

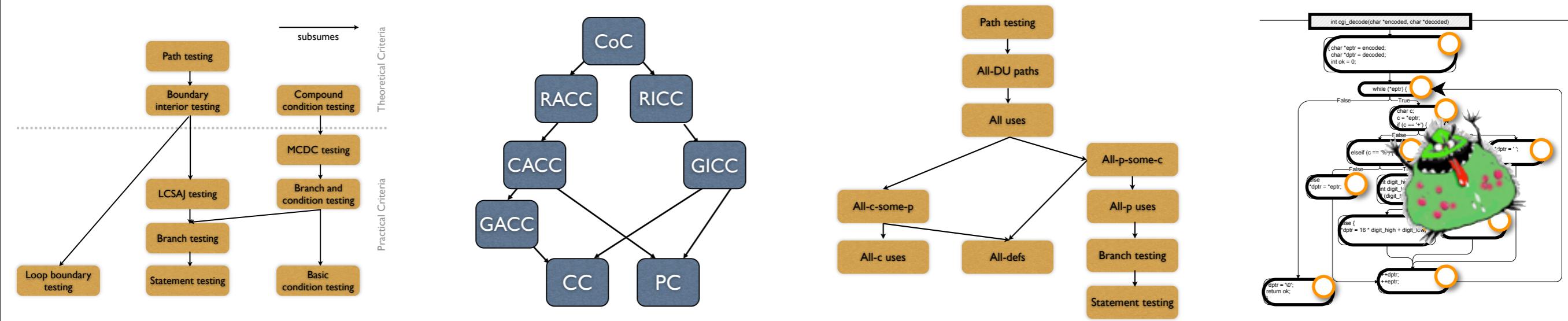
A photograph of a person from behind, wearing a cap and a dark jacket, looking through binoculars at a landscape of rolling hills under a clear blue sky.

# Search-based Testing

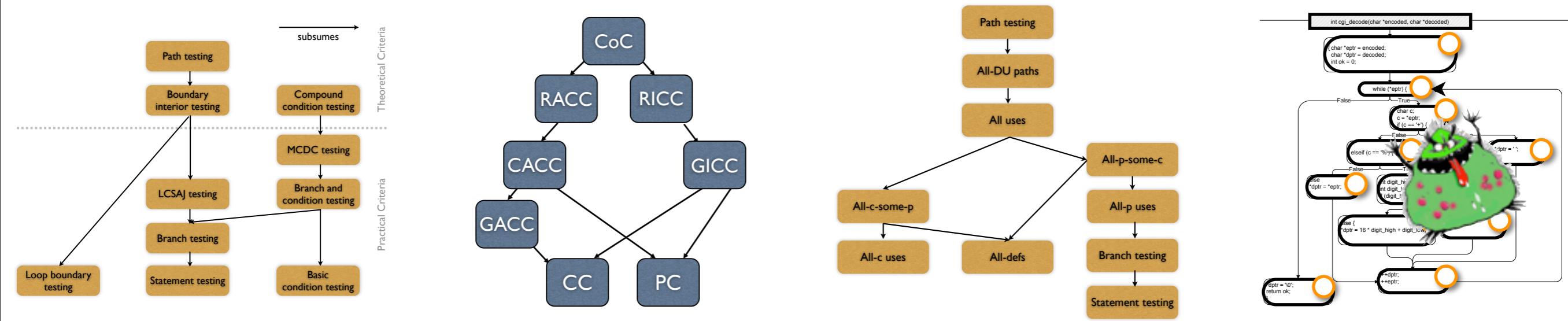
Software Engineering  
Gordon Fraser • Saarland University

# So many different test objectives...

# So many different test objectives...



# So many different test objectives...



But how to create the tests?

# Test Data Generation

Given a function and a location we want to reach, how do we derive inputs to the function that lead the control flow to the desired statement?



# Search-based Software Engineering

- Cast problems of software engineering as optimization problems
- Search problems in software engineering are **BIG**
- Apply meta-heuristic search algorithms to solve these problems

# Search Algorithms



# Random Testing

- (Totally) uninformed search
- If a systematic approach is not better than random testing, it is not useful
- Cheap & easy to implement
- Works pretty well in many cases

# Heuristics

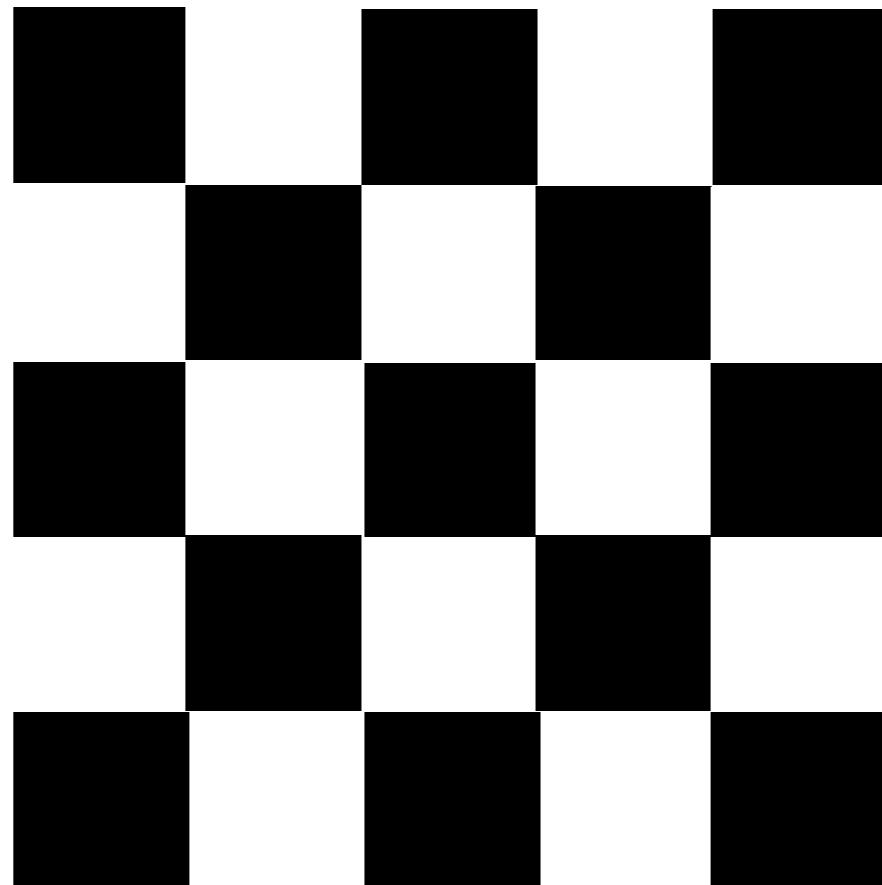
- Might not always find the best solution
- But finds a good solution in reasonable time
- Sacrifices completeness but increases efficiency
- Useful in solving tough problems
  - Which could not be solved any other
  - Which would take very long time to compute
- Use problem specific metrics

# Meta-Heuristics

- Problem-independent algorithms to solve optimization problems
- Problem-specific implementation of certain parts (as heuristic)

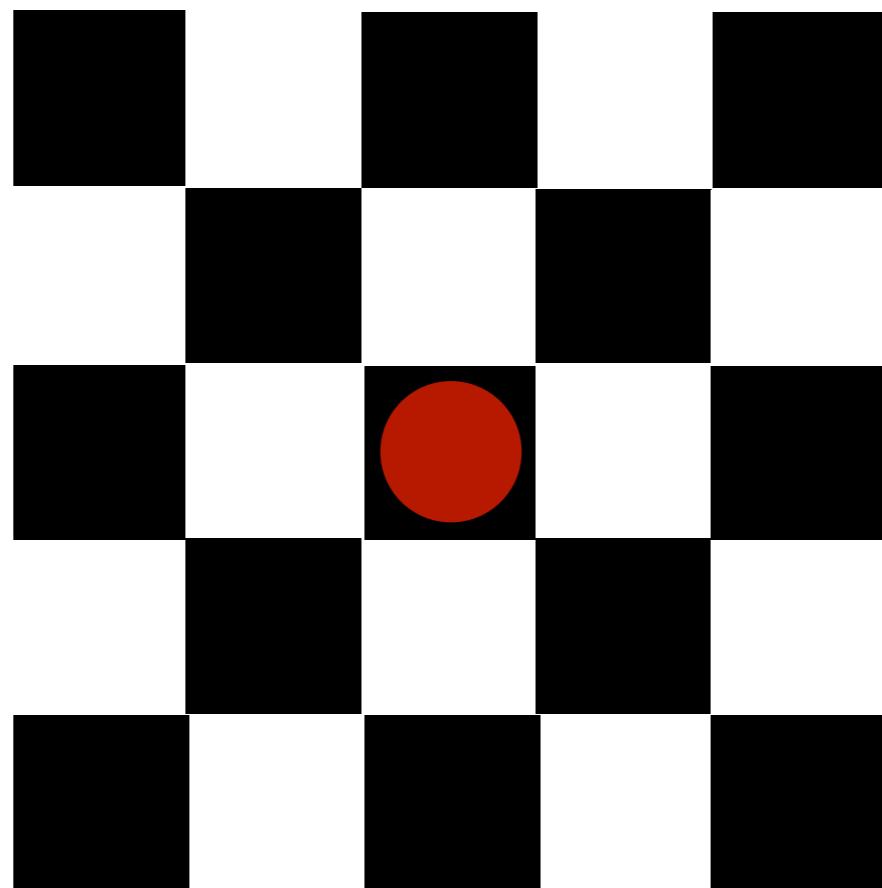
# Search Space

Search space every  
possible position (x,y)



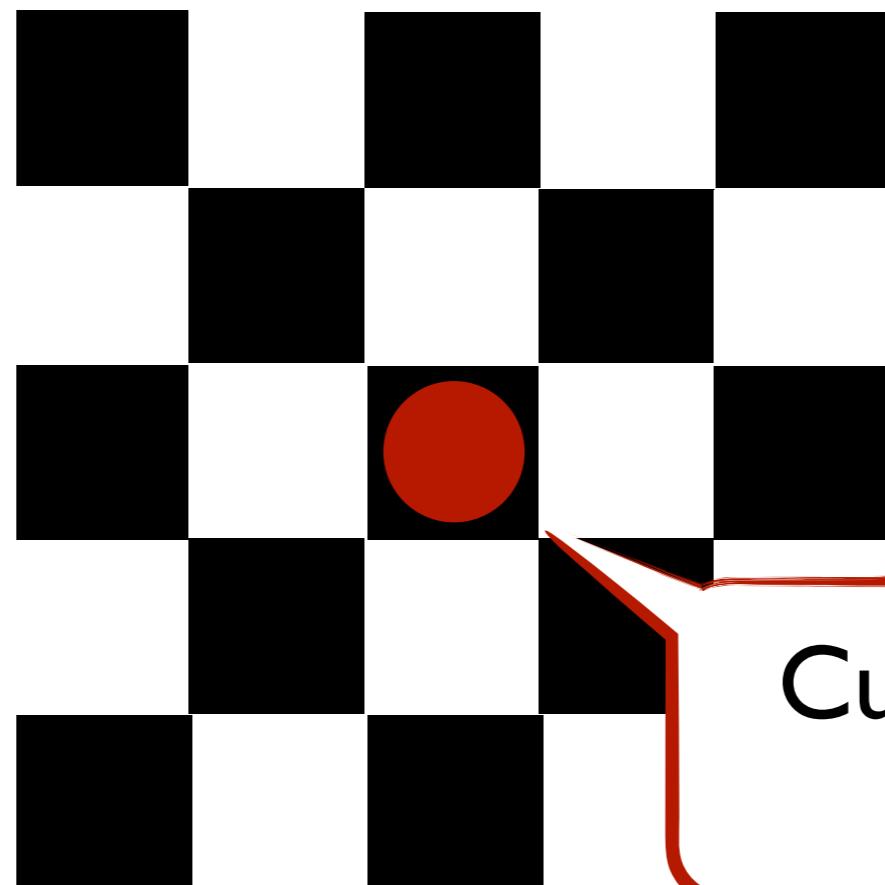
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# Search Space

Search space every  
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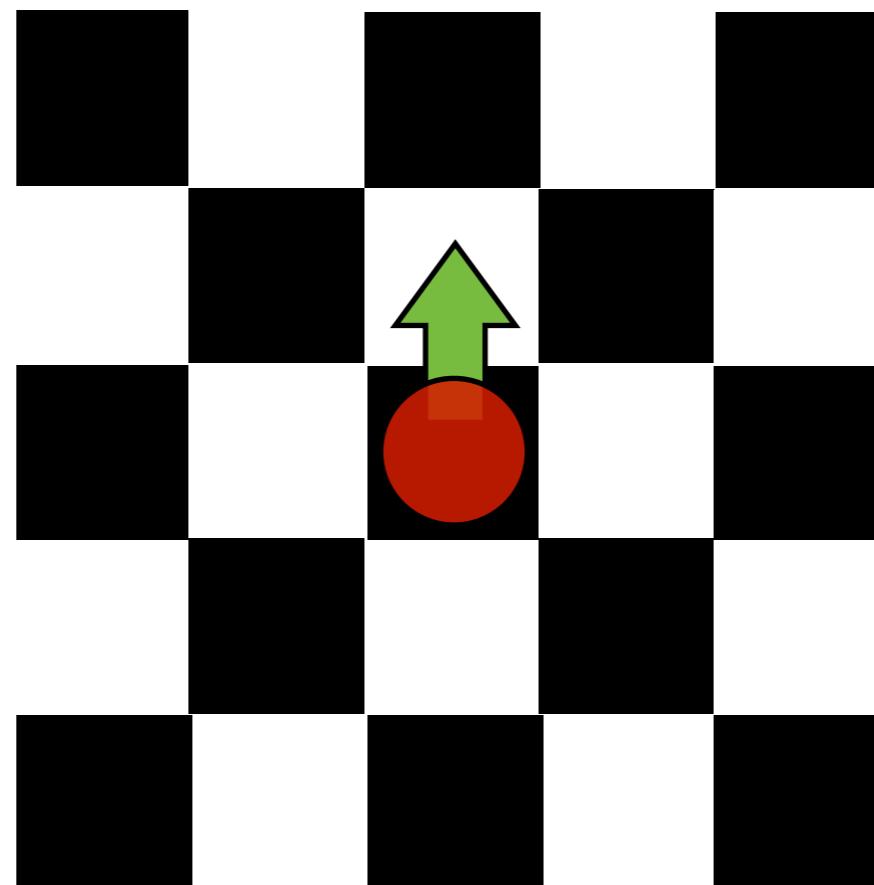


Current solution:  
(3,3)

# Search Space

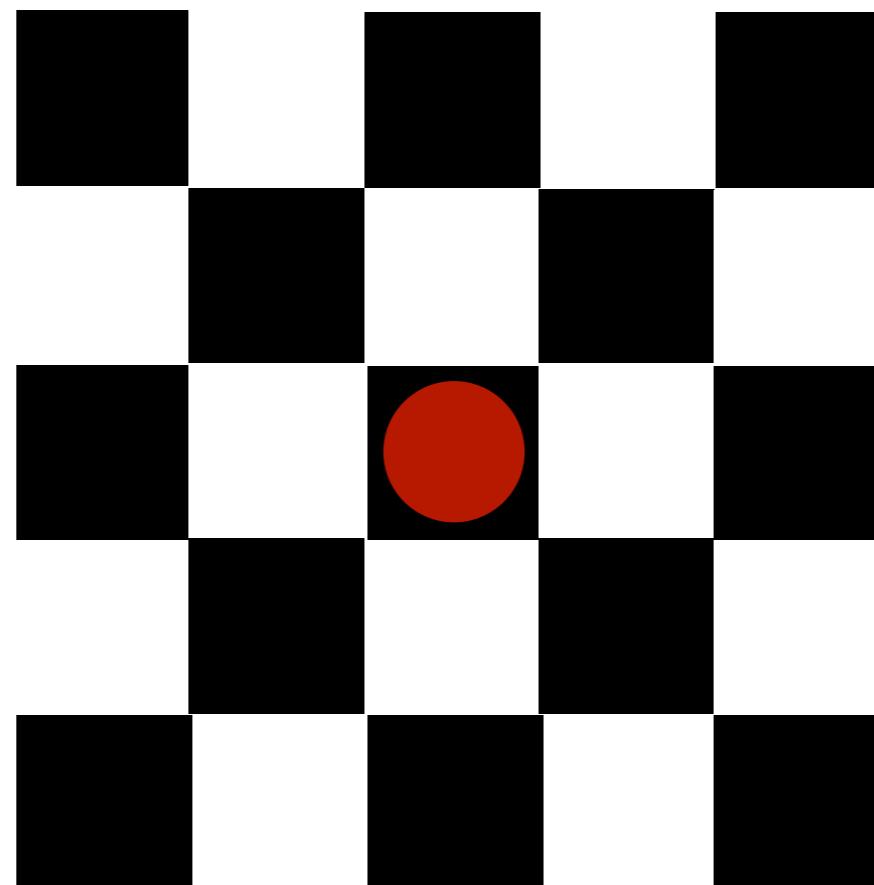
Possible move:

(3,3), (3,4)



