bullet q\) Unique existential quantification

let \(a \equiv x; \ldots; p\) Local definition

Sets and expressions

- \(x = y\) Equality of expressions
- \(x \neq y\) Inequality (\(\neg (x = y)\))
- \(x \in A\) Set membership
- \(x \notin A\) Non-membership (\(\neg (x \in A)\))
- \(\emptyset\) Empty set
- \(A \subseteq B\) Set inclusion
- \(A \subset B\) Strict set inclusion (\(A \subseteq B \land A \neq B\))
- \(\{x, y, \ldots\}\) Set of elements
- \(\{X \bullet x\}\) Set comprehension
- \(\lambda X \bullet x\) Lambda-expression – function
- \(\mu X \bullet x\) Mu-expression – unique value

let \(a \equiv x; \ldots; b\) Local definition

if \(p\) then \(x\) else \(y\) Conditional expression

\((x, y, \ldots)\) Ordered tuple

\(A \times B \times \ldots\) Cartesian product

\(P A\) Power set (set of subsets)

\(P_1 A\) Non-empty power set

\(F A\) Set of finite subsets

\(F_1 A\) Non-empty set of finite subsets

\(A \cap B\) Set intersection

\(A \cup B\) Set union

\(A \setminus B\) Set difference

\(\bigcup A\) Generalized union of a set of sets

\(\bigcap A\) Generalized intersection of a set of sets

\(\text{first } x\) First element of an ordered pair

\(\text{second } x\) Second element of an ordered pair

\(# A\) Size of a finite set

Relations

- \(A \leftrightarrow B\) Relation (\(P(A \times B)\))
- \(a \mapsto b\) Maplet (\(\langle a, b\rangle\))
- \(\text{dom } R\) Domain of a relation
- \(\text{ran } R\) Range of a relation
- \(\text{id } A\) Identity relation
- \(Q \circ R\) Forward relational composition
- \(Q \circ R\) Backward relational composition (\(R \circ Q\))
- \(A \restriction R\) Domain restriction
- \(A \restriction R\) Domain anti-restriction
- \(R \triangleright A\) Range restriction
- \(R \triangleright A\) Range anti-restriction
- \(R\langle A\rangle\) Relational image
- \(\text{iter } n\ R\) Relation composed \(n\) times
- \(R^n\) Same as \(\text{iter } n\ R\)
- \(R^\sim\) Inverse of relation (\(R^{-1}\))
- \(R^*\) Reflexive-transitive closure
- \(R^+\) Irreflexive-transitive closure
- \(Q \oplus R\) Relational overriding (\(\langle \text{dom } R \subseteq Q \cup R\rangle\))

\(a \mathbin{R} b\) Infix relation

Functions

- \(A \rightarrow B\) Partial functions
- \(A \rightarrow B\) Total functions
- \(A \twoheadrightarrow B\) Partial injections
- \(A \twoheadrightarrow B\) Total injections
- \(A \rightarrow B\) Partial surjections
- \(A \rightarrow B\) Total surjections
- \(A \mapsto B\) Bijective functions
- \(A \mapsto B\) Finite partial functions
- \(A \mapsto B\) Finite partial injections
- \(f\ x\) Function application (or \(f(x)\))
Numbers

\[ \mathbb{Z} \] Set of integers
\[ \mathbb{N} \] Set of natural numbers \{0, 1, 2, \ldots\}
\[ \mathbb{N}_1 \] Set of non-zero natural numbers \(\mathbb{N} \setminus \{0\}\)
\[ m + n \] Addition
\[ m - n \] Subtraction
\[ m \times n \] Multiplication
\[ m \div n \] Division
\[ m \mod n \] Modulo arithmetic
\[ m \leq n \] Less than or equal
\[ m < n \] Less than
\[ m \geq n \] Greater than or equal
\[ m > n \] Greater than
\[ \text{successor } n \] Successor function \(\{0 \mapsto 1, 1 \mapsto 2, \ldots\}\)
\[ m \ldots n \] Number range
\[ \operatorname{min} A \] Minimum of a set of numbers
\[ \operatorname{max} A \] Maximum of a set of numbers