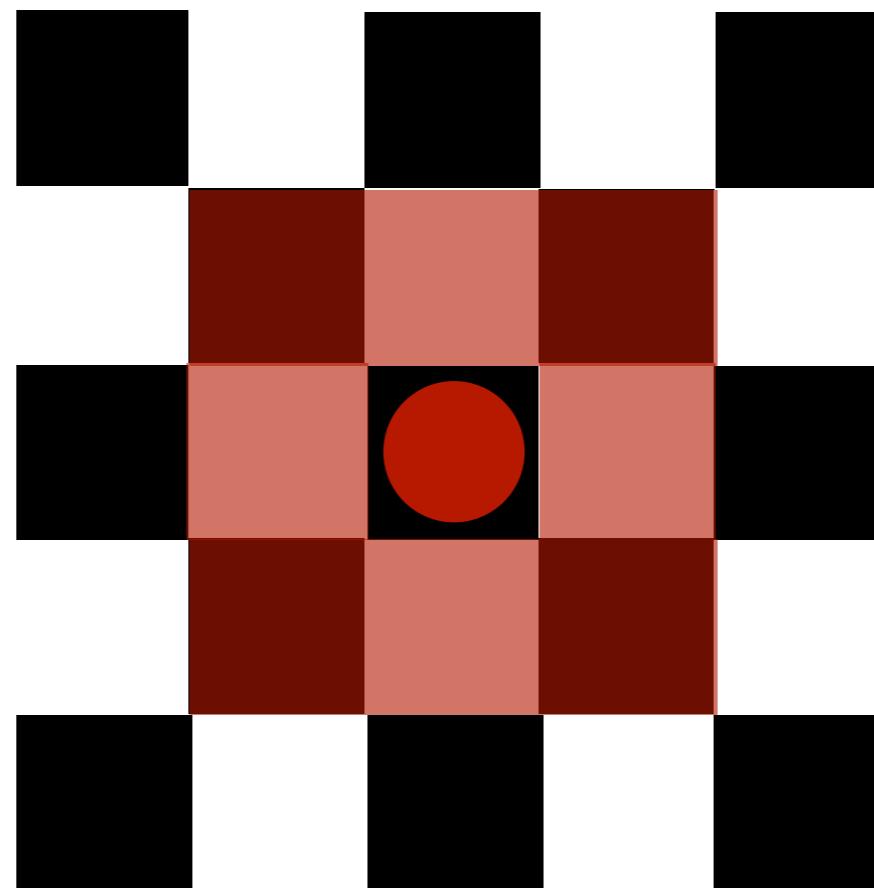
# Neighborhood

Any possible move

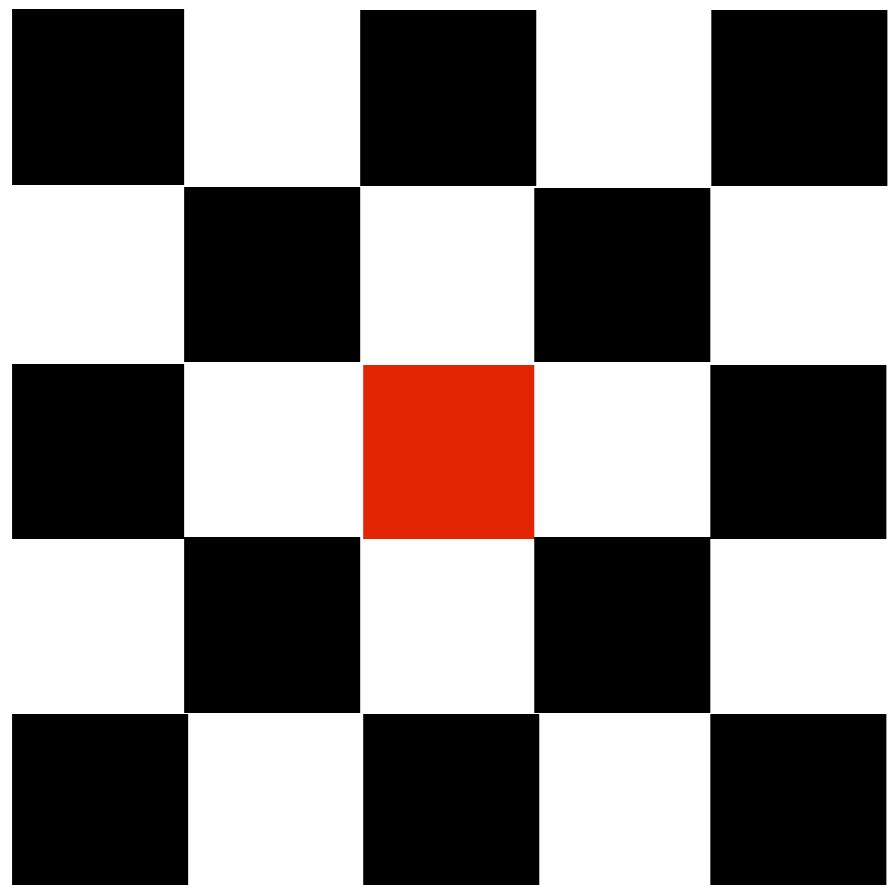


# Neighborhood

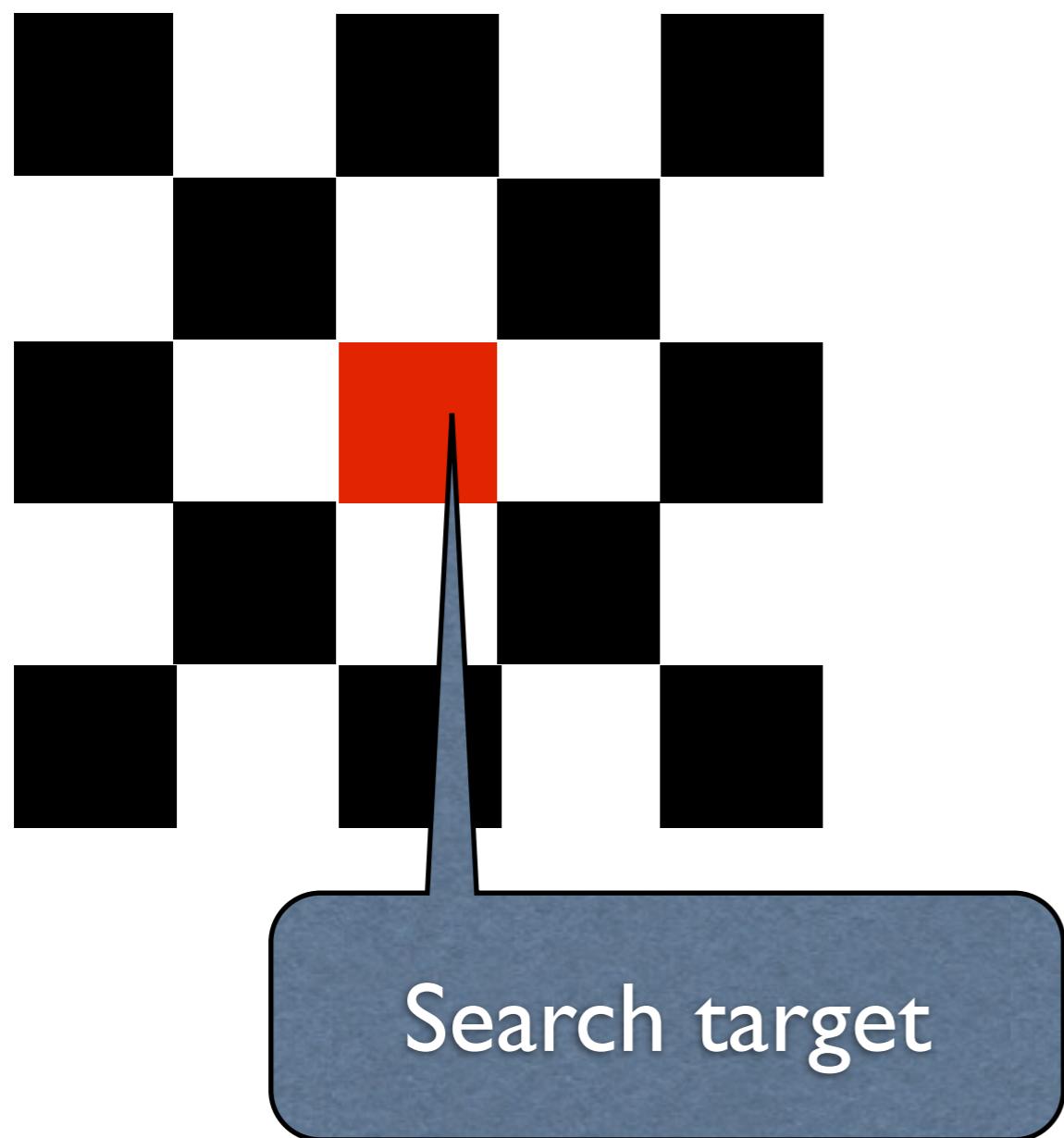
Any possible move



# Search Space

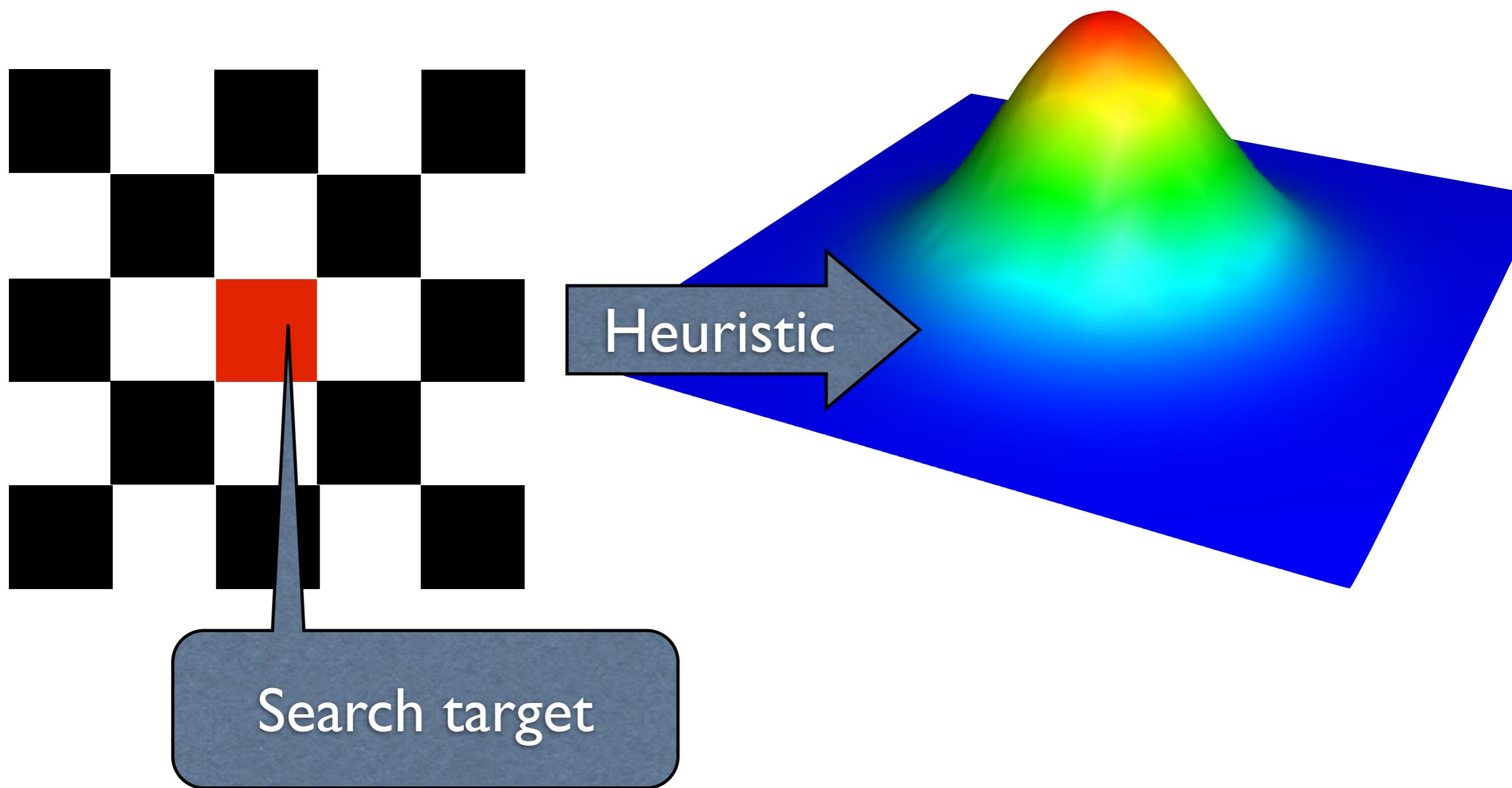


# Search Space



Search target

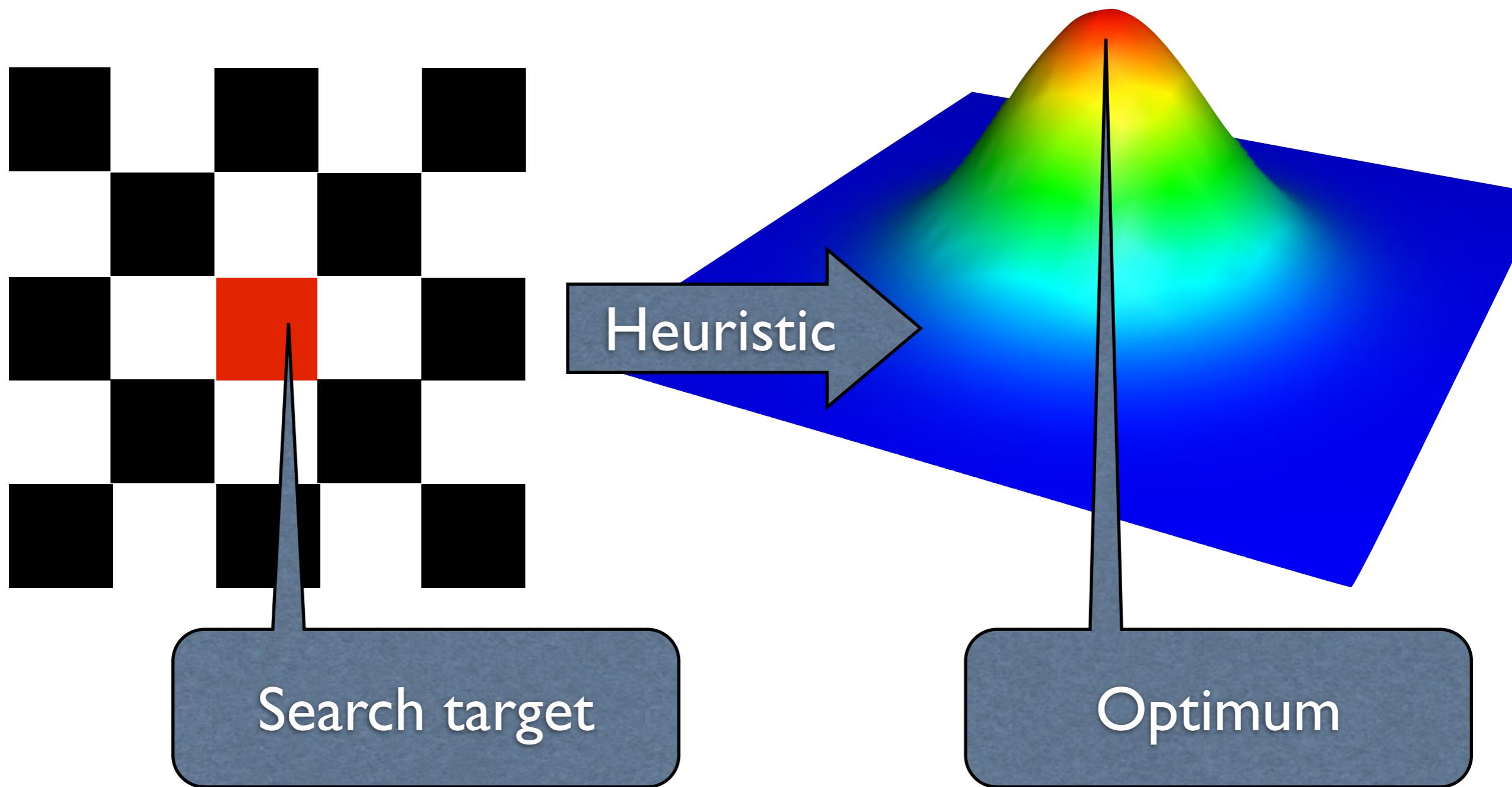
# Search Space



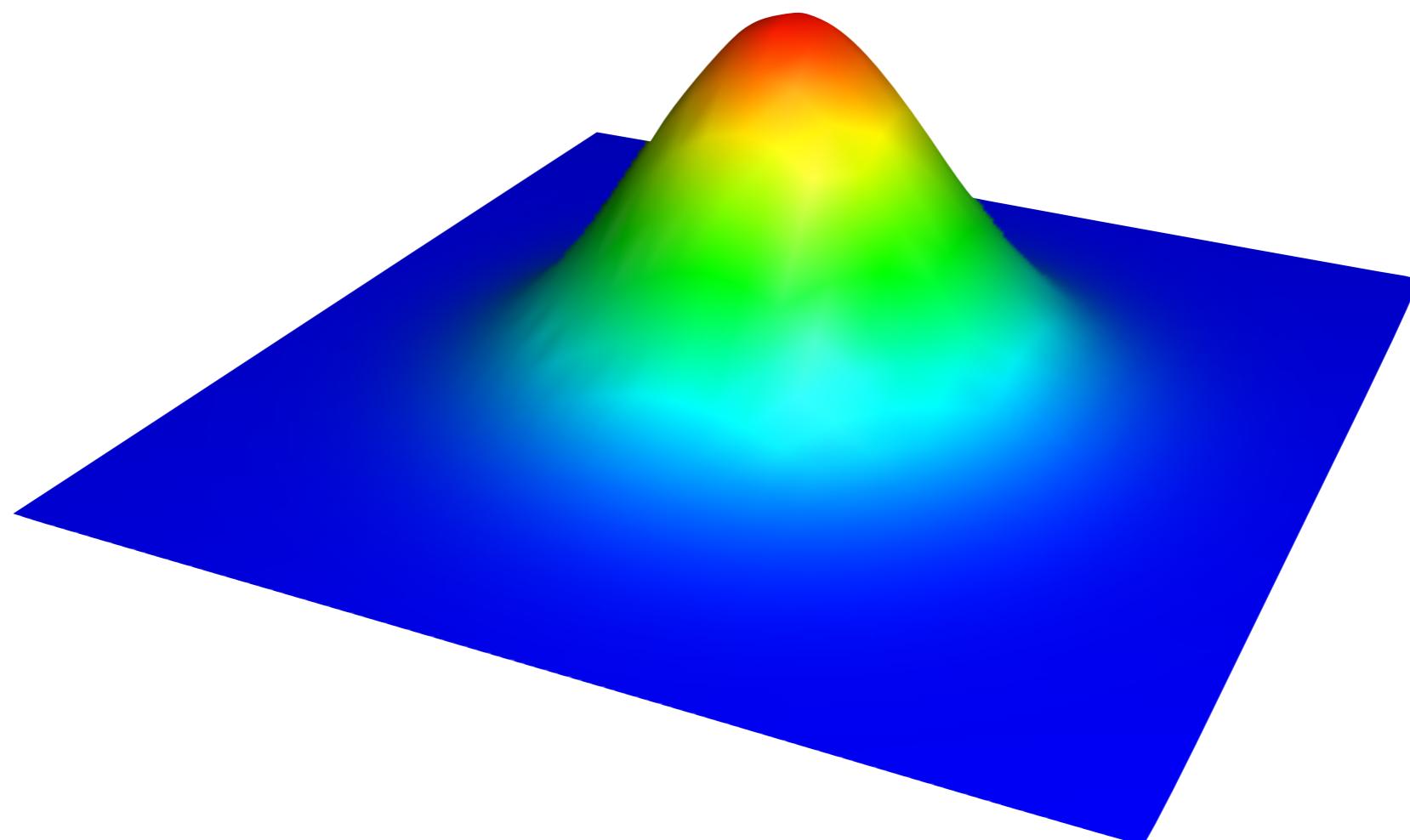
Search target

Heuristic

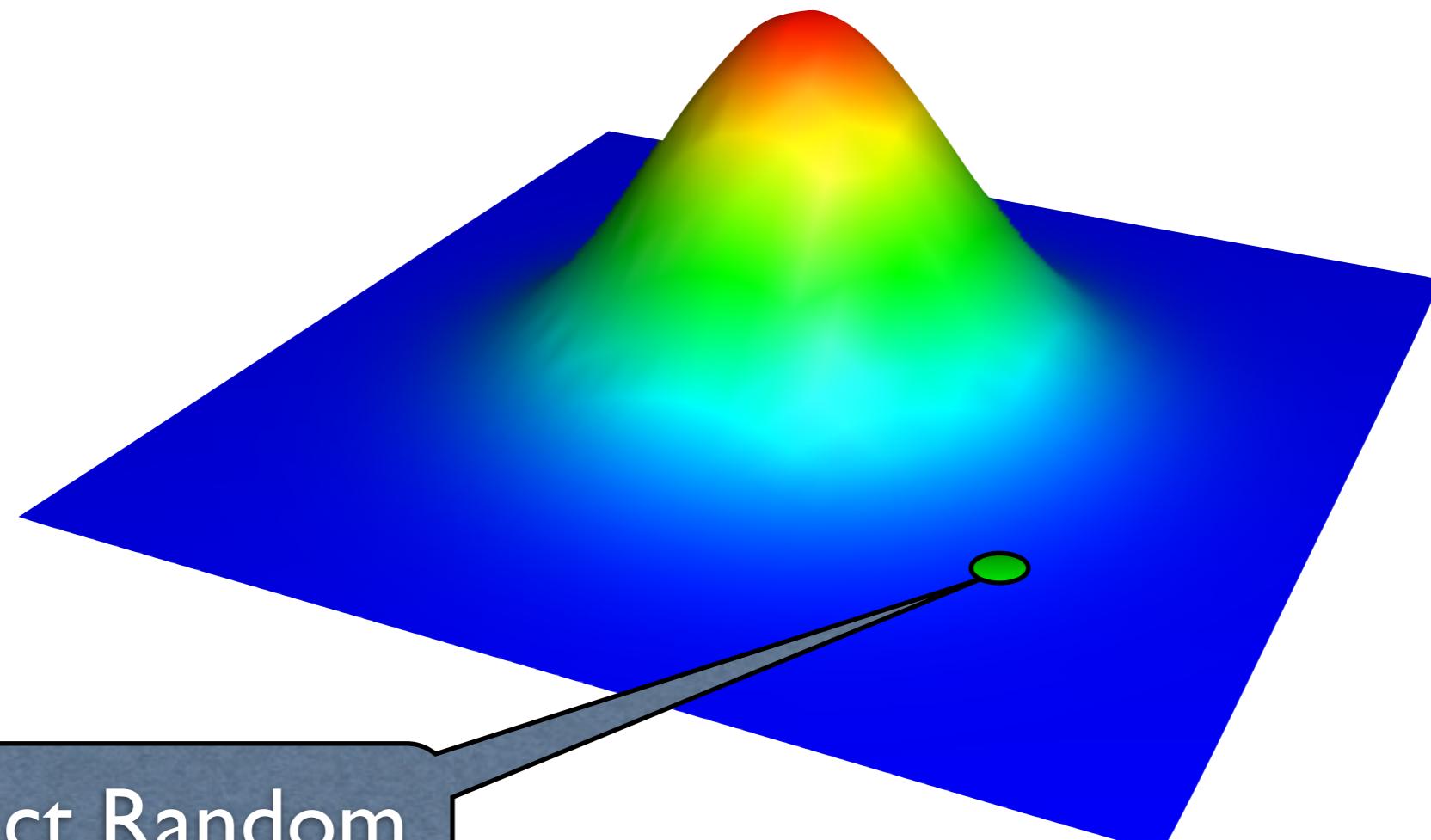
# Search Space



# Hill Climbing

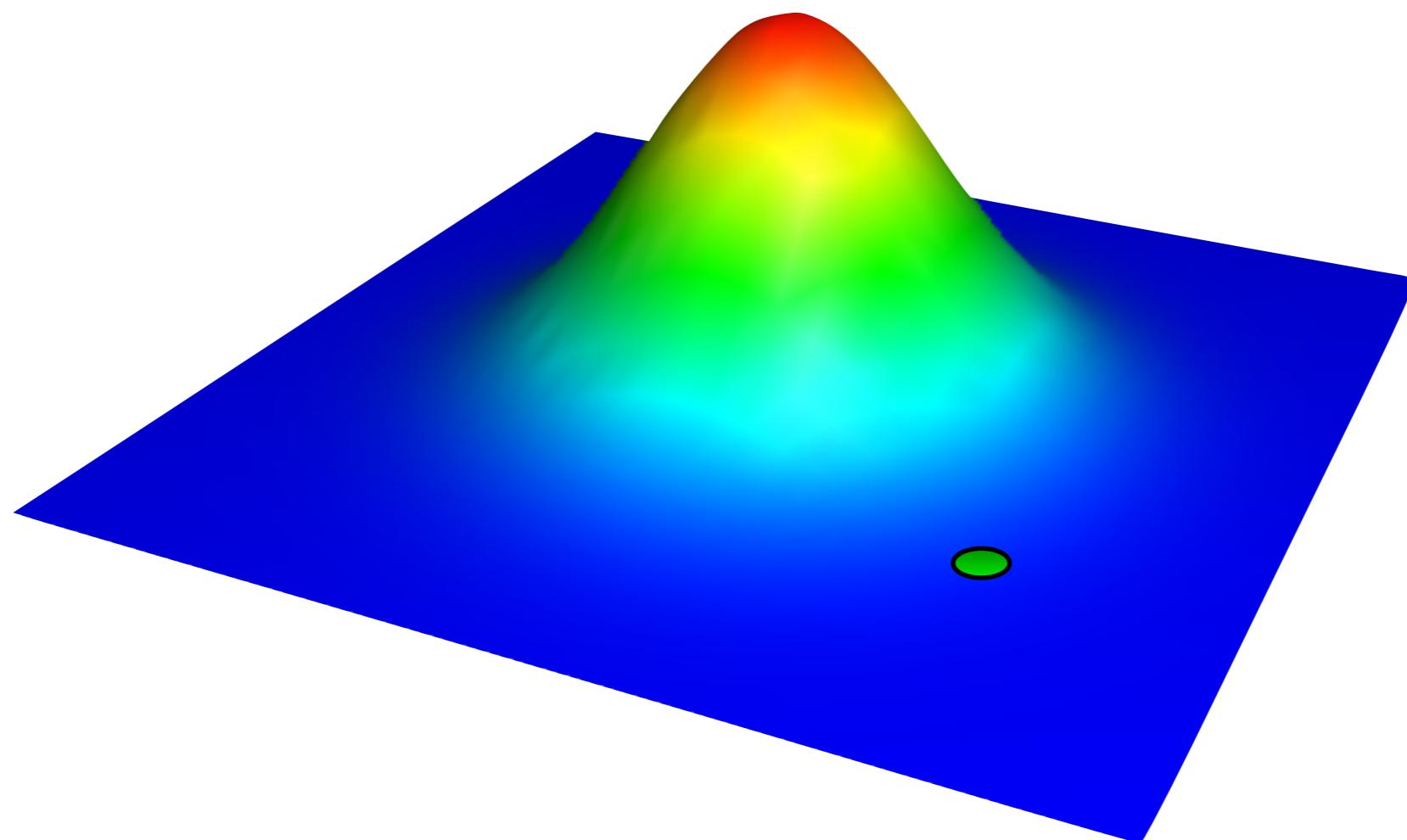


# Hill Climbing

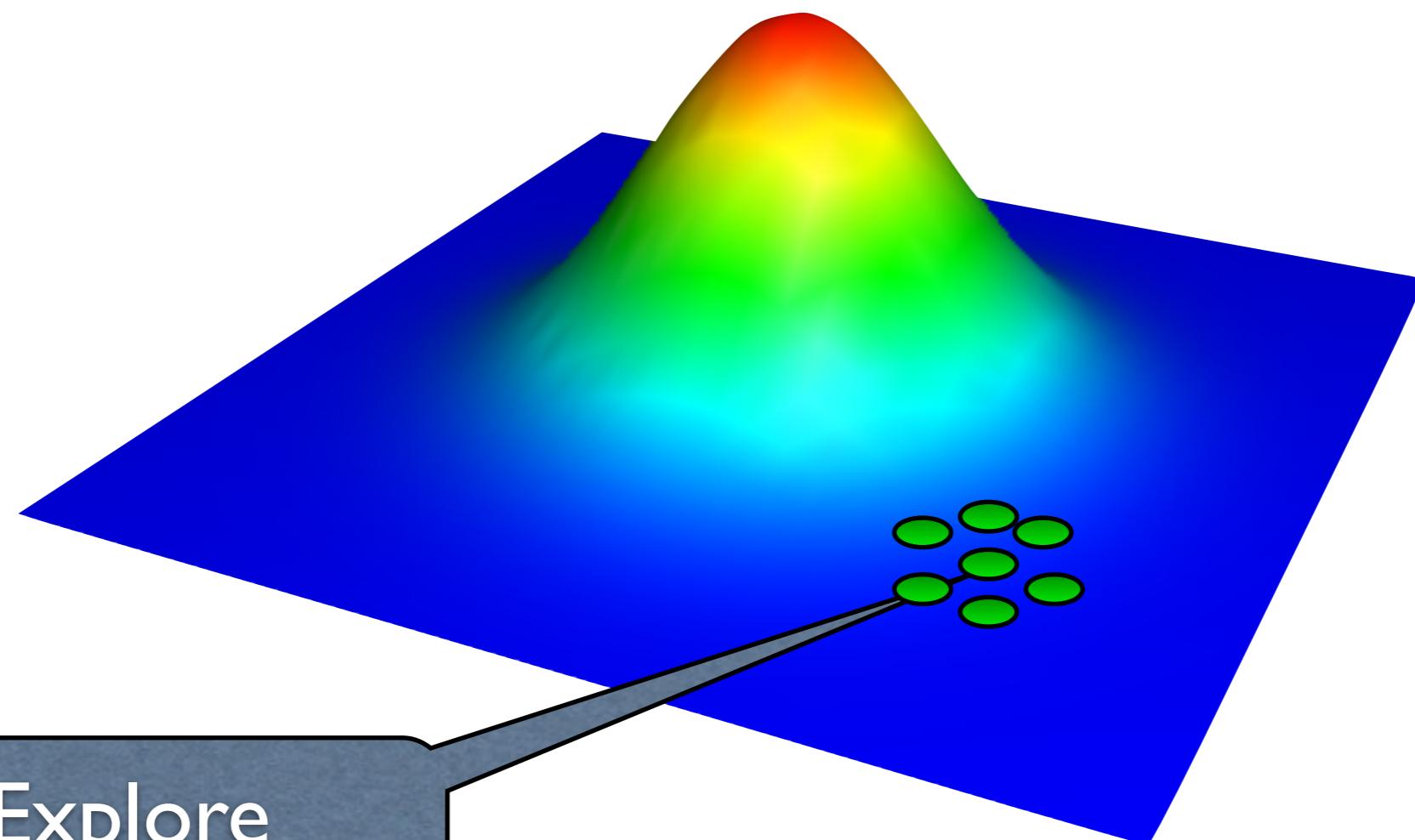


I. Select Random  
Value

# Hill Climbing

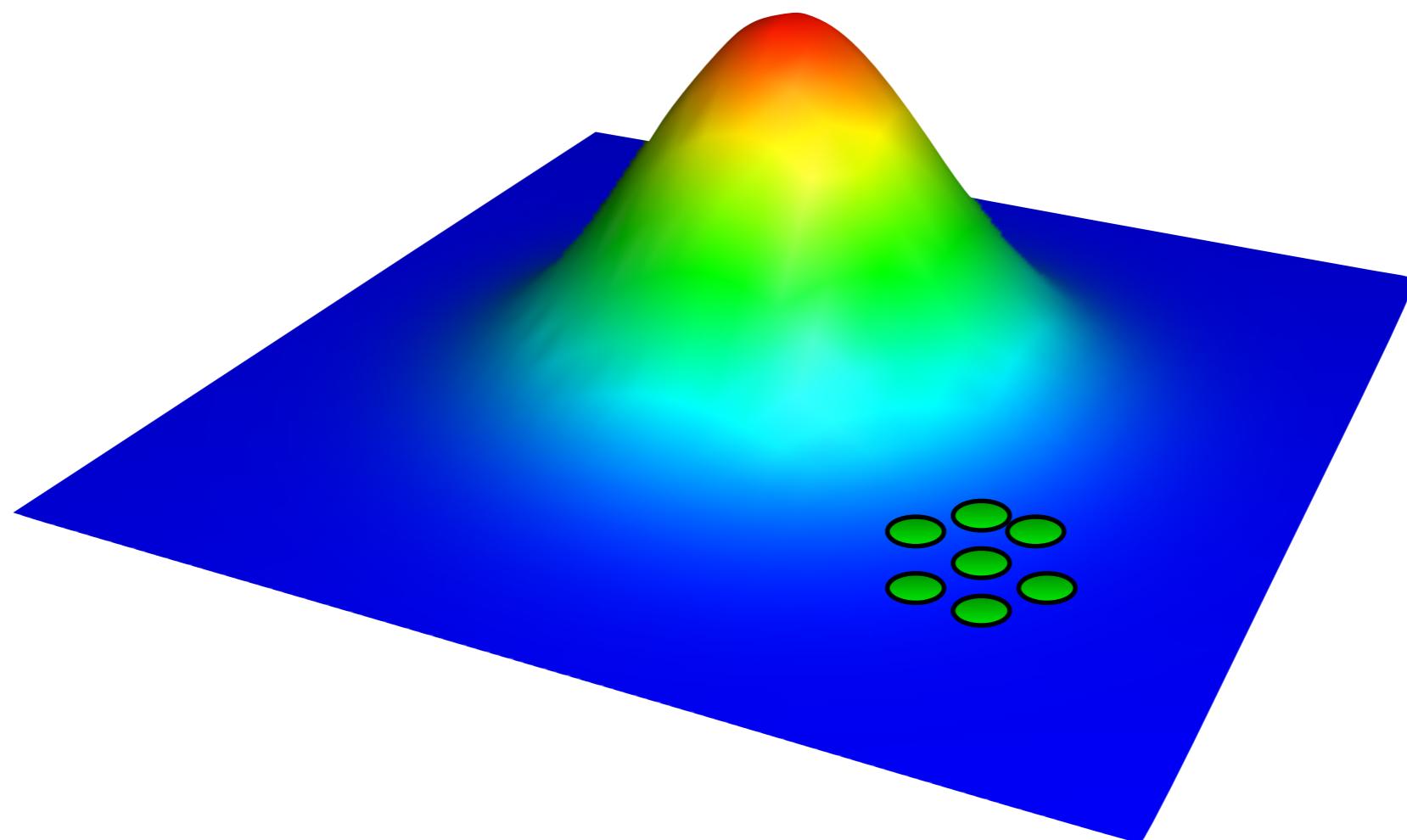


# Hill Climbing

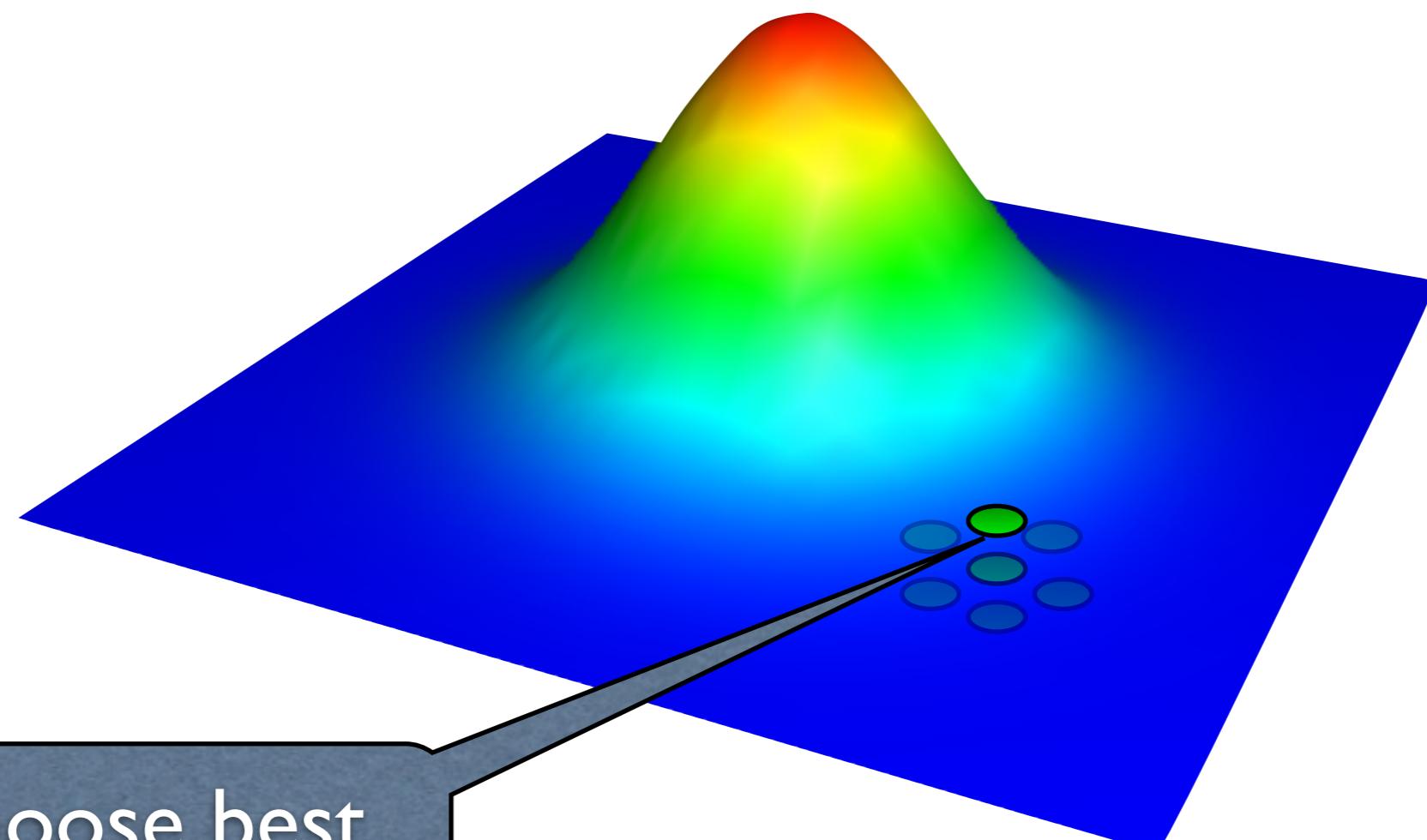


2. Explore  
Neighborhood

# Hill Climbing

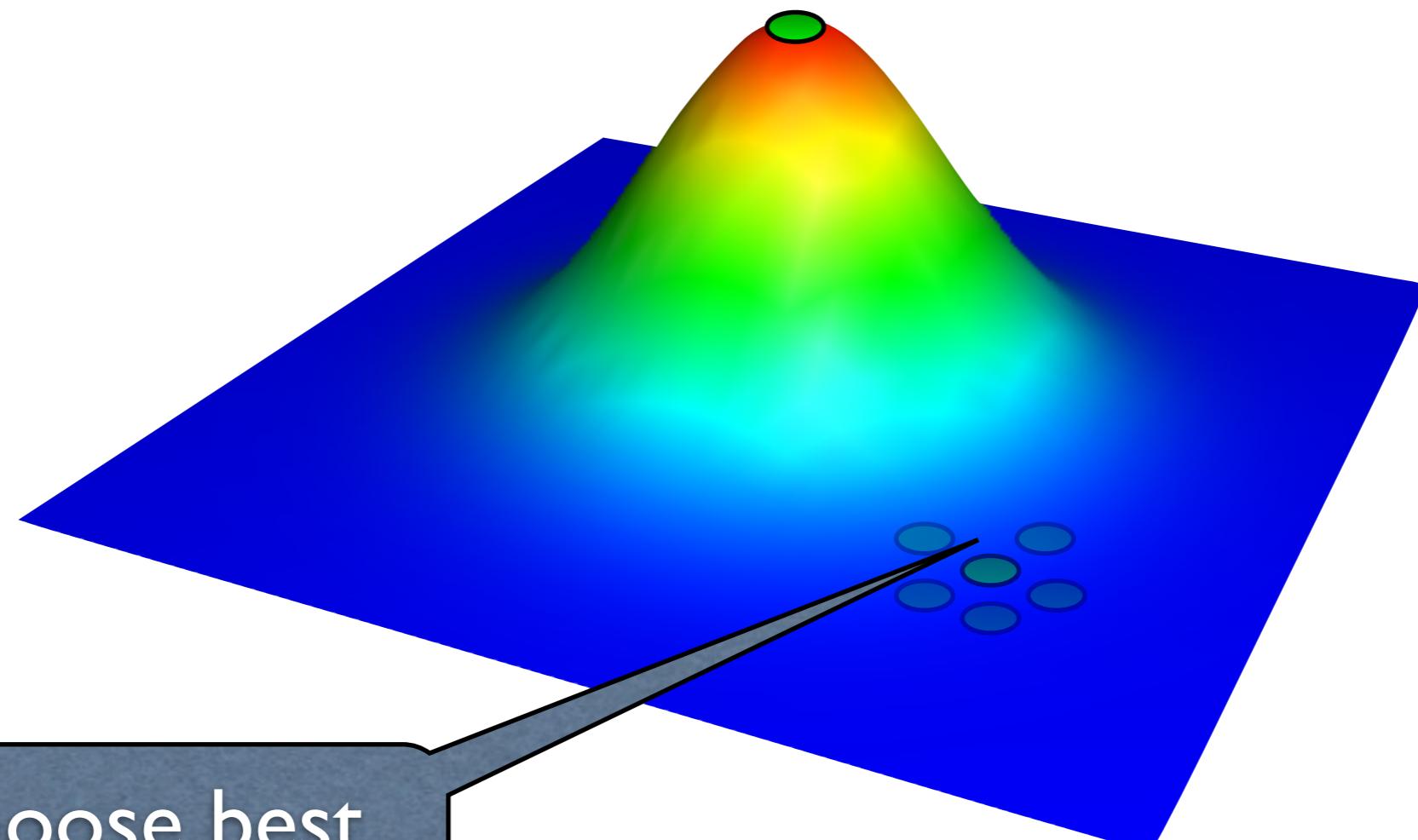


# Hill Climbing



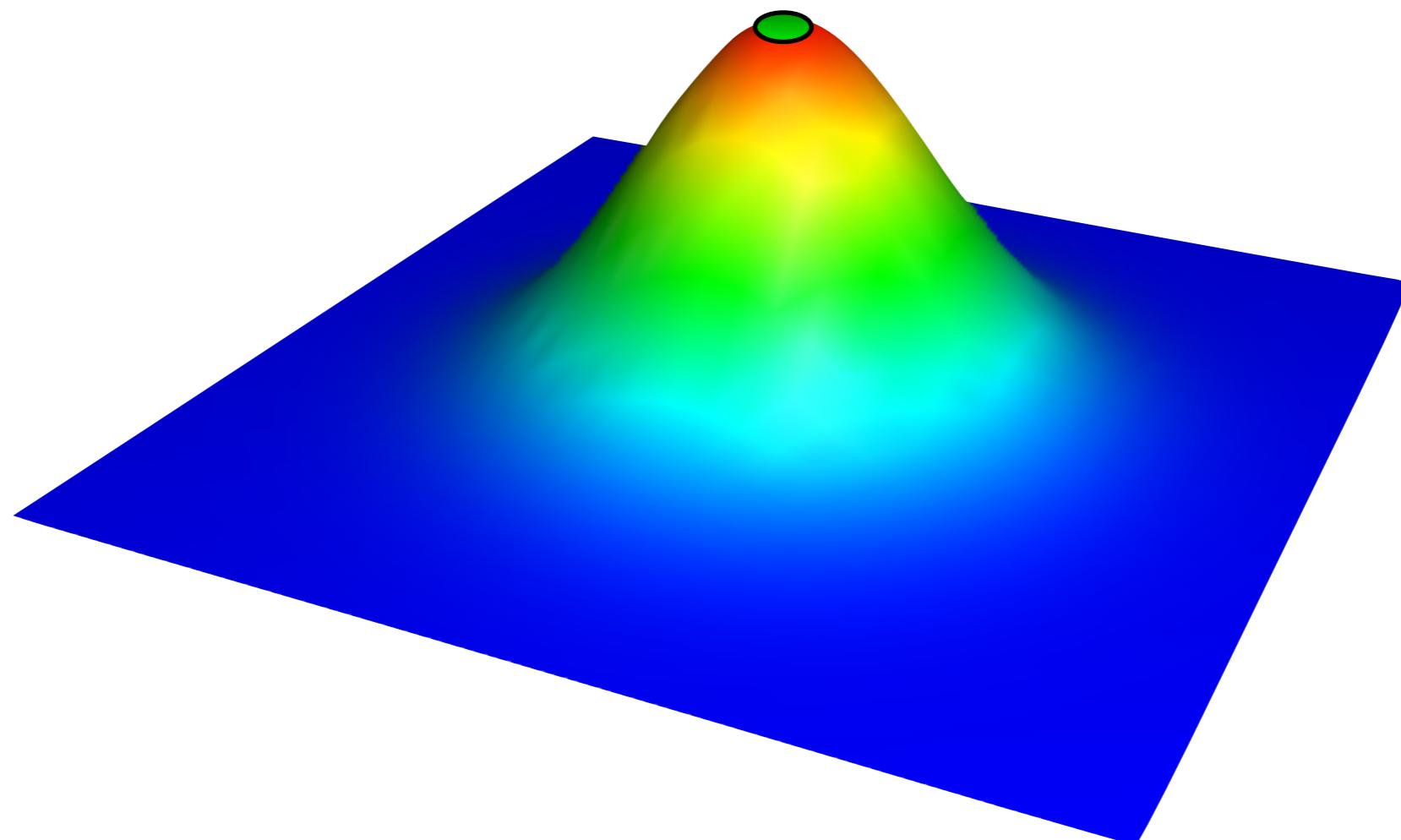
3. Choose best  
neighbor

# Hill Climbing

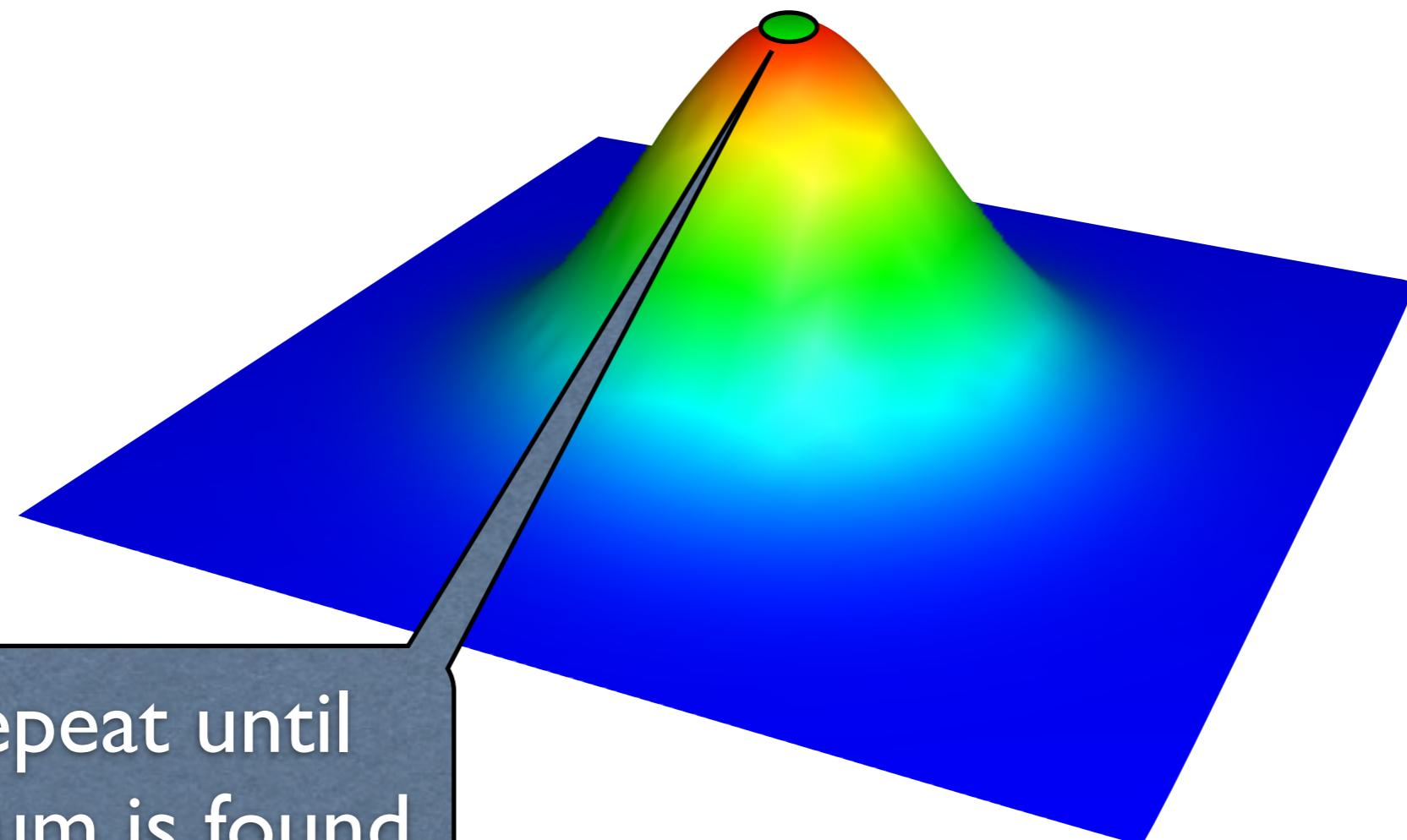


3. Choose best  
neighbor

# Hill Climbing

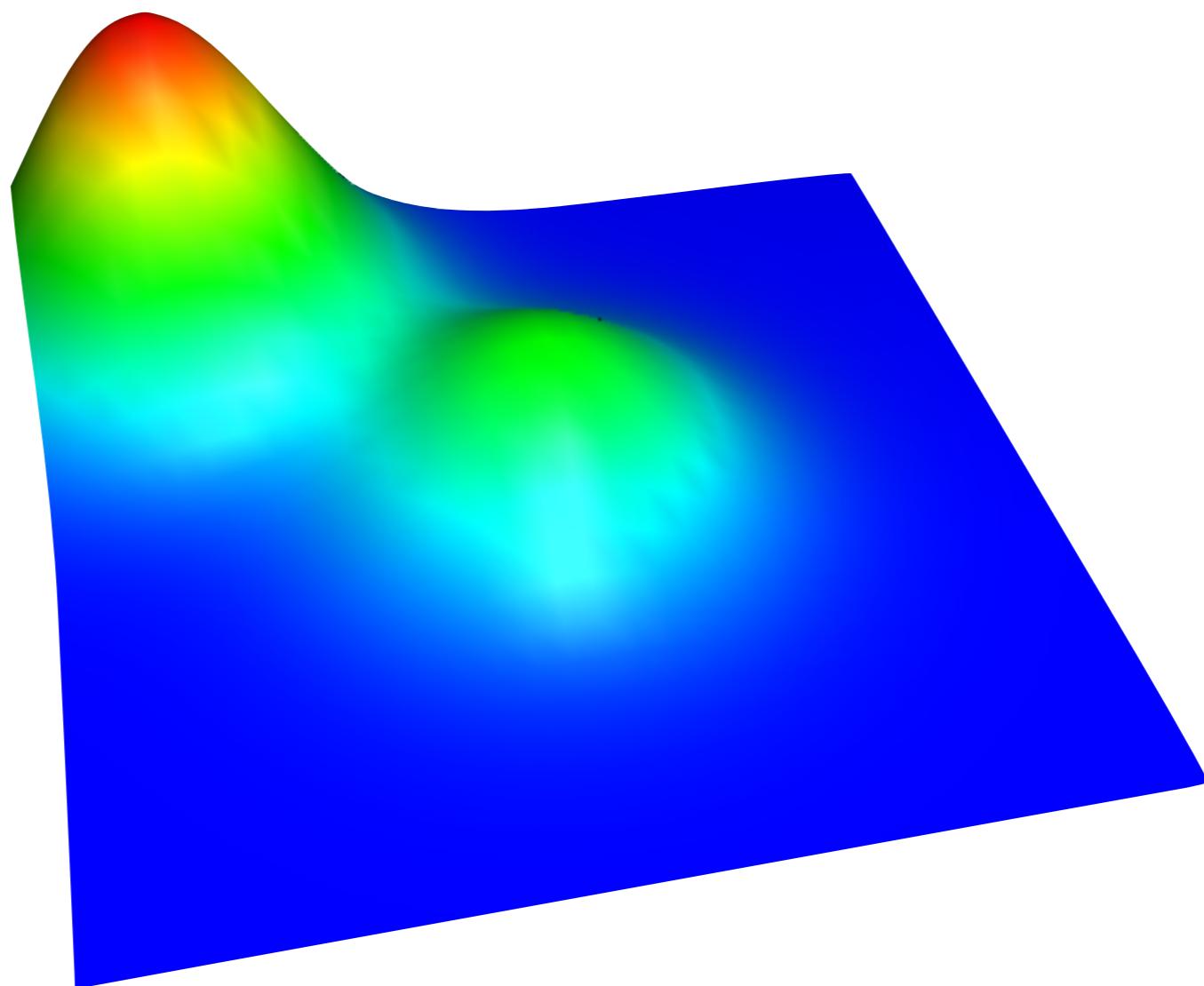


# Hill Climbing

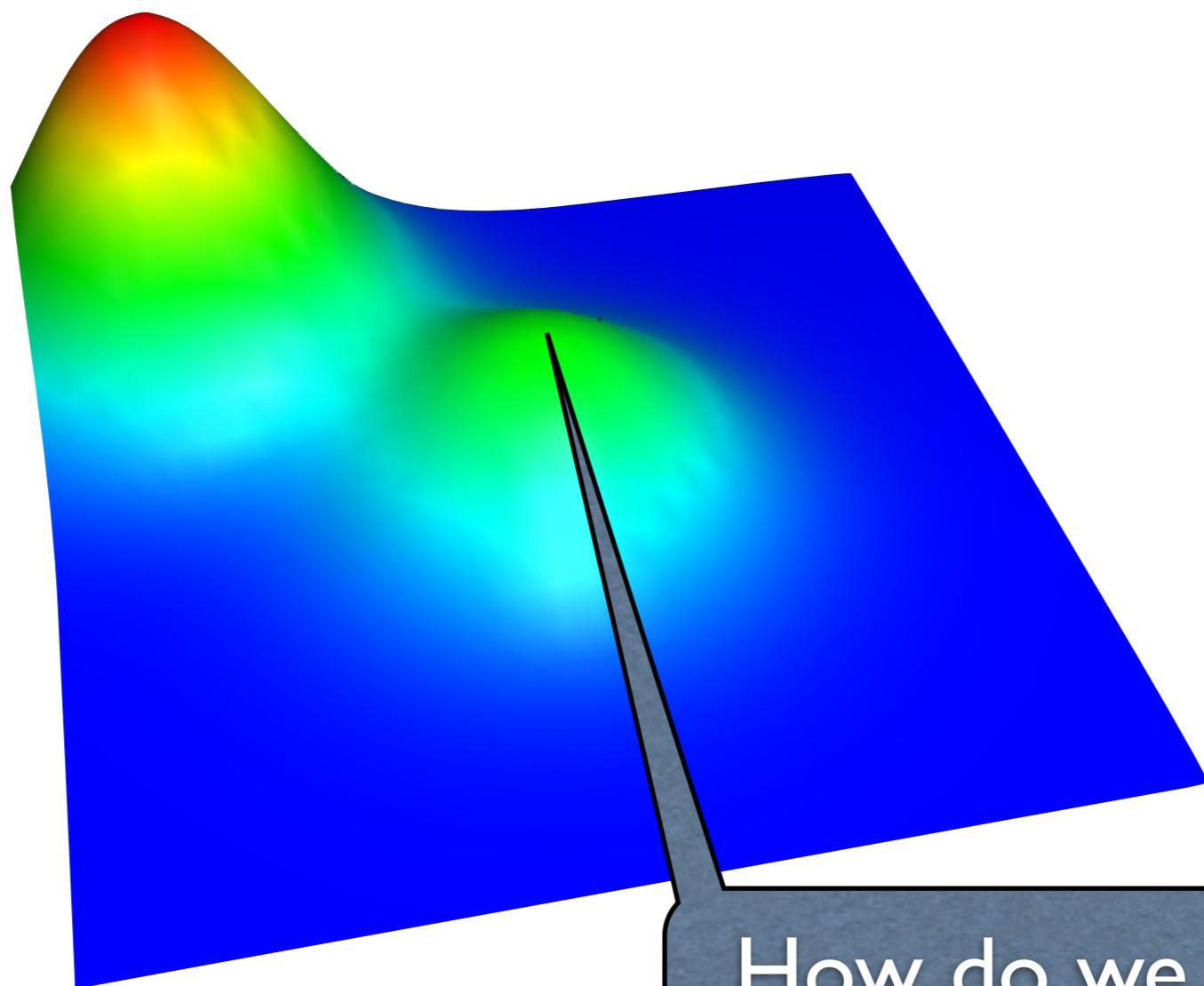


4. Repeat until  
optimum is found

# Local Optima

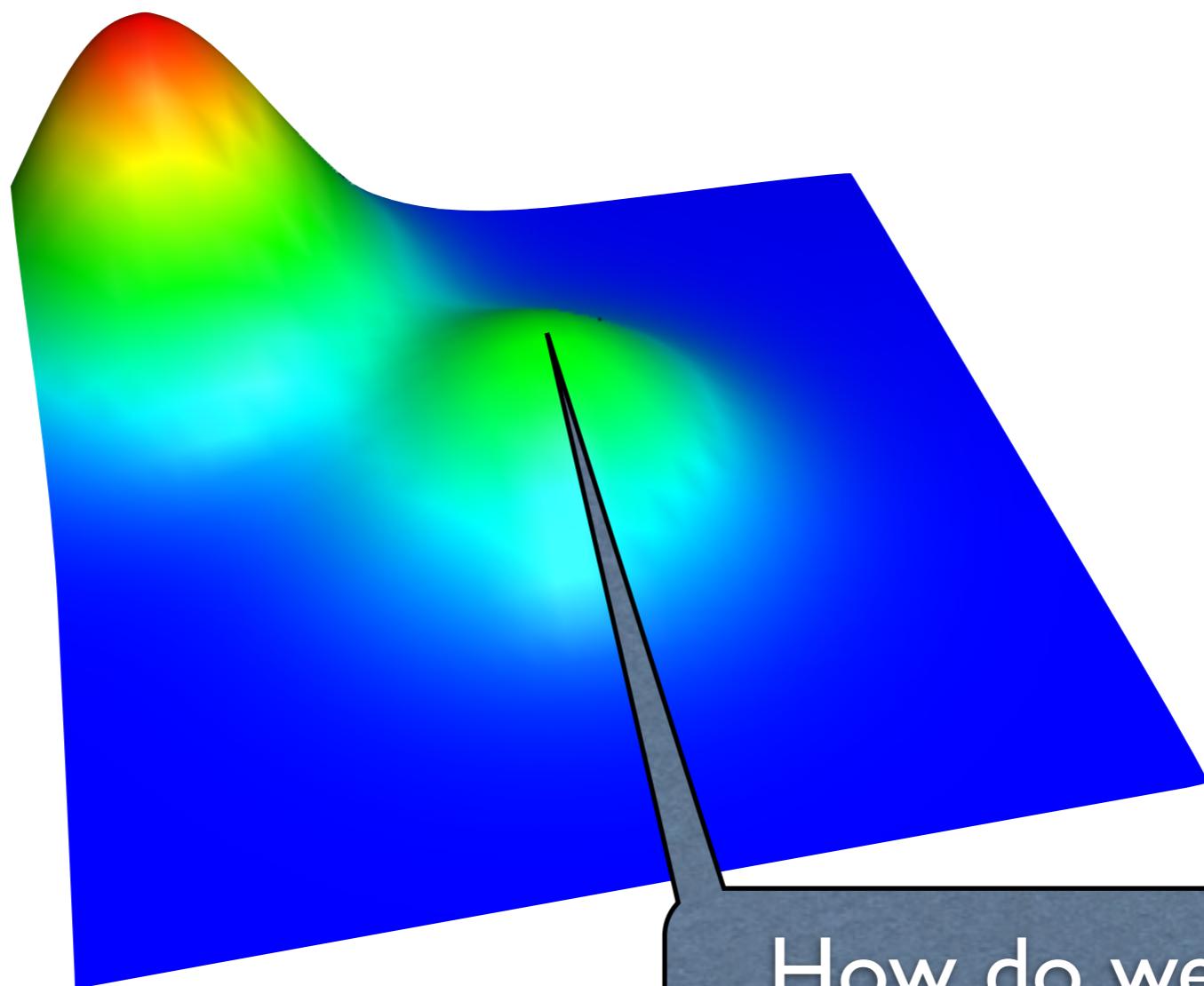


# Local Optima



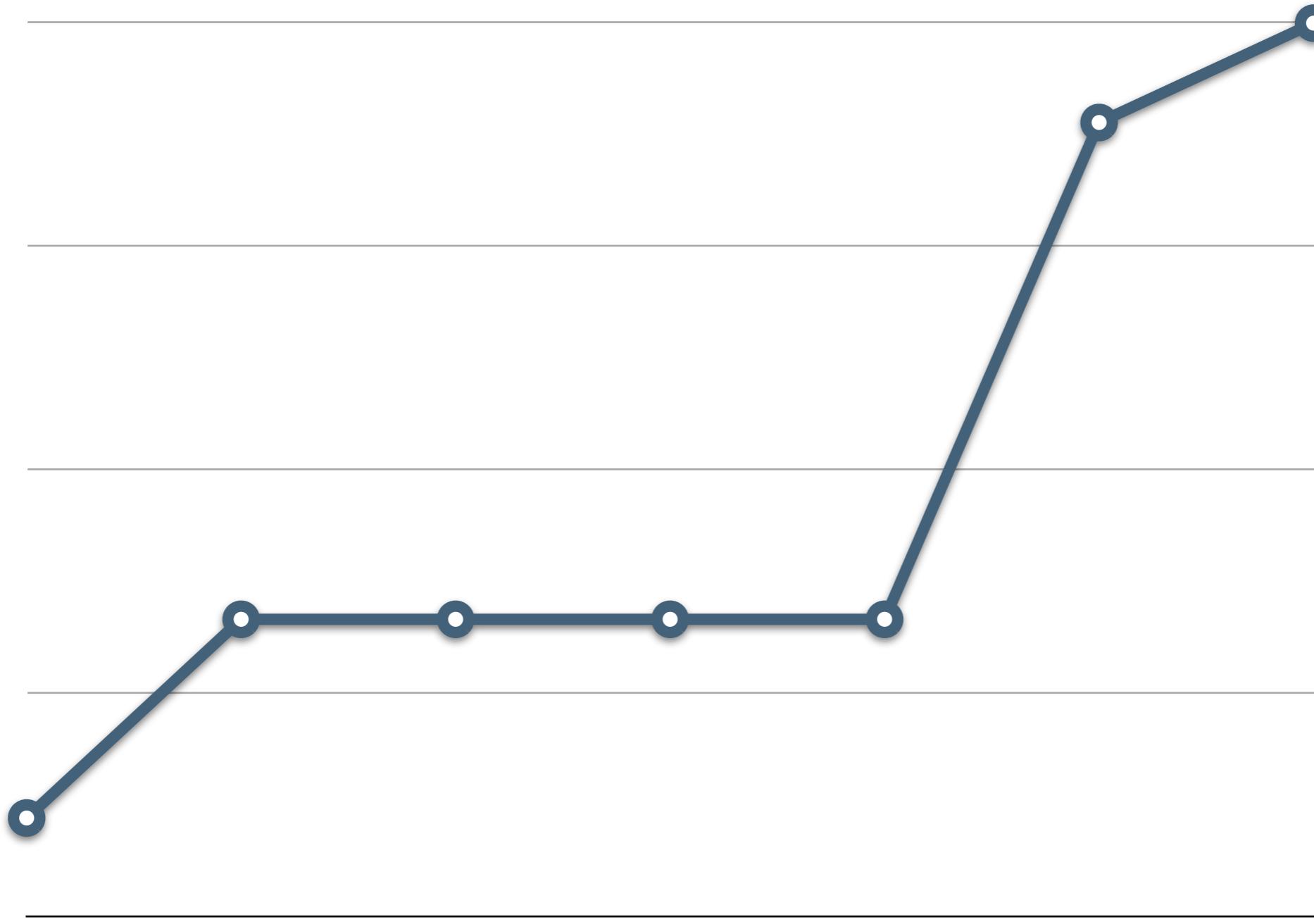
How do we know we're here?

# Local Optima



How do we get out of  