Sequences

\[ \text{seq} A \] Set of finite sequences
\[ \text{seq}_A \] Set of non-empty finite sequences
\[ \text{iseq} A \] Set of finite injective sequences
\[ \emptyset \] Empty sequence
\[ \langle x, y, \ldots \rangle \] Sequence \(\{1 \mapsto x, 2 \mapsto y, \ldots\}\)
\[ \setminus t \] Sequence concatenation
\[ \setminus{s} \] Distributed sequence concatenation
\[ \text{head } s \] First element of sequence \(s(1)\)
\[ \text{tail } s \] All but the head element of a sequence
\[ \text{last } s \] Last element of sequence \(s(#s)\)
\[ \text{front } s \] All but the last element of a sequence
\[ \text{rev } s \] Reverse a sequence
\[ \text{squash } f \] Compact a function to a sequence
\[ A \mid s \] Sequence extraction \((\text{squash}(A \setminus s))\)
\[ s \mid A \] Sequence filtering \((\text{squash}(s \triangleright A))\)
\[ s\text{ prefix } t \] Sequence prefix relation \((s \triangleright v = t)\)
\[ s\text{ suffix } t \] Sequence suffix relation \((u \triangleright s = t)\)
\[ s \text{ in } t \] Sequence segment relation \((u \triangleright s \triangleright v = t)\)
\[ \text{disjoint } A \] Disjointness of an indexed family of sets
\[ A \text{ partition } B \] Partition an indexed family of sets

Bags

\[ \text{bag } A \] Set of bags or multisets \((A \mapsto \mathbb{N}_1)\)
\[ \emptyset \] Empty bag
\[ [x, y, \ldots ] \] Bag \(\{x \mapsto 1, y \mapsto 1, \ldots\}\)
\[ \text{count } C x \] Multiplicity of an element in a bag
\[ C \# x \] Same as \(\text{count } C x\)
\[ n \otimes C \] Bag scaling of multiplicity
\[ x \in C \] Bag membership
\[ C \sqsubseteq D \] Sub-bag relation
\[ C \sqsupset D \] Bag union
\[ C \sqsubseteq D \] Bag difference
\[ \text{items } s \] Bag of elements in a sequence

Schema notation

**Vertical schema.**
\[
\begin{array}{c}
\vdots \\
S \\
\vdots \\
d \\
\vdots \\
p \\
\vdots
\end{array}
\]
New lines denote ‘;’ and ‘\&’. The schema name and predicate part are optional. The schema may subsequently be referenced by name in the document.

**Axiomatic definition.**
\[
\begin{array}{c}
\vdots \\
S \\
\vdots \\
d \\
\vdots \\
p \\
\vdots
\end{array}
\]
The definitions may be non-unique. The predicate part is optional. The definitions apply globally in the document.

**Generic definition.**
\[
\begin{array}{c}
\vdots \\
S \\
\vdots \\
d \\
\vdots \\
p \\
\vdots
\end{array}
\]
The generic parameters are optional. The definitions must be unique. The definitions apply globally in the document.

\[ S \triangleq [X] \] Horizontal schema
\[ [T; \ldots ] \] Schema inclusion
\[ z.a \] Component selection (given \(z : S\))
\[ \theta S \] Tuple of components
\[ \neg S \] Schema negation
\[ \pre S \] Schema precondition
\[ S \land T \] Schema conjunction
\[ S \lor T \] Schema disjunction
\[ S \Rightarrow T \] Schema implication
\[ S \Leftrightarrow T \] Schema equivalence
\[ S \backslash (a, \ldots) \] Hiding of component(s)
\[ S \mid T \] Projection of components
\[ S \triangleright T \] Schema composition \((S \text{ then } T)\)
\[ S \triangleright\triangleright T \] Schema piping \((S \text{ outputs to } T \text{ inputs})\)
\[ S[a/b, \ldots] \] Schema component renaming \((b \text{ becomes } a, \text{ etc.})\)
\[ \forall X \bullet S \] Schema universal quantification
\[ \exists X \bullet S \] Schema existential quantification
\[ \exists_1 X \bullet S \] Schema unique existential quantification

Conventions

\[ \alpha? \] Input to an operation
\[ \alpha! \] Output from an operation
\[ \alpha \] State component before an operation
\[ \alpha' \] State component after an operation
\[ S \] State schema before an operation
\[ S' \] State schema after an operation
\[ \Delta S \] Change of state (normally \(S \land S'\))
\[ \Xi S \] No change of state (normally \([S \land S'] \theta S = \theta S'\))

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