here?

# Plateaux



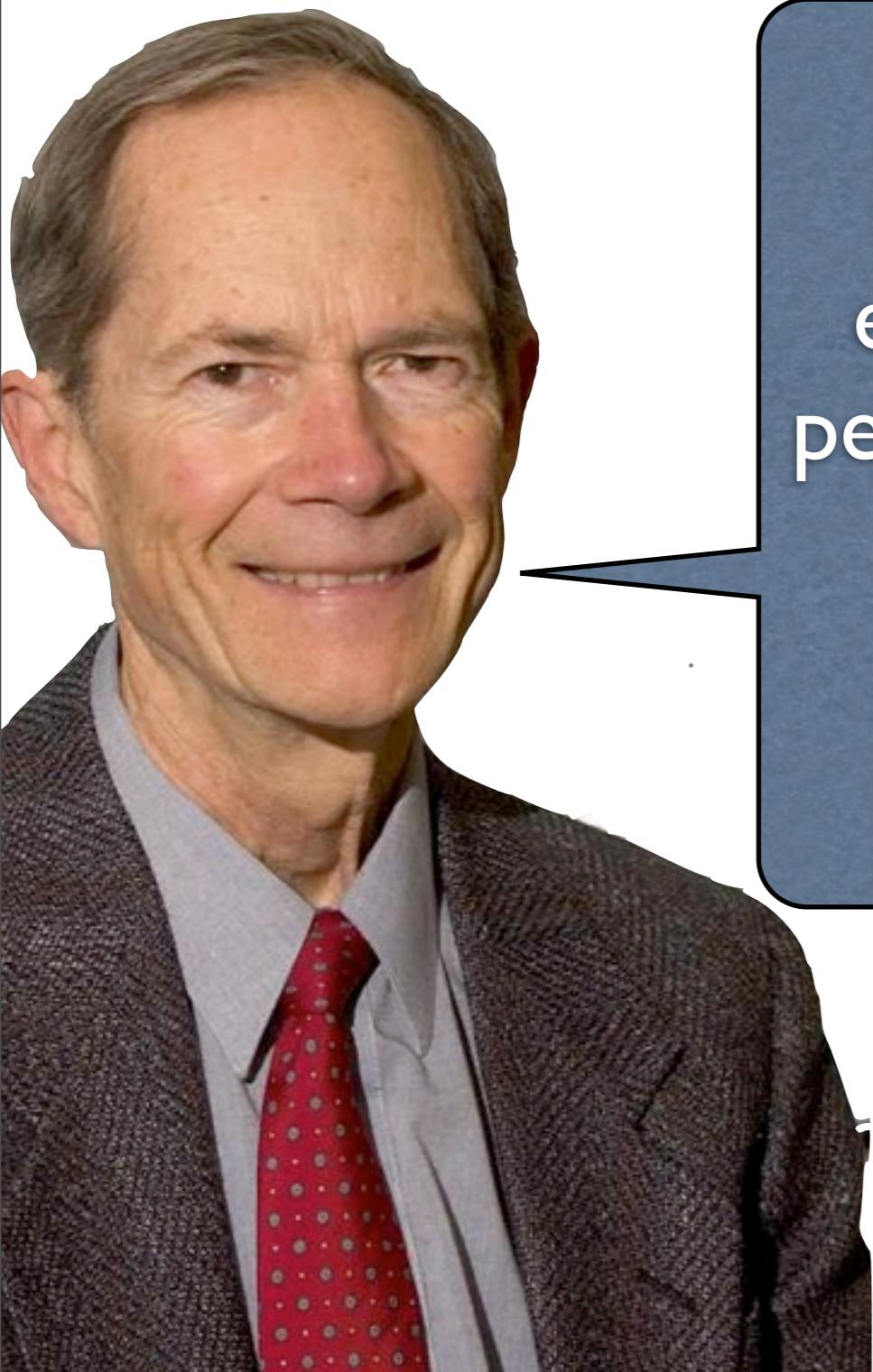
# Plateaux



# Hill Climbing

- Simple hill climbing  
First better neighbor is chosen
- Steepest ascent  
Best neighbor is chosen
- Random-restart  
Randomly reset and start with a different random value
- Resets can lead out of local optima

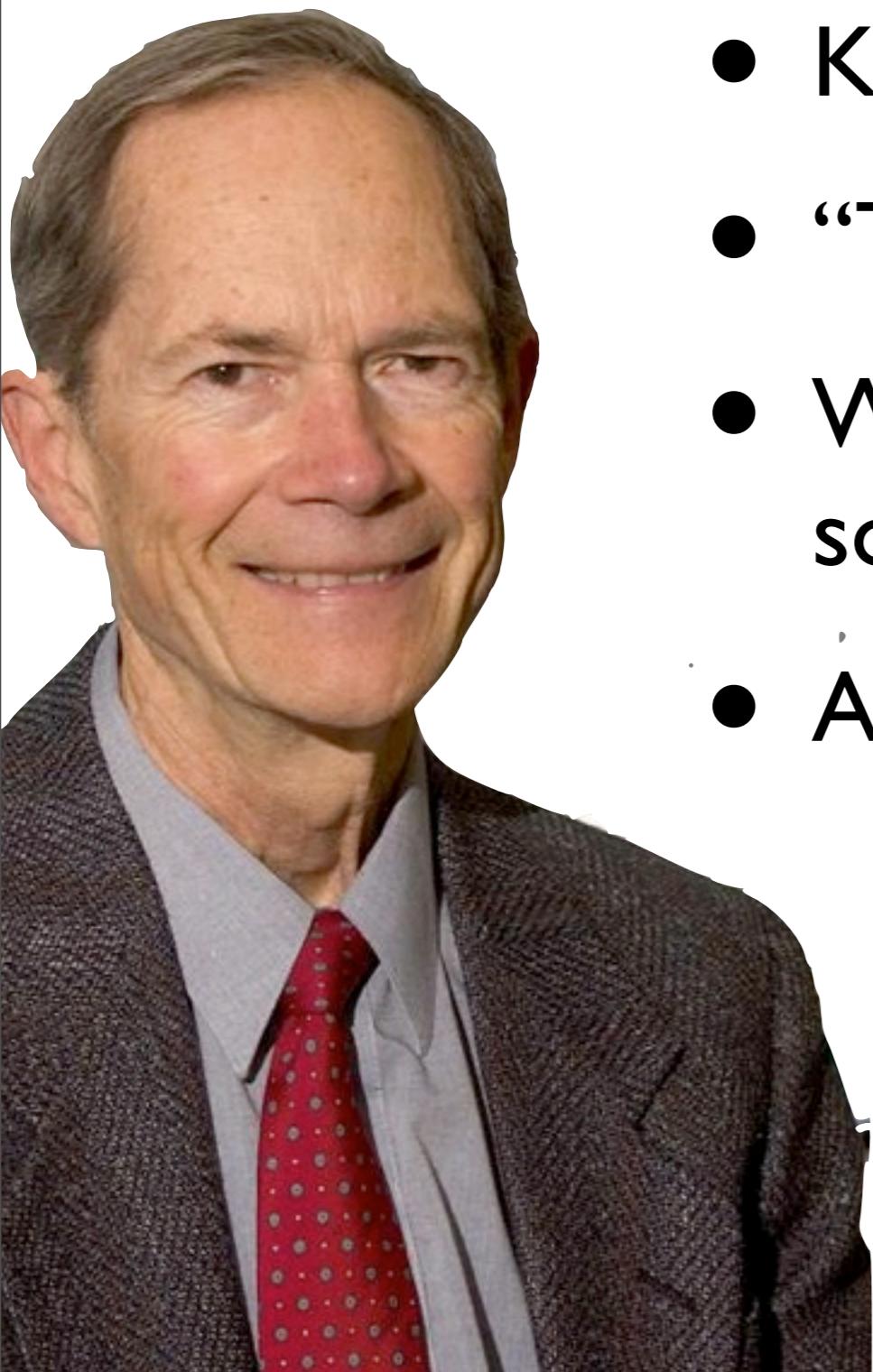
# Tabu Search



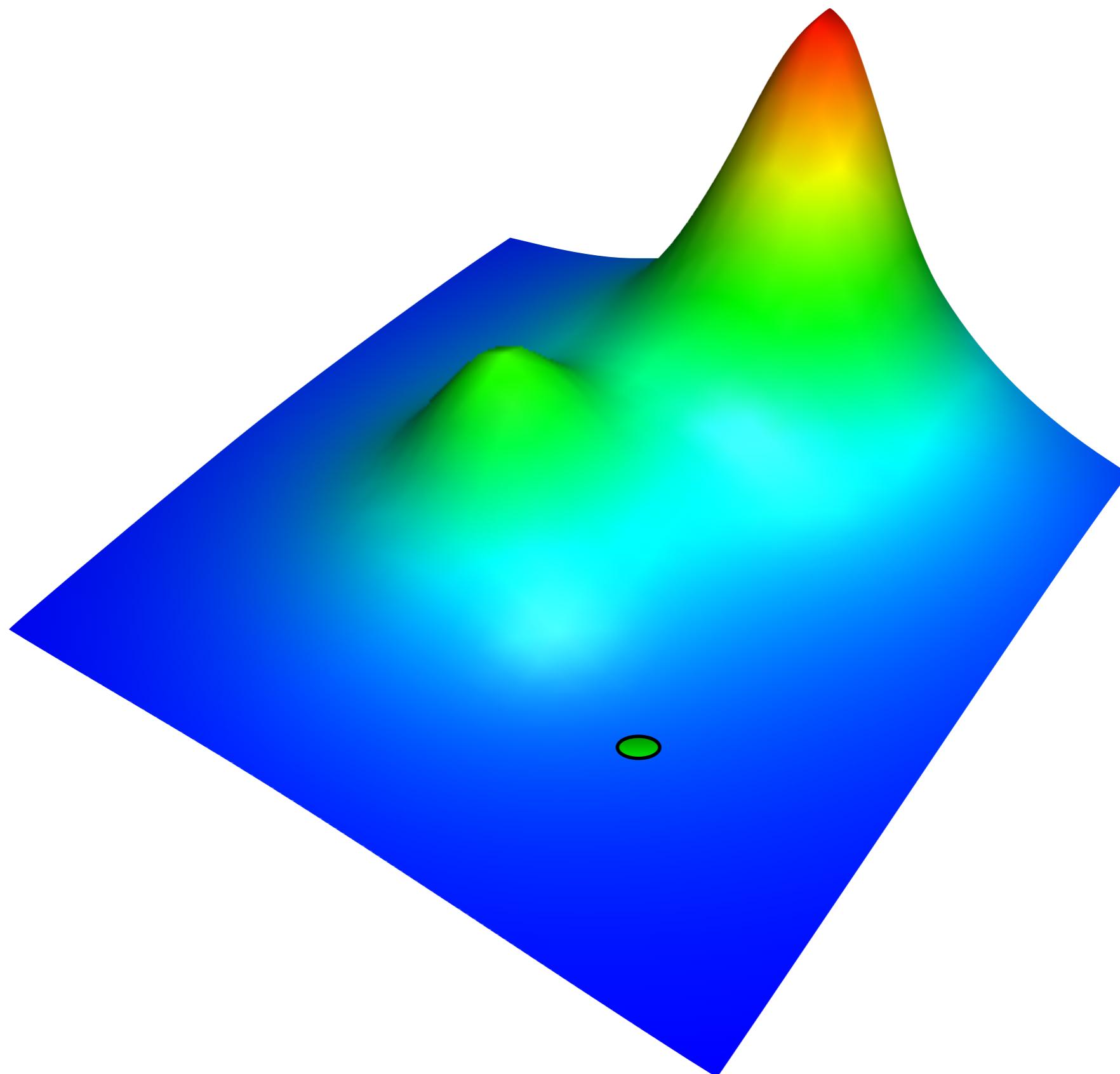
The overall approach is to avoid entrainment in cycles by forbidding or penalizing moves which take the solution, in the next iteration, to points in the solution space previously visited.

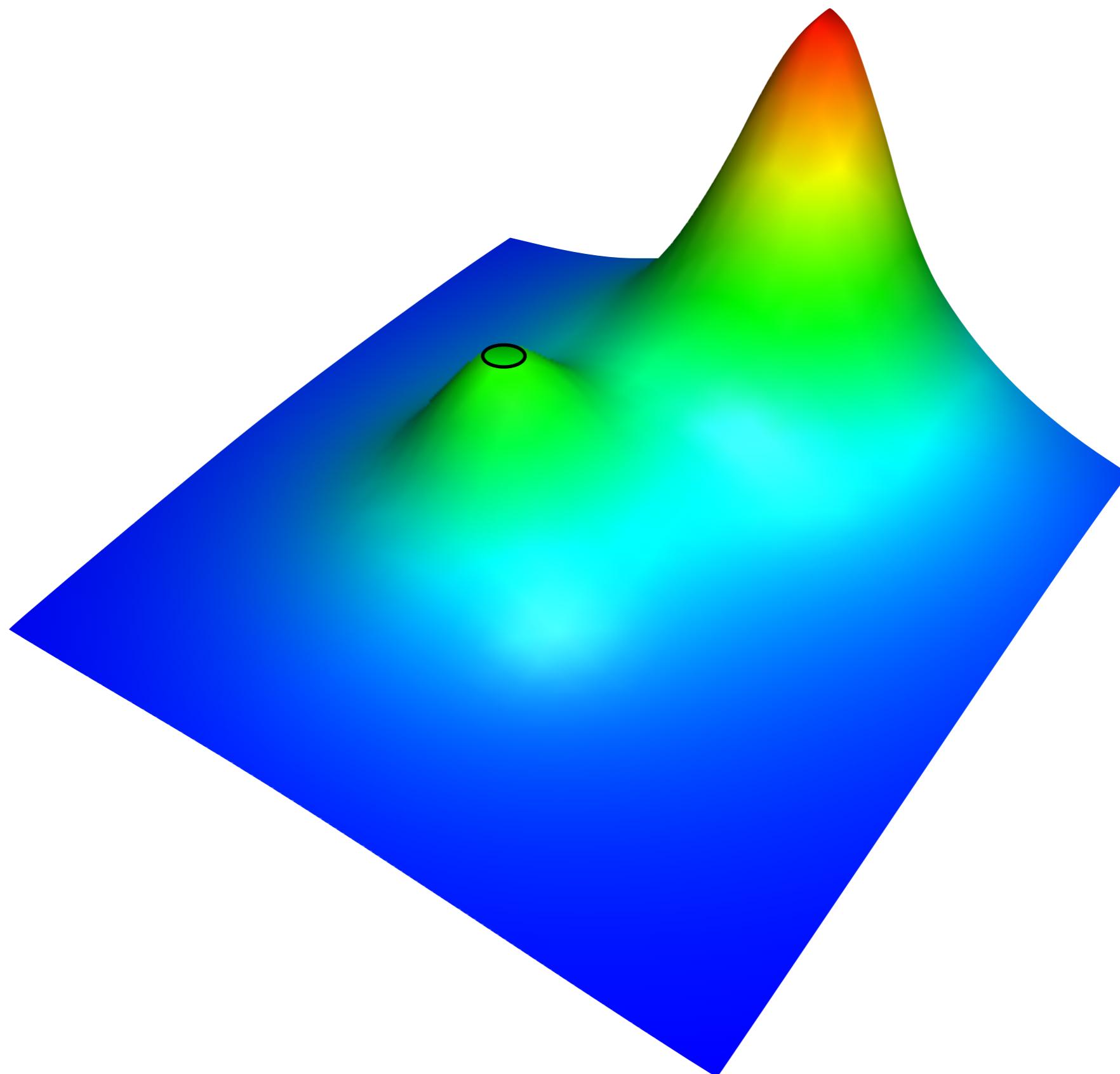
(Fred Glover 1987)

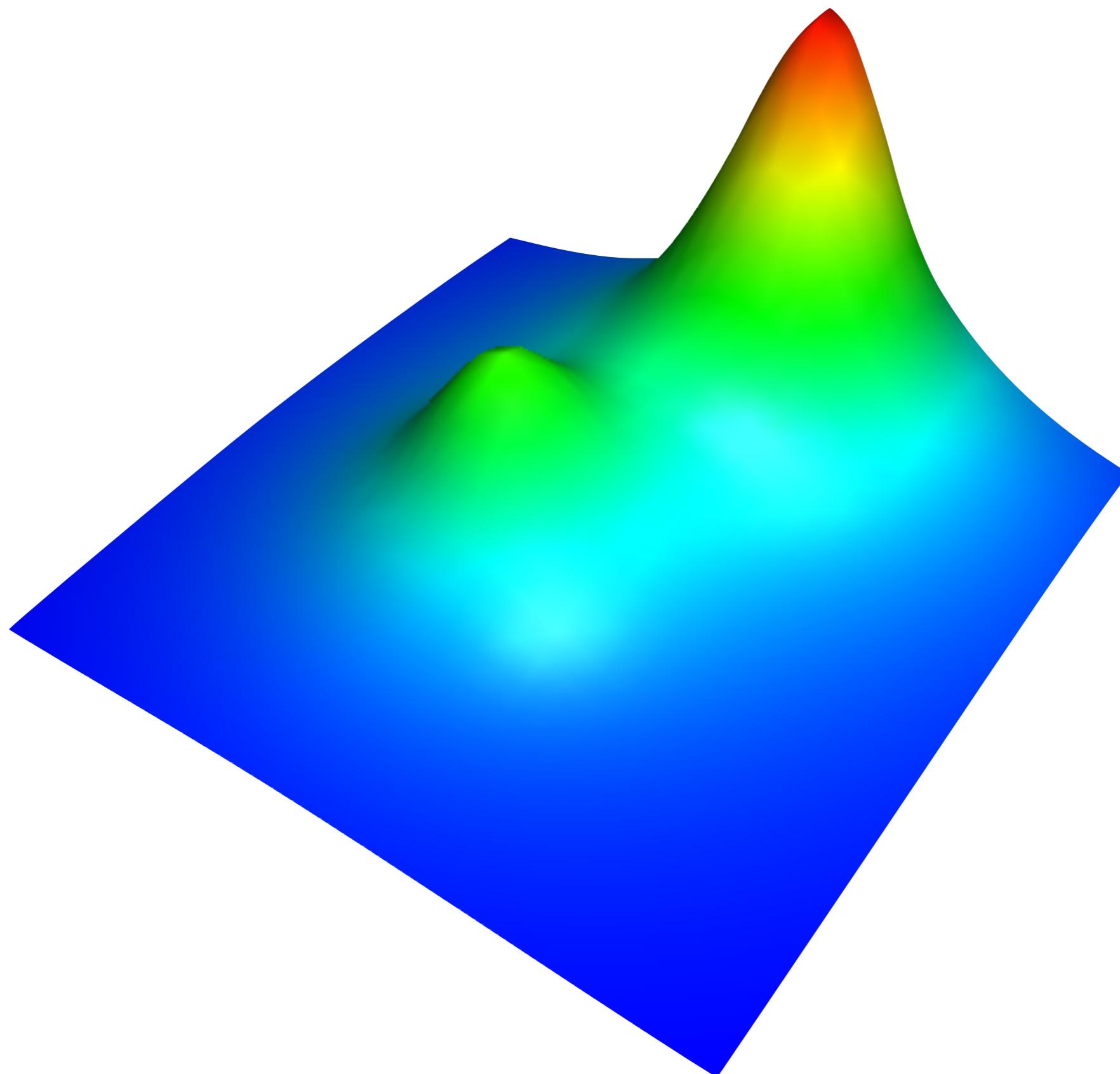
# Tabu Search

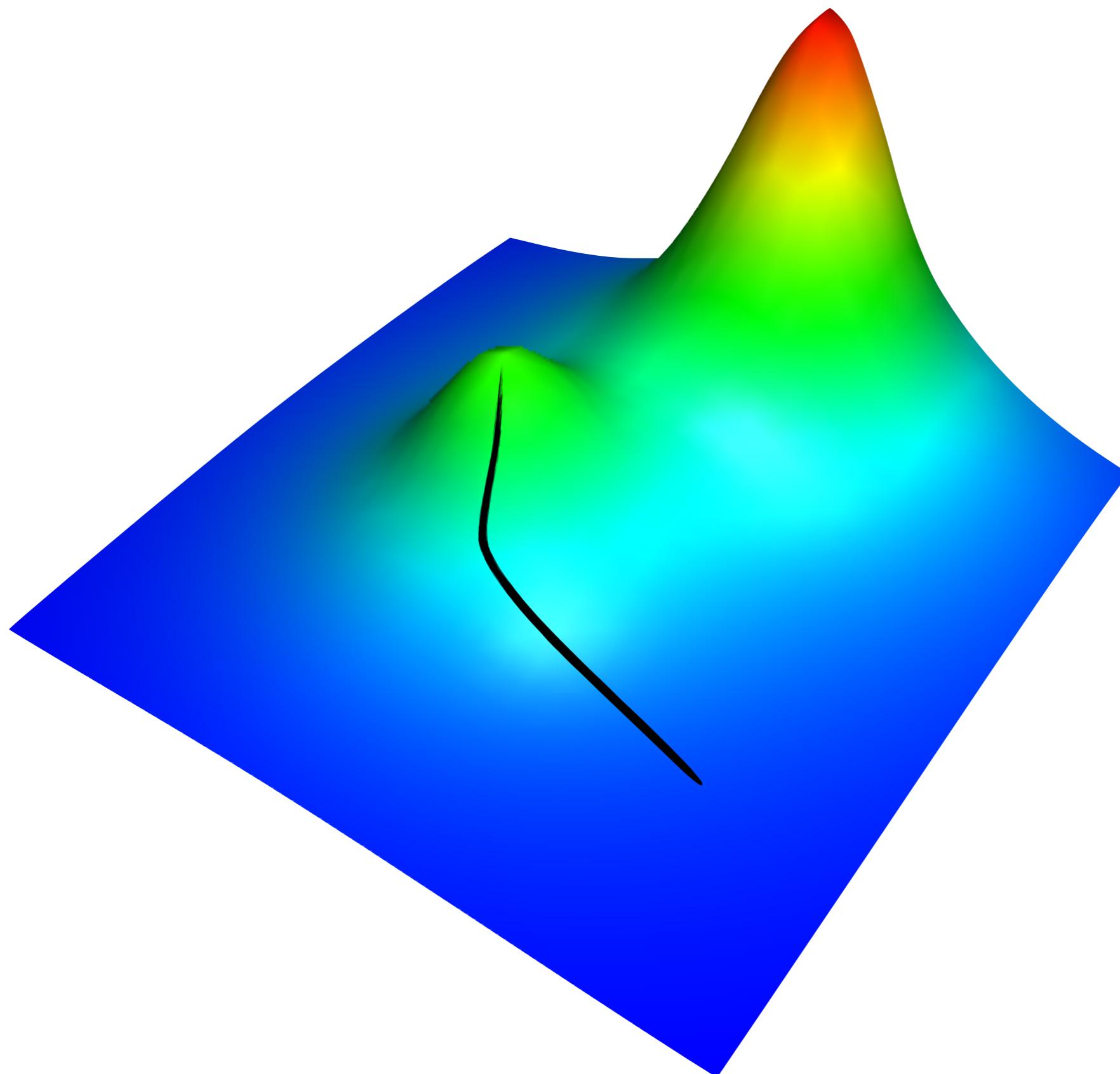


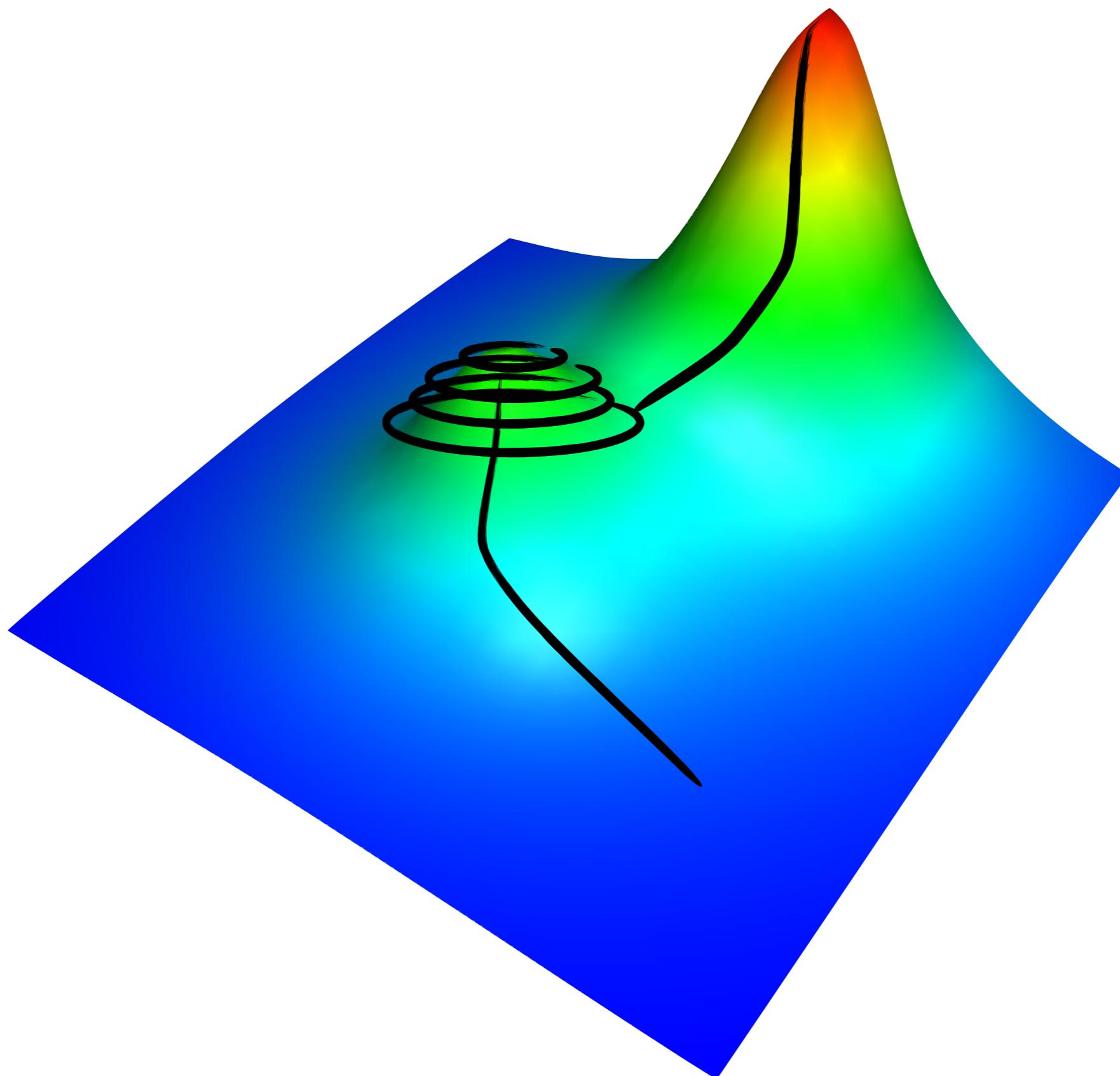
- Keep track of last  $k$  moves
- “Tabu list”
- When choosing next move, take no solution that is on tabu list
- Avoids cycles of length  $k$



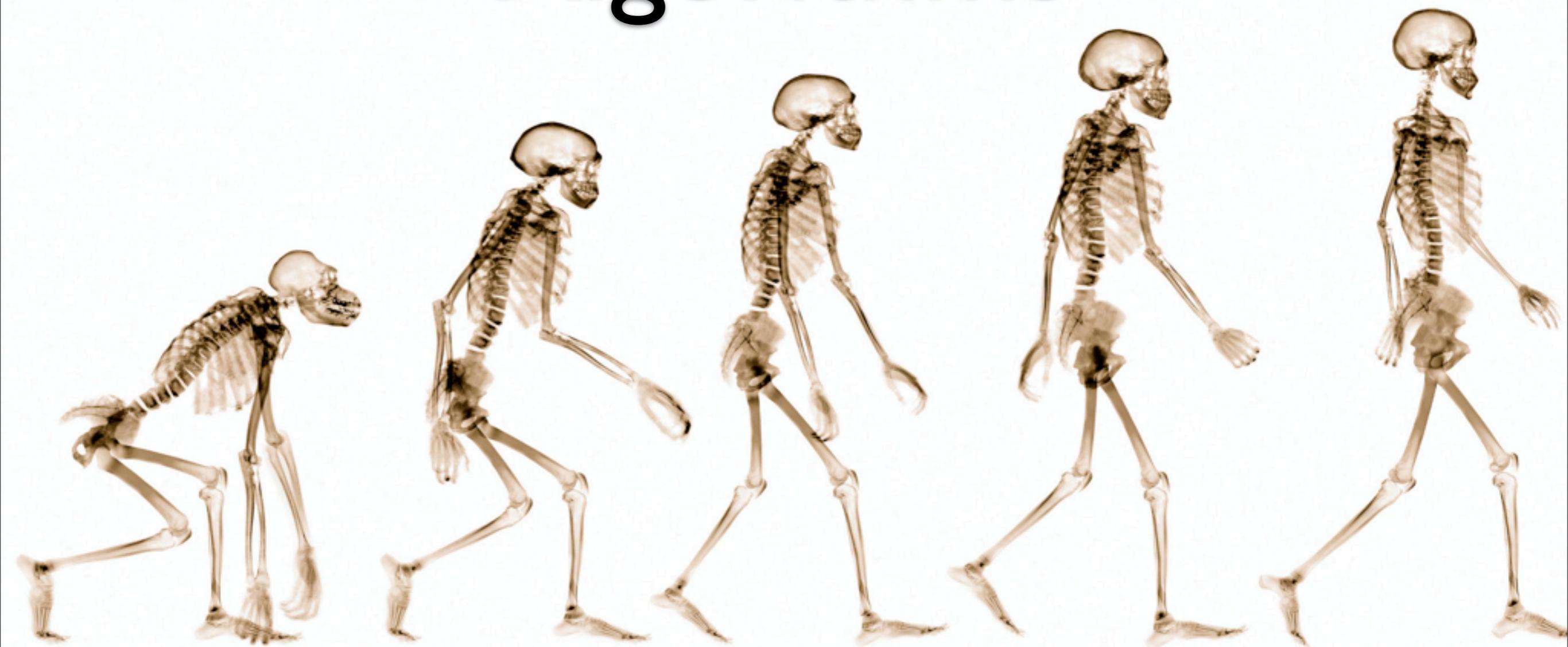






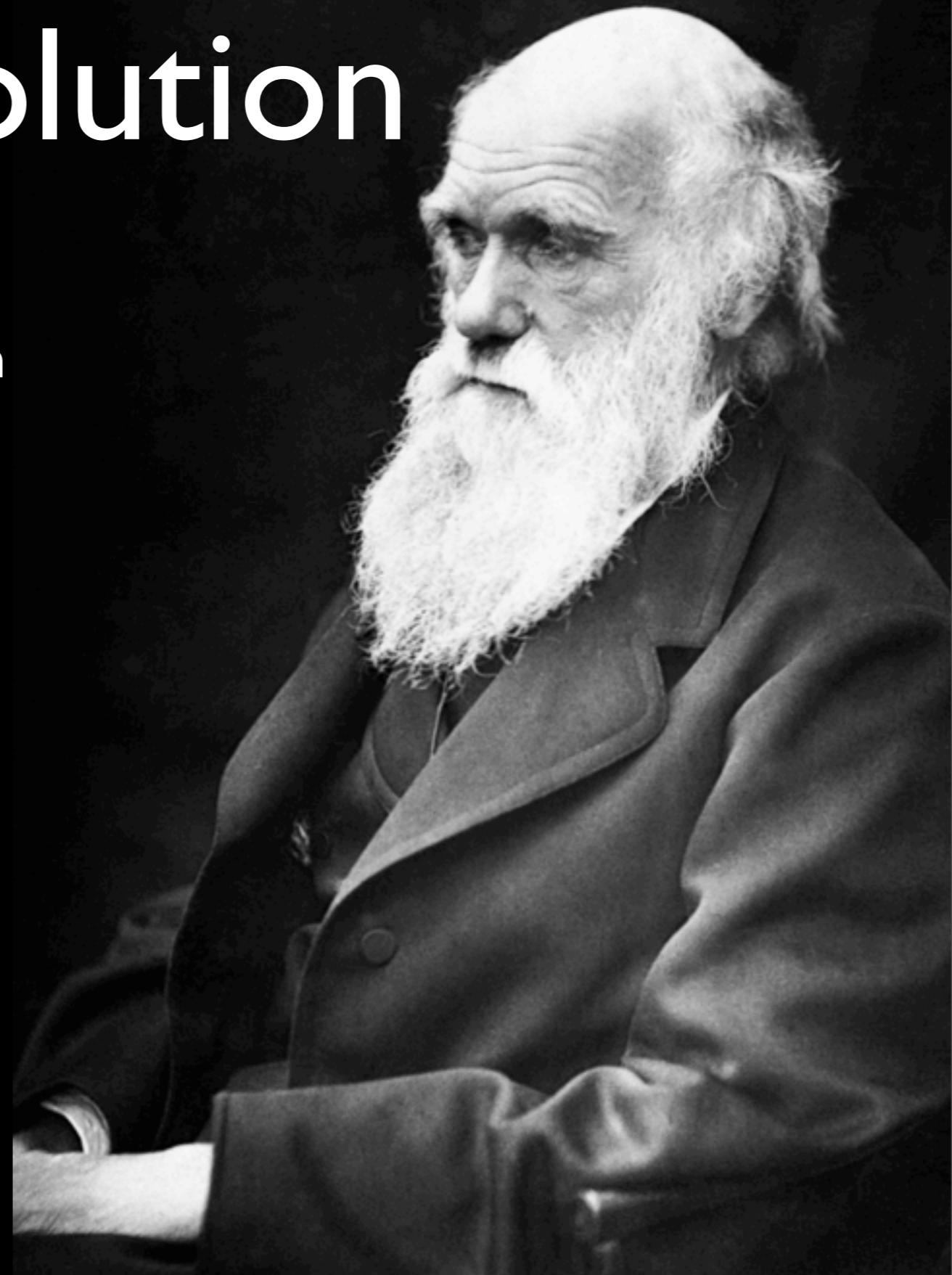


# Evolutionary Algorithms



# Biological Evolution

- Gene  
Unit of information • passed from one generation to another
- Natural Selection
- Survival of The Fittest
- Origin of New Species

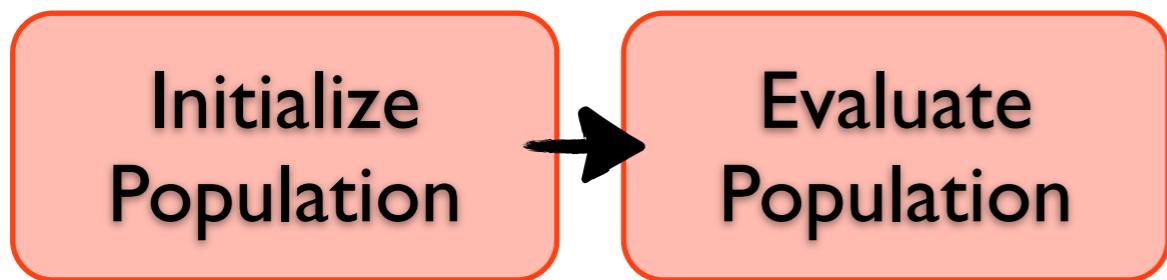


# Genetic Algorithms

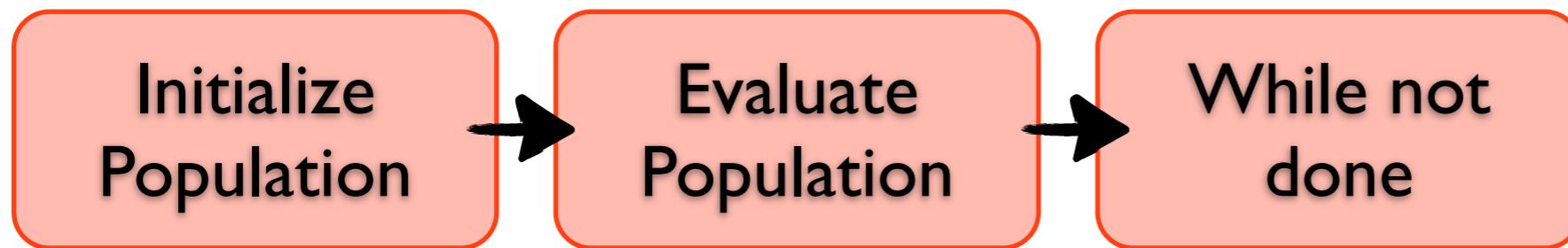
# Genetic Algorithms

Initialize  
Population

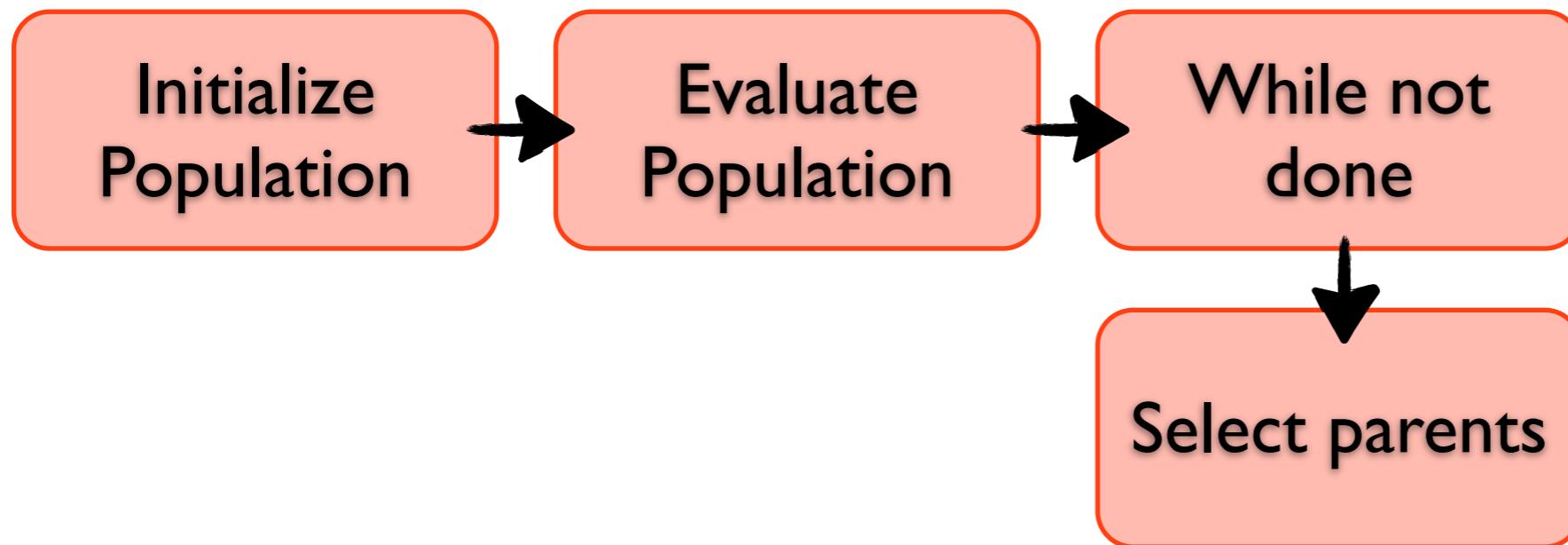
# Genetic Algorithms



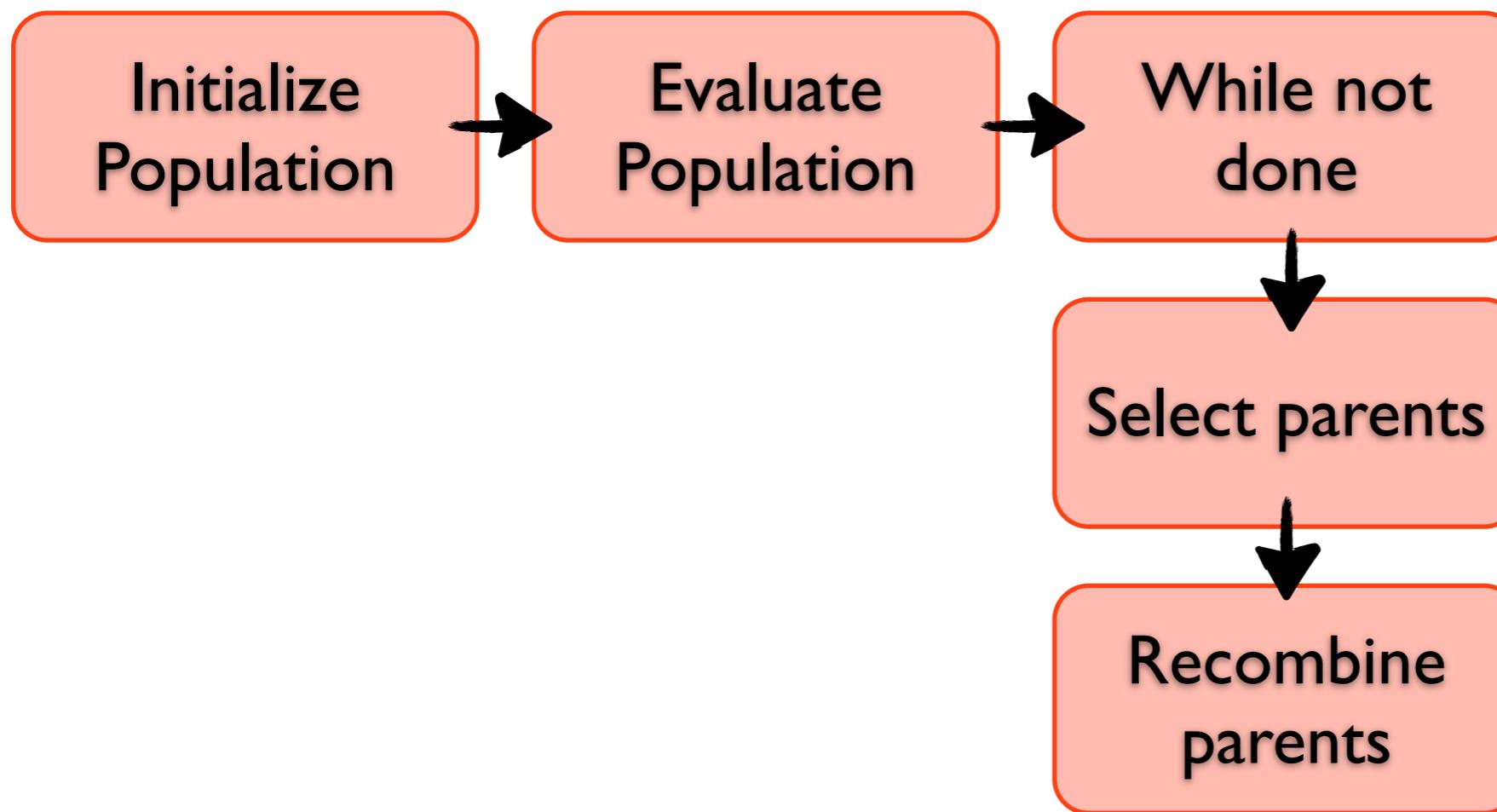
# Genetic Algorithms



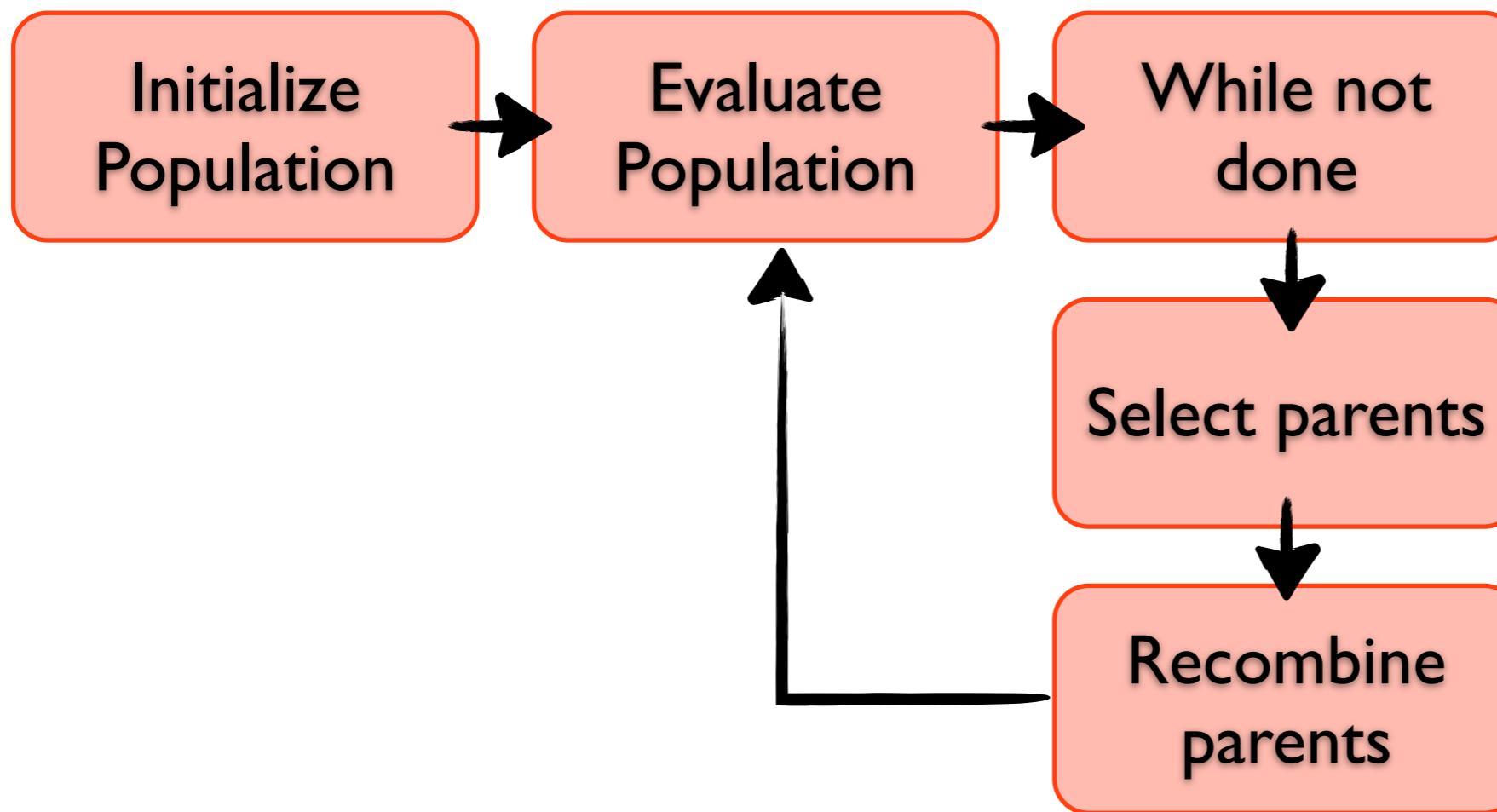
# Genetic Algorithms



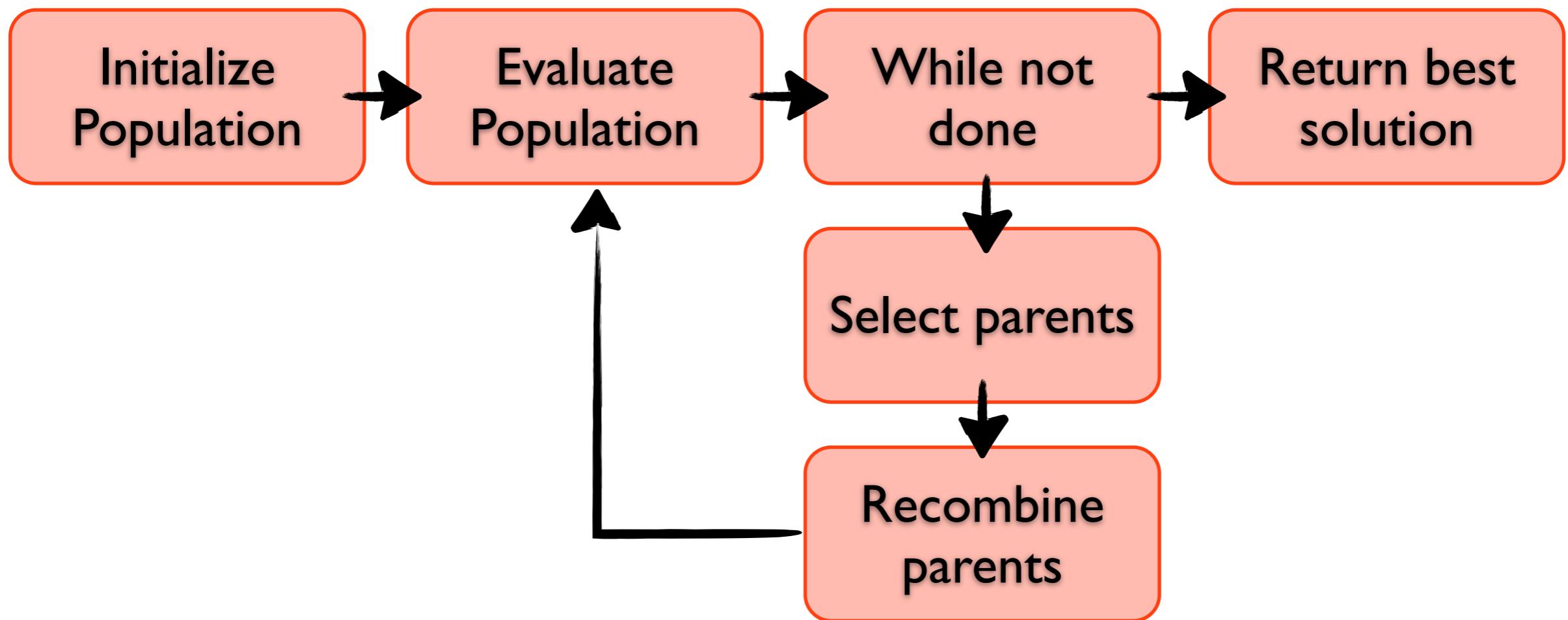
# Genetic Algorithms

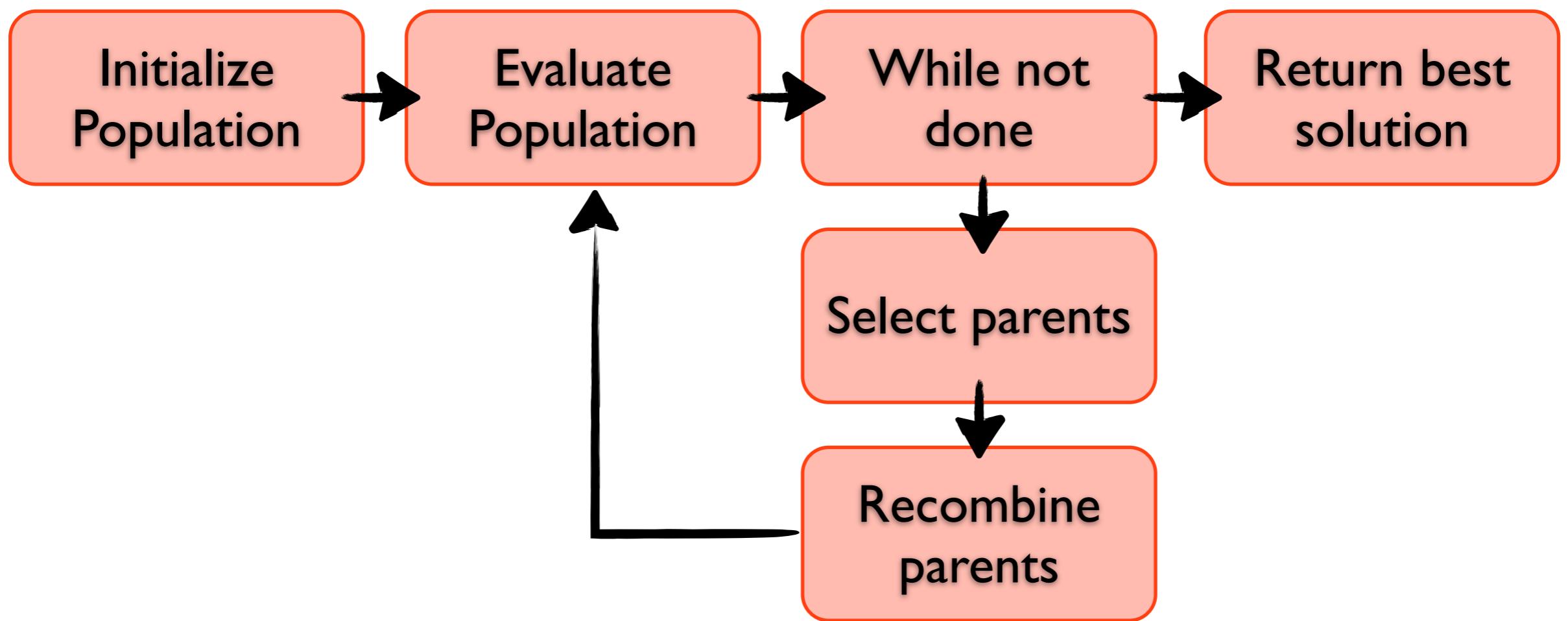


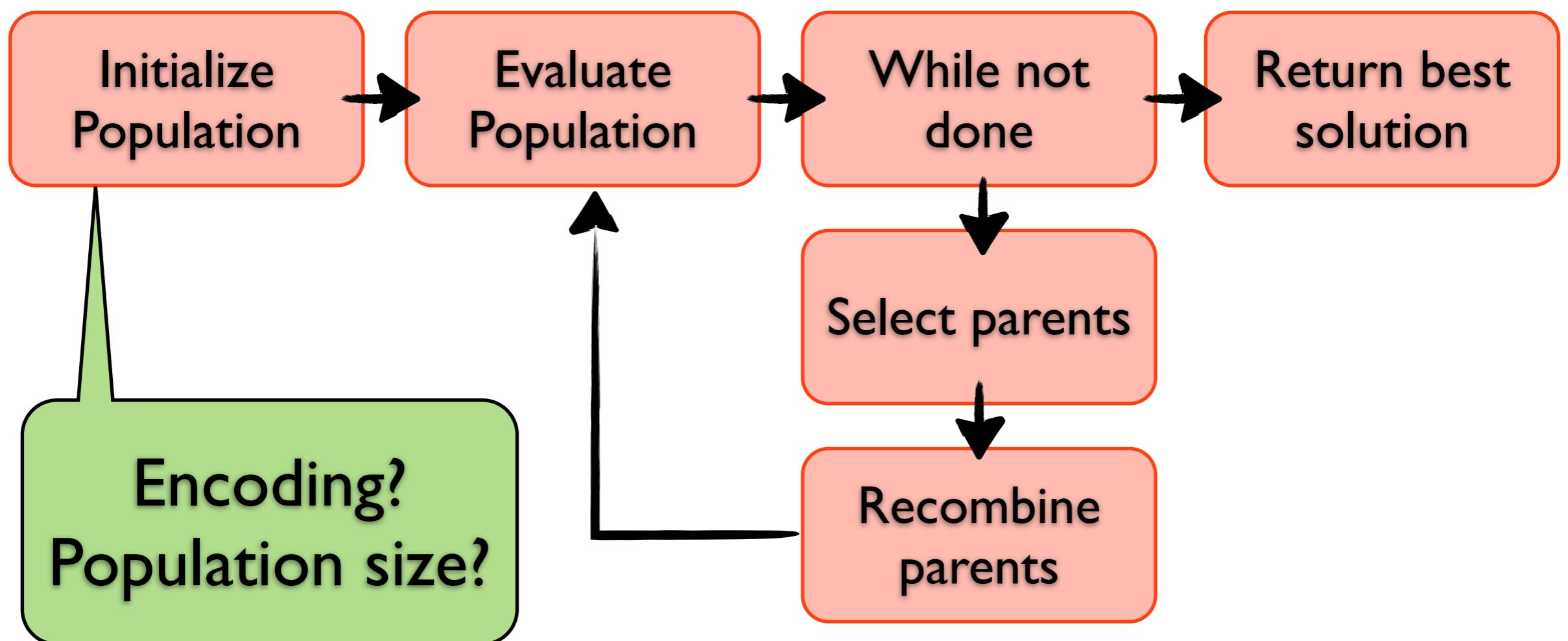
# Genetic Algorithms

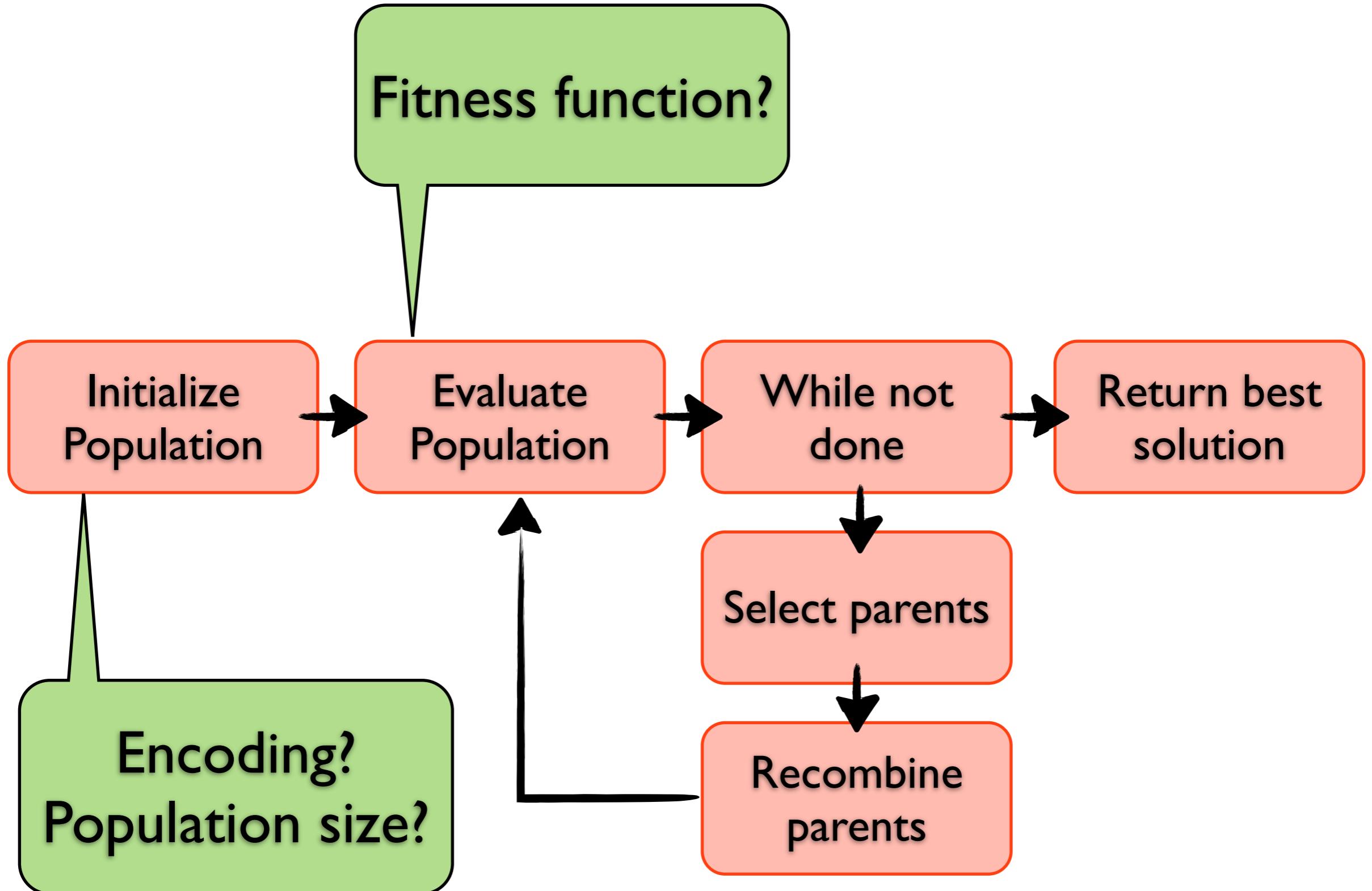


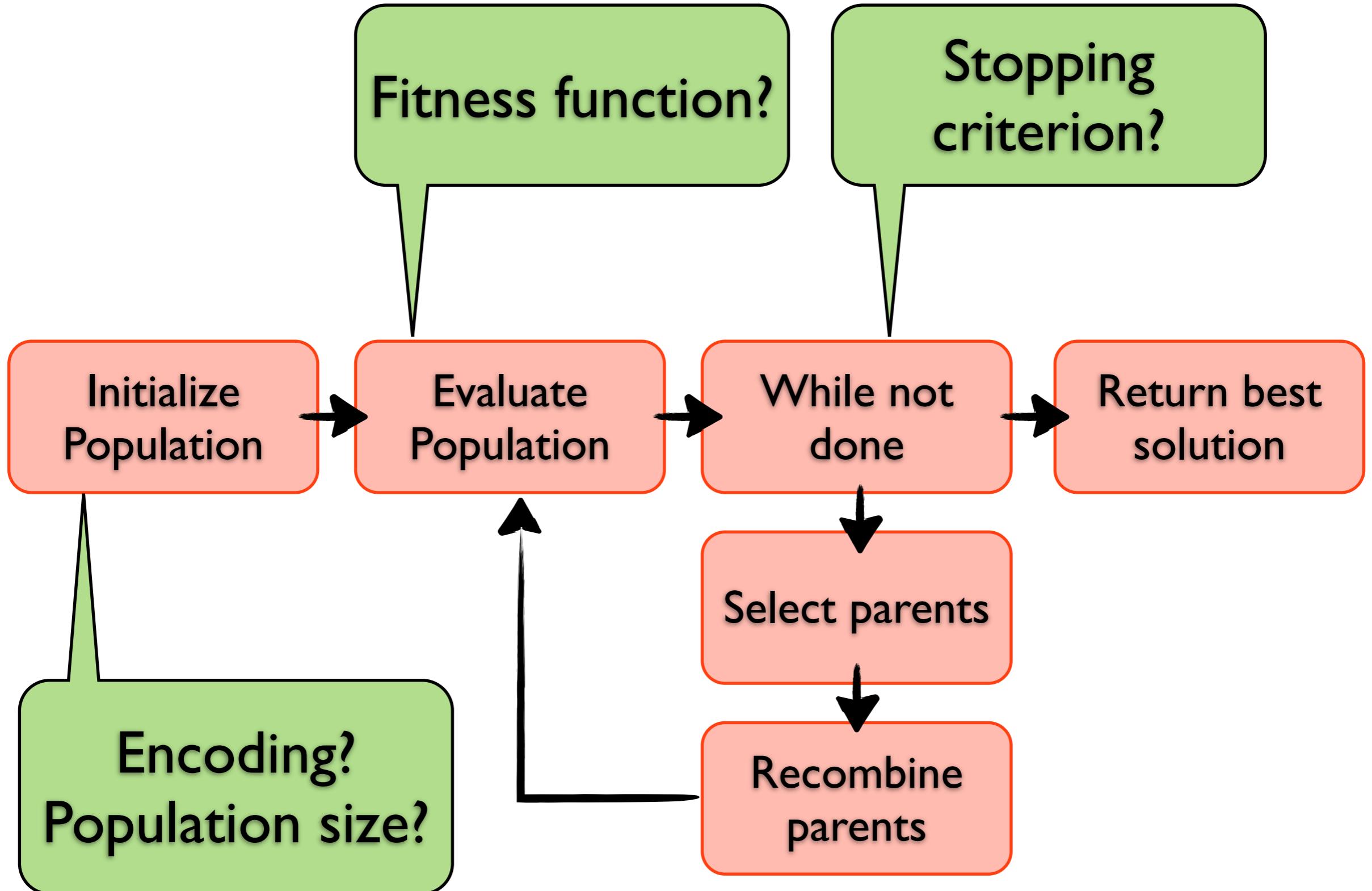
# Genetic Algorithms

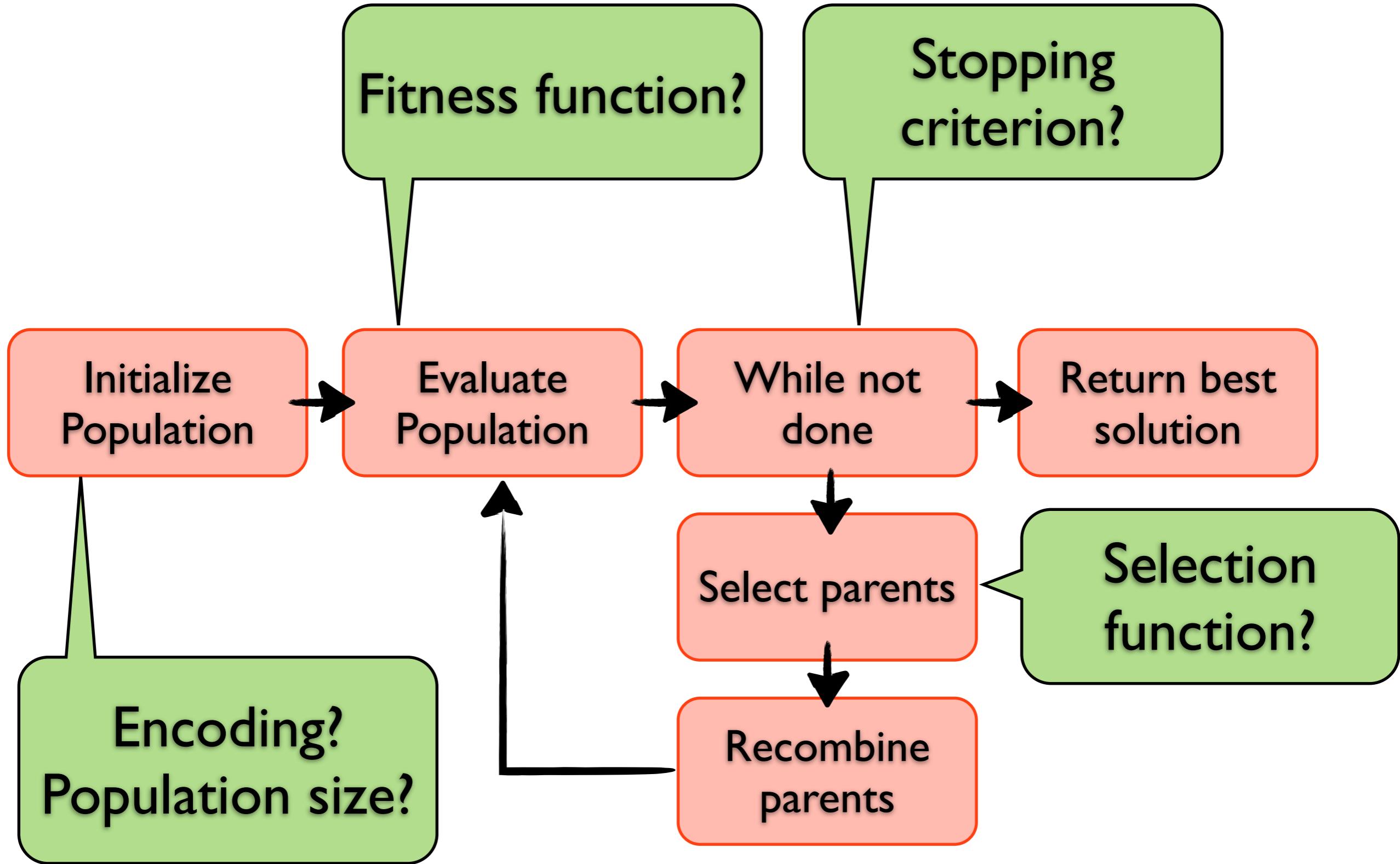


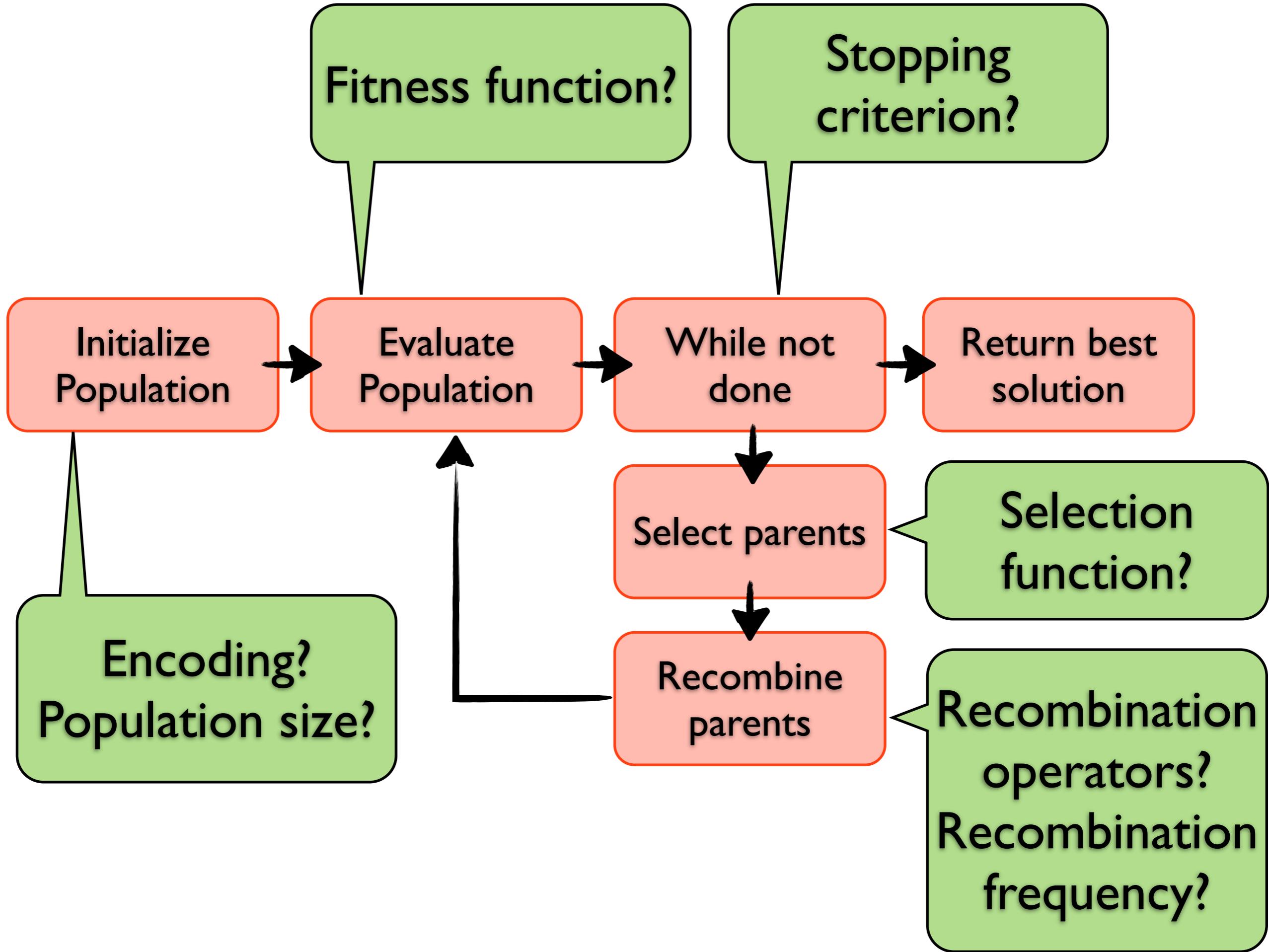


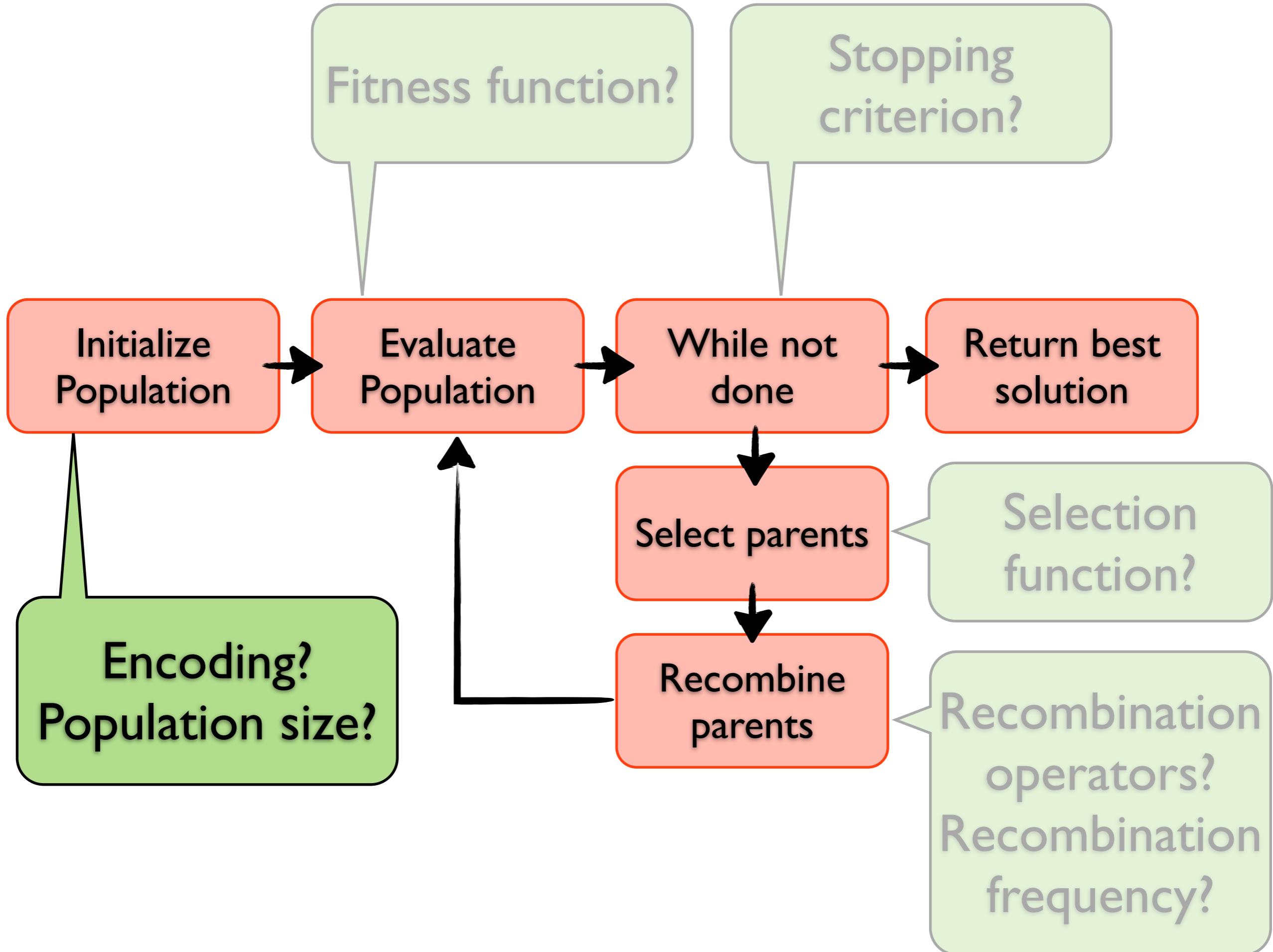












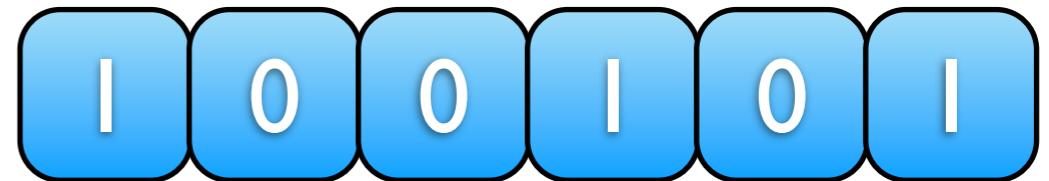
# Encoding

# Encoding

- Binary encoding

# Encoding

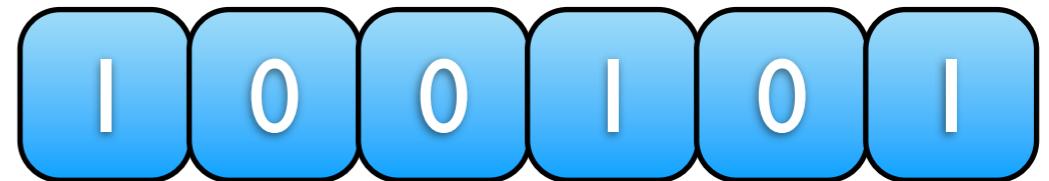
- Binary encoding



A binary sequence consisting of six blue rounded rectangular boxes, each containing a black vertical bar symbol (|). The sequence is 100101.

# Encoding

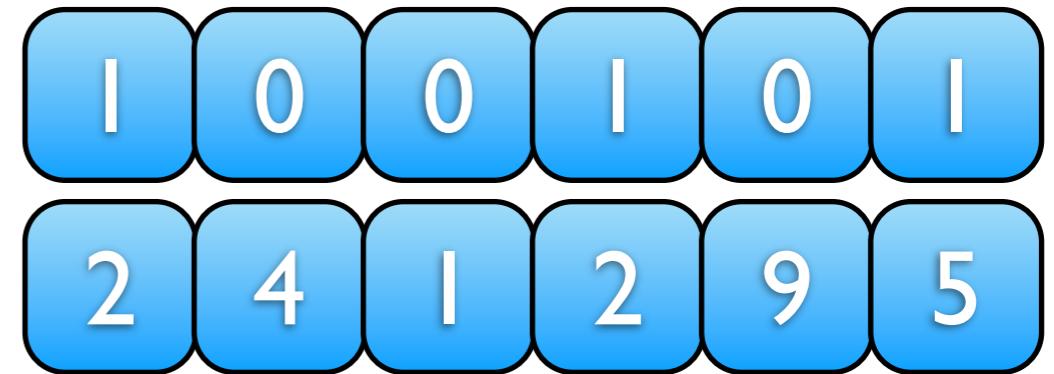
- Binary encoding
- Number encoding



1 0 0 1 0 1

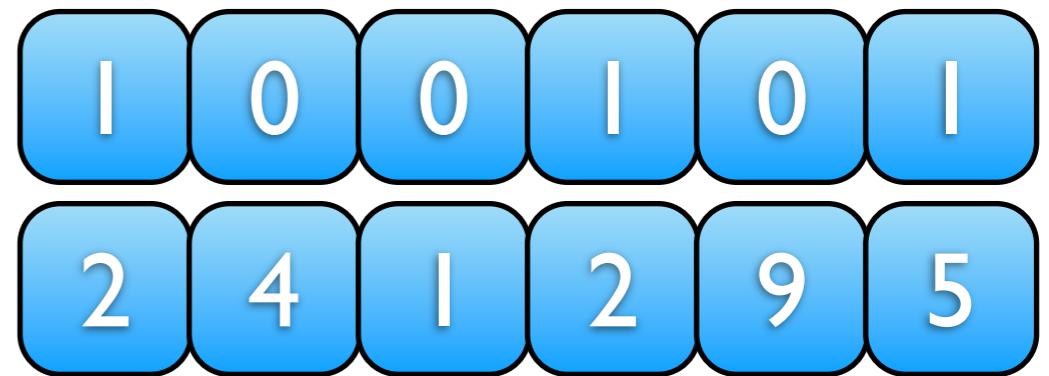
# Encoding

- Binary encoding
- Number encoding



# Encoding

- Binary encoding
- Number encoding
- Any encoding really...



# Encoding

- Binary encoding
- Number encoding
- Any encoding really...

I	0	0	I	0	I
2	4	I	2	9	5
a	b	c	d	e	f

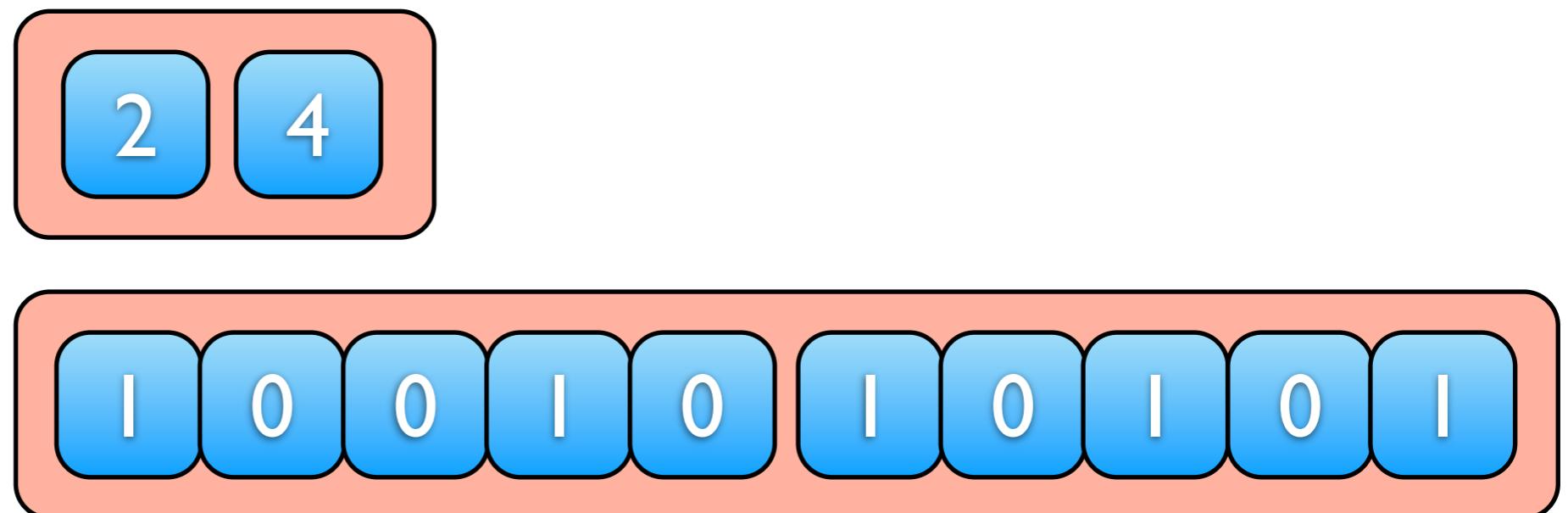
# Encoding

- Binary encoding
- Number encoding
- Any encoding really...
- Encoding is problem specific

I	0	0	I	0	I
2	4	I	2	9	5
a	b	c	d	e	f

# Encoding

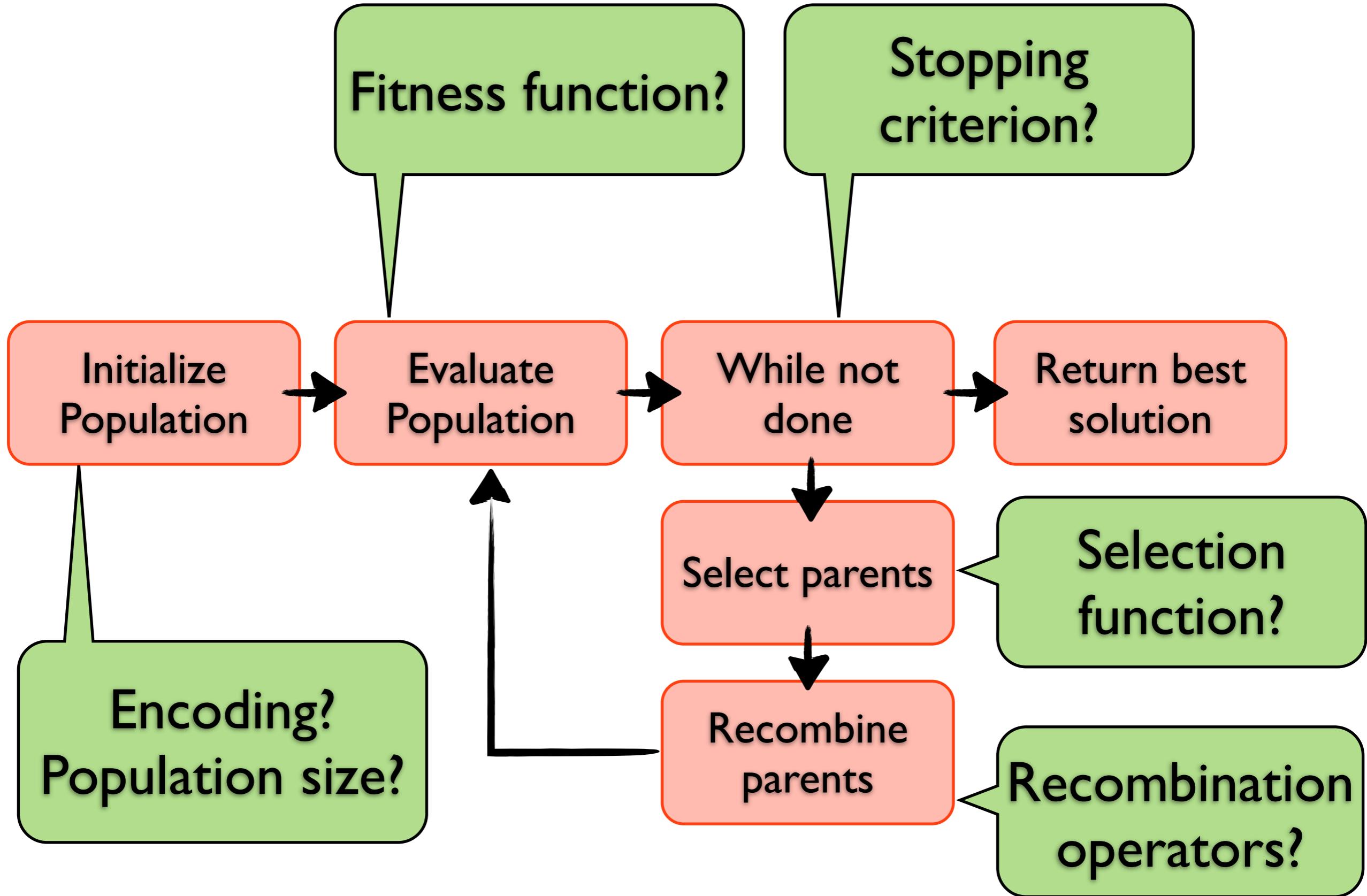
- Search space: (x,y)

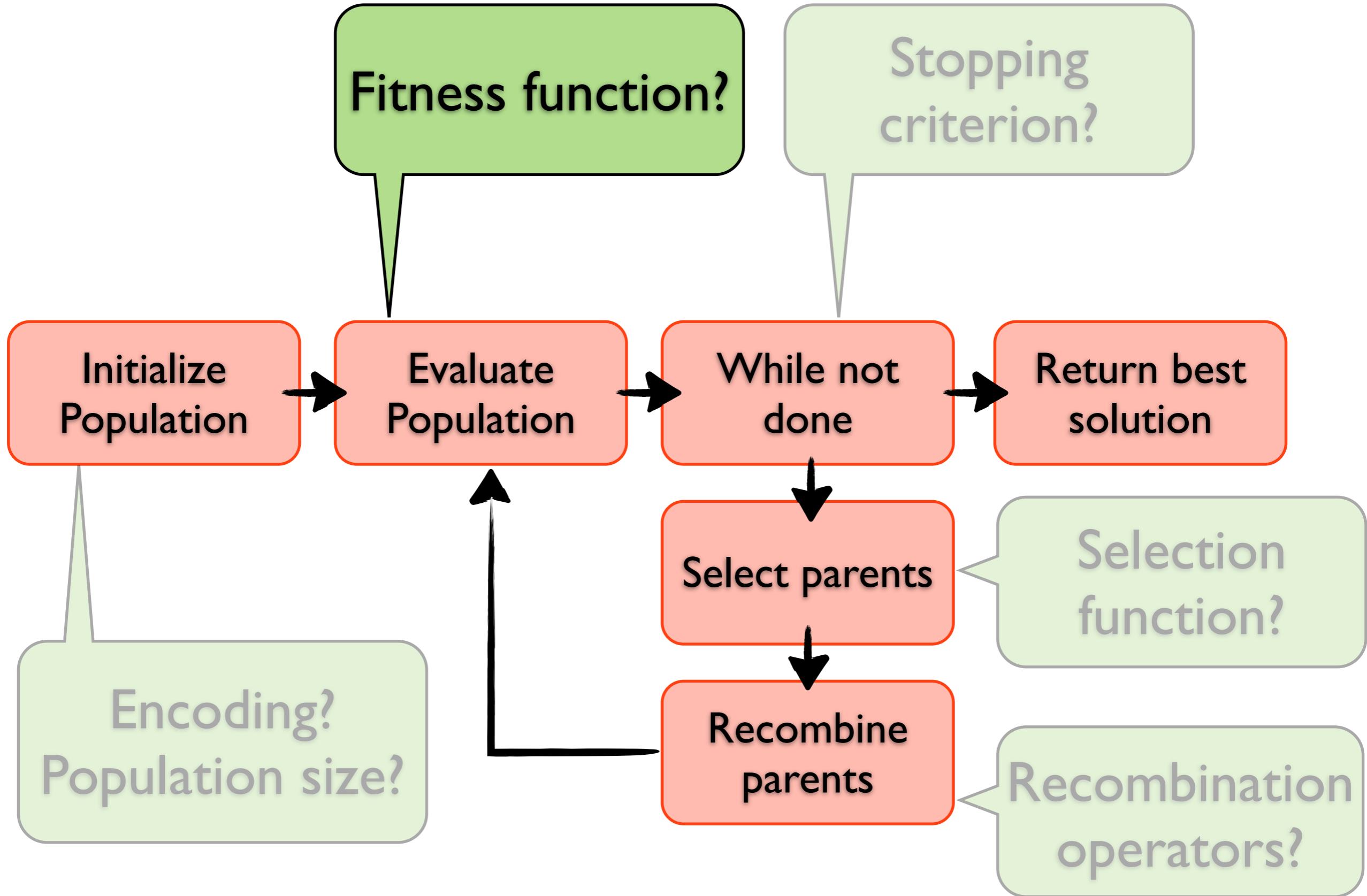


# Initial Population

- Randomly chosen
- Existing solutions
- Coverage test suite for weaker criterion
- Manually generated test cases





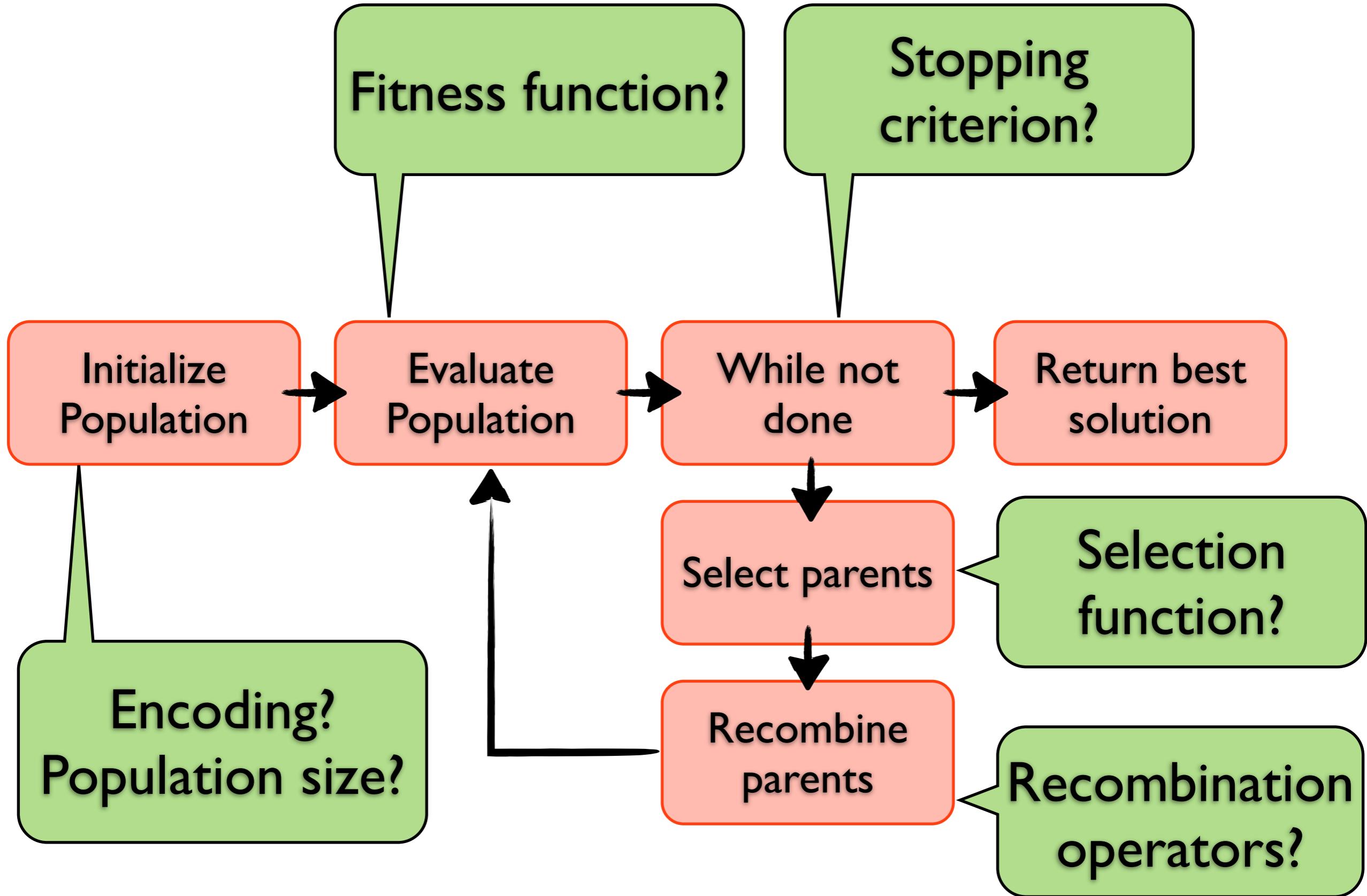


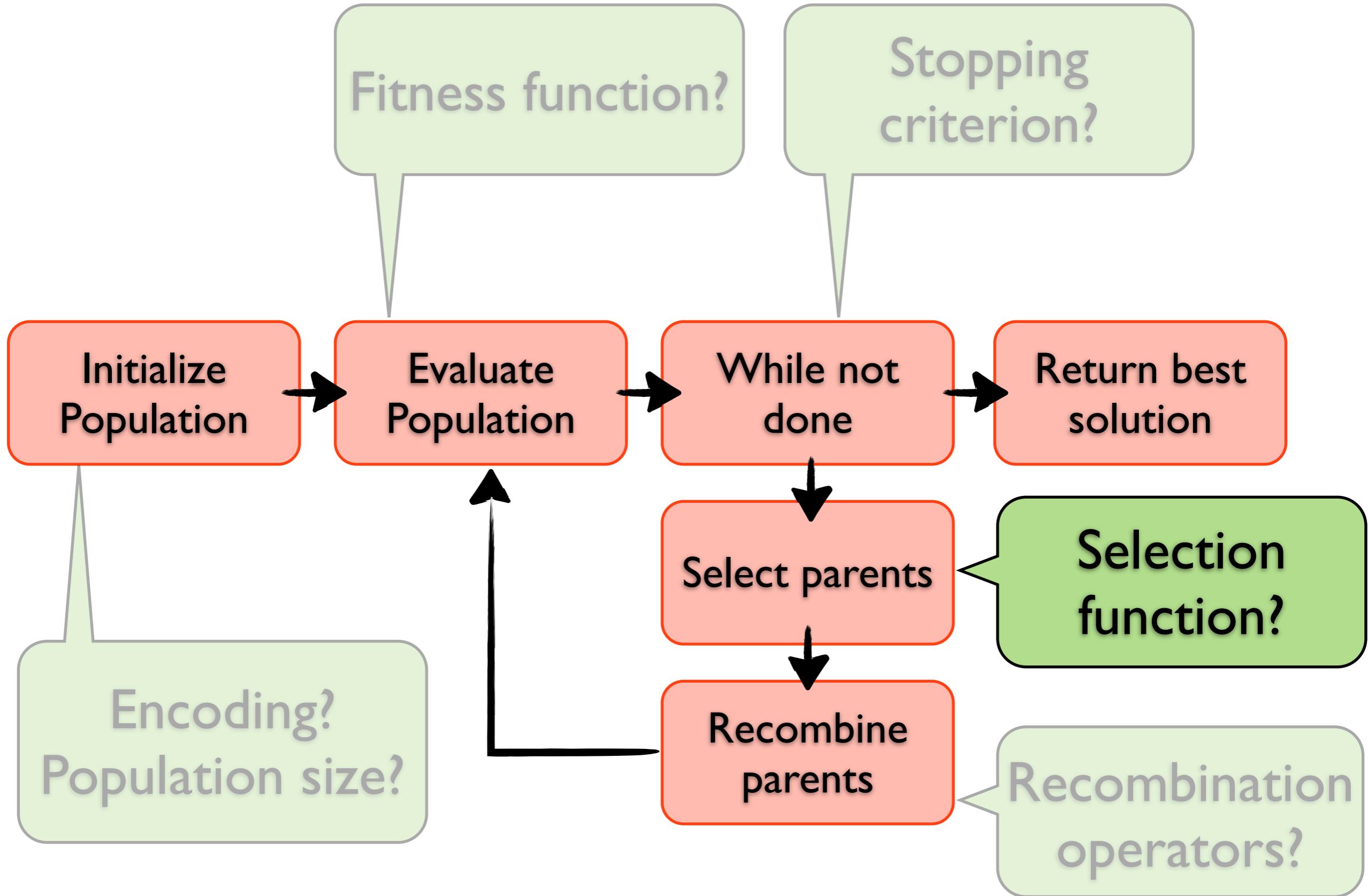
# Fitness

- How good is a candidate solution?
- Is solution A better than solution B?
- Fitness quantifies the “goodness” of a solution
- Fitness is problem specific
- Fitness function determines the search landscape

# Kin Compensation

- Diversity in the population is key to successful search
- Kin compensation:
  - Fitness penalty if identical individual already exists



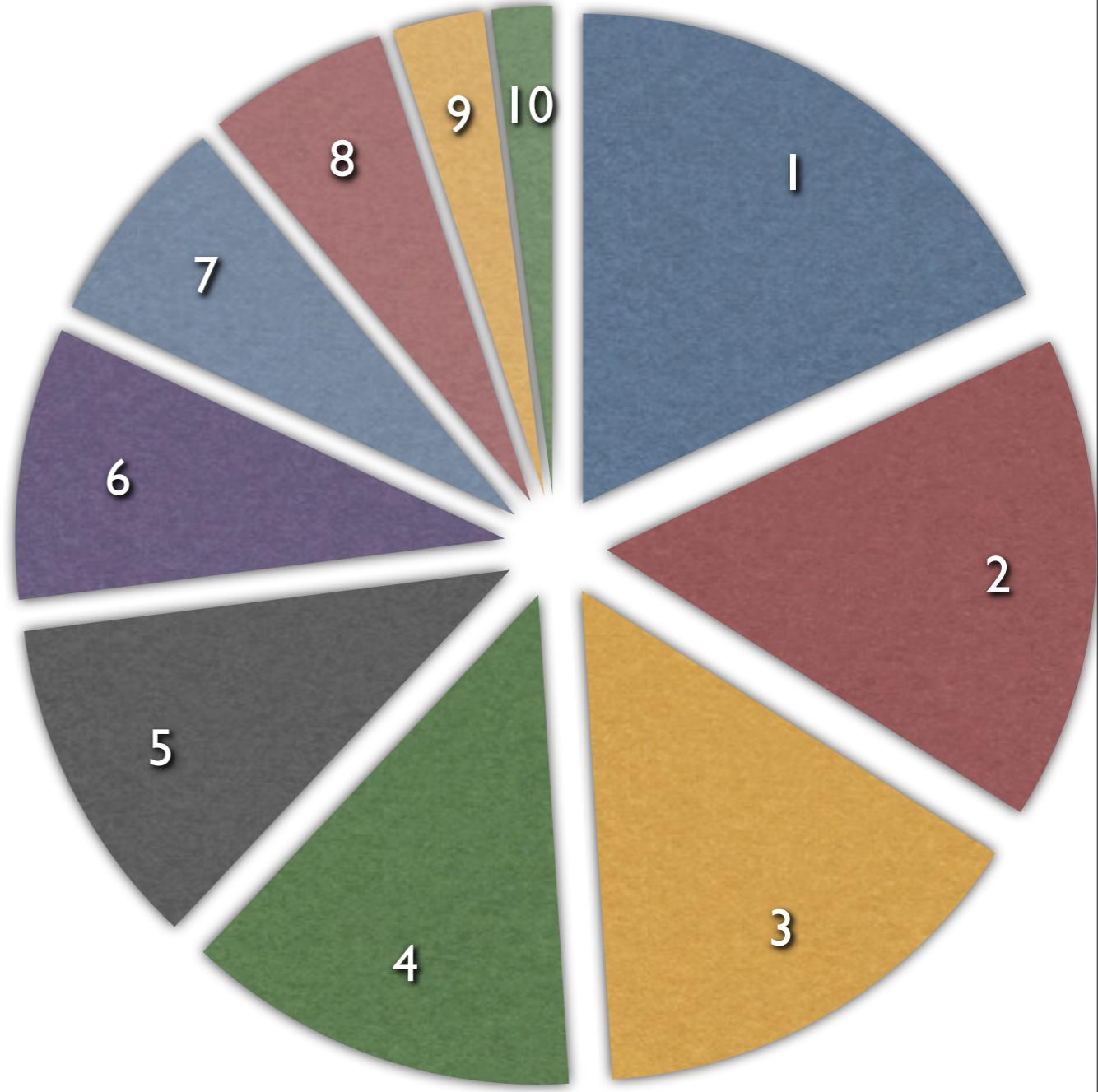


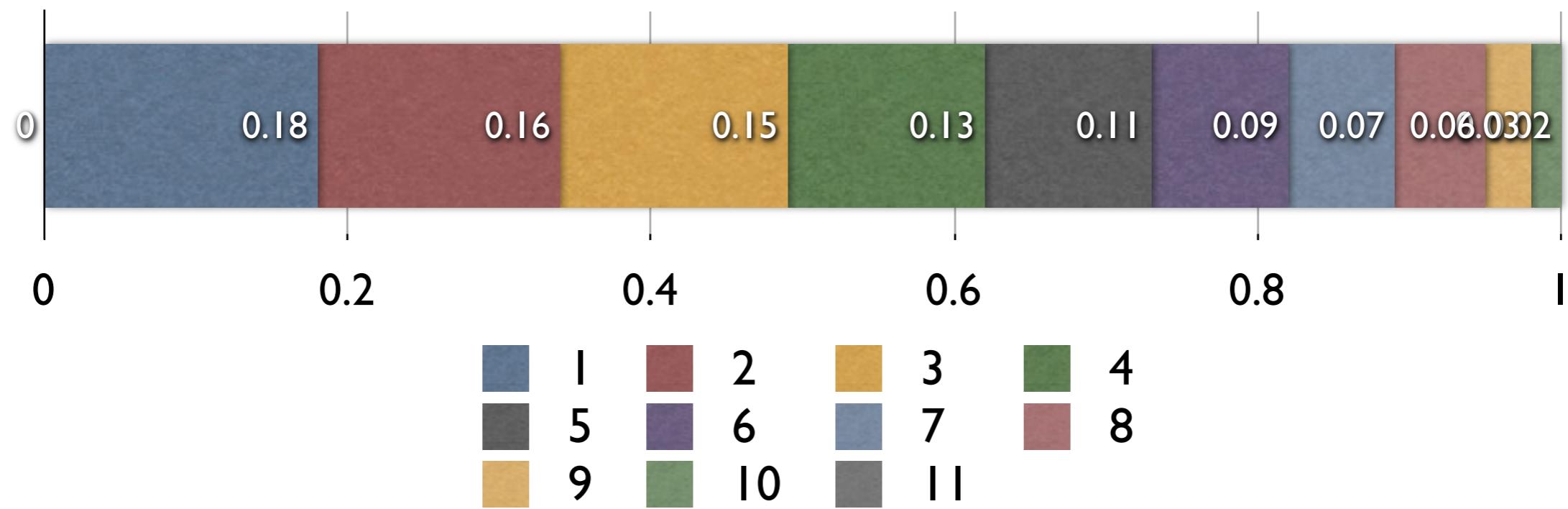
# Fitness Proportionate Selection

- A.k.a. Roulette wheel selection
- Probability of choosing an individual is proportional to its fitness

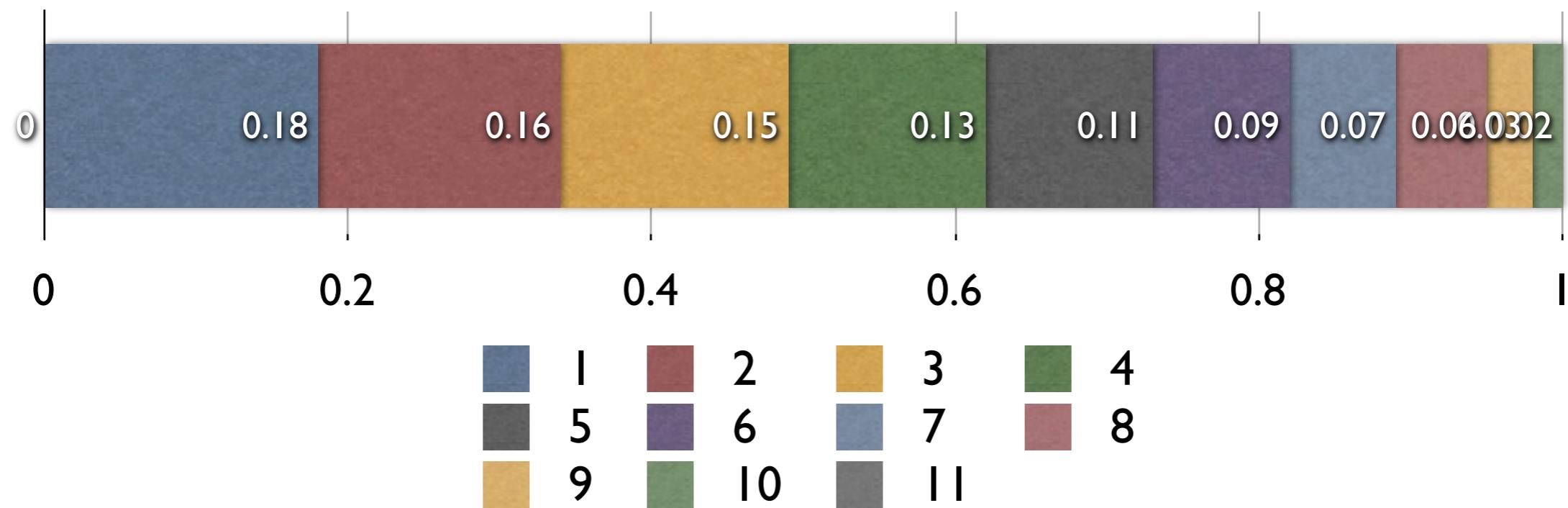
Individual	Fitness
1	2.0
2	1.8
3	1.6
4	1.4
5	1.2
6	1.0
7	0.8
8	0.6
9	0.4
10	0.2
11	0.0

Individual	Fitness
1	2.0
2	1.8
3	1.6
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7	0.8
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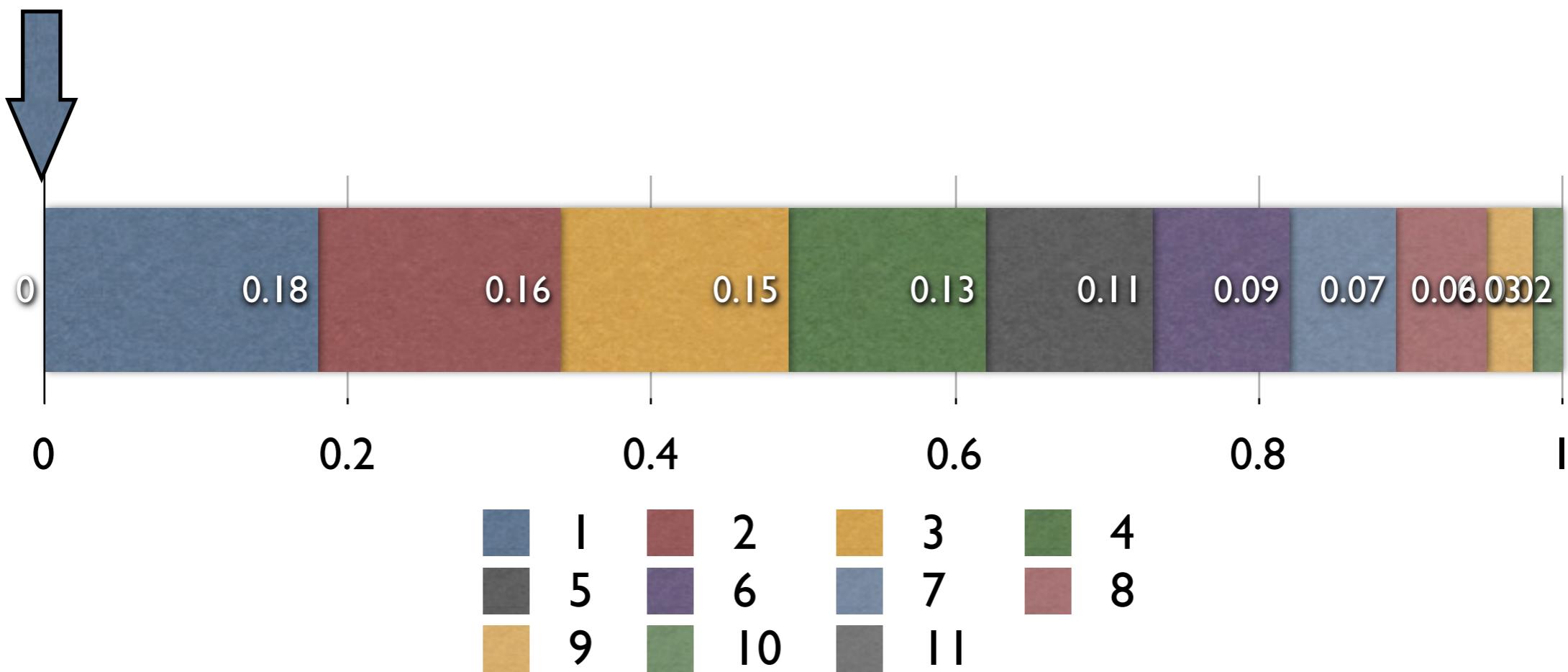




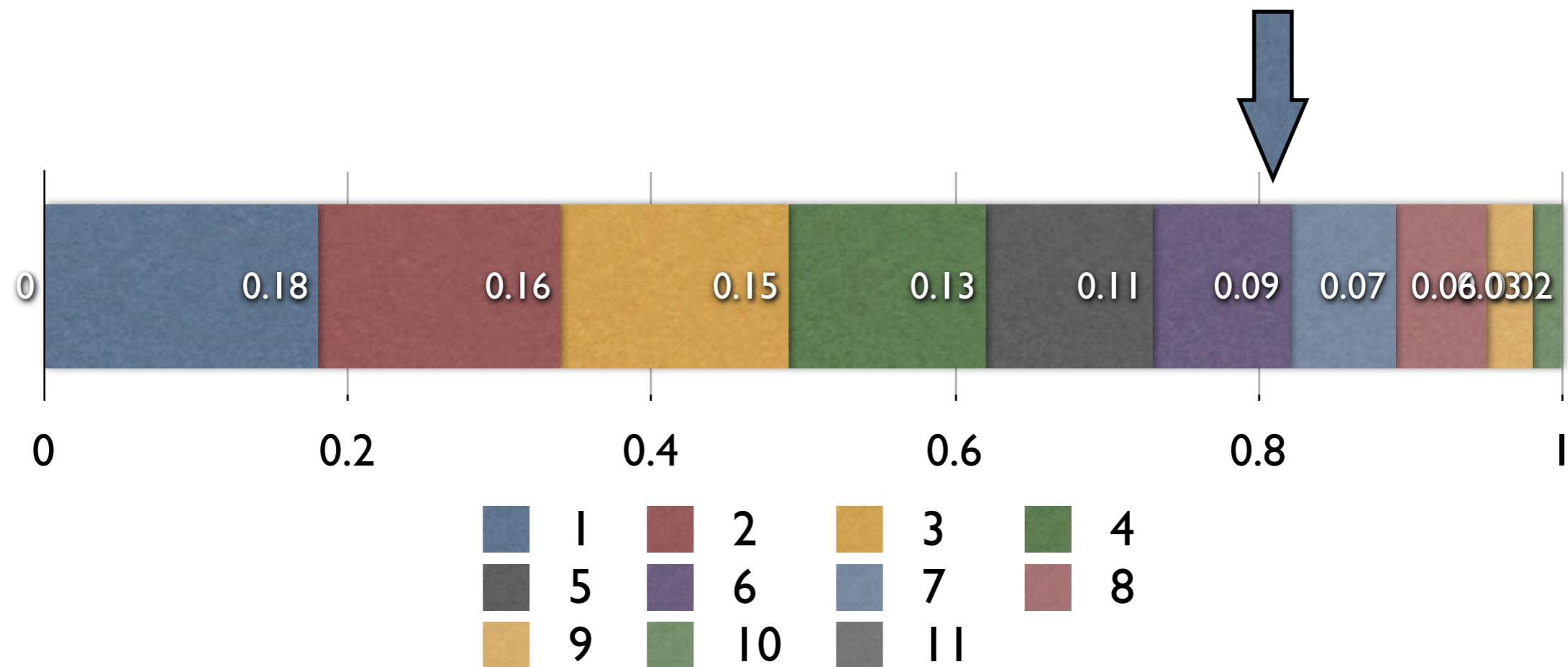
- Chosen value: 0,8 |



- Chosen value: 0,8 |

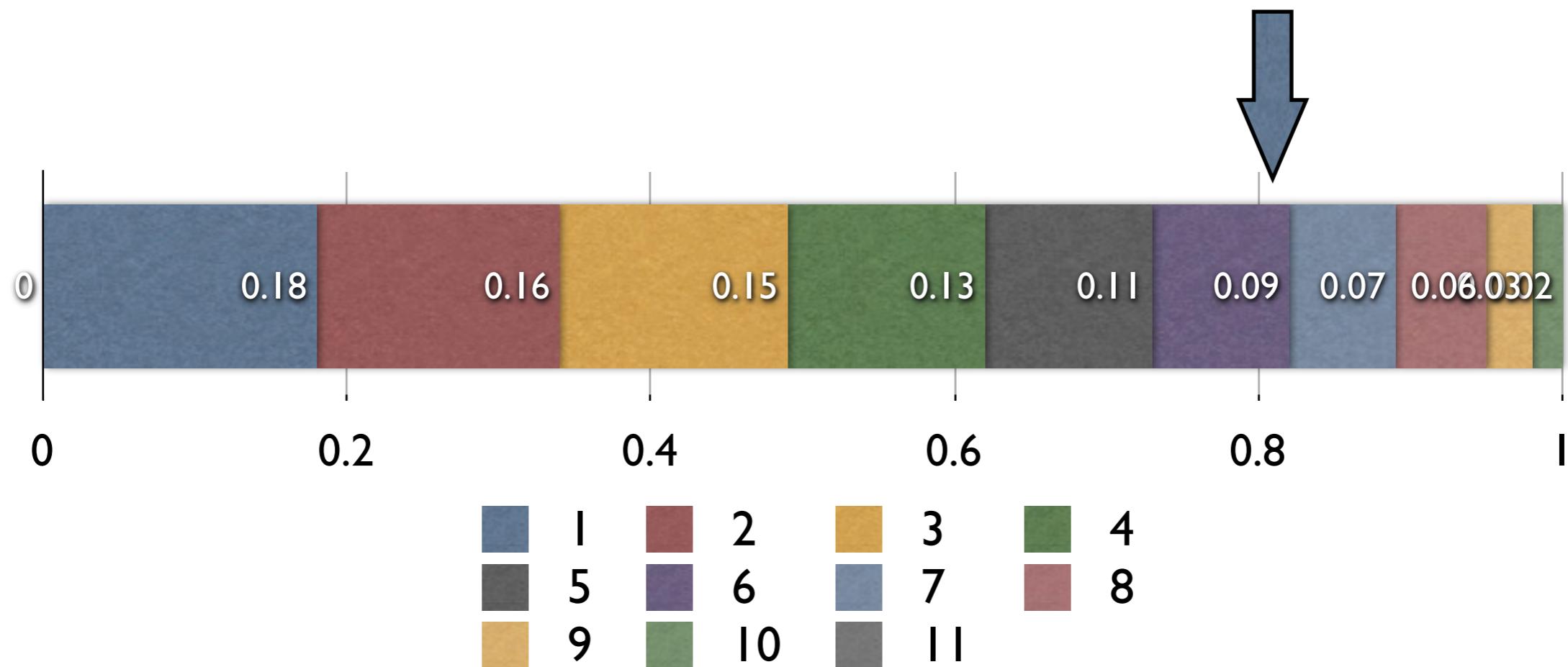


- Chosen value: 0,8 |



- Chosen value: 0,8 |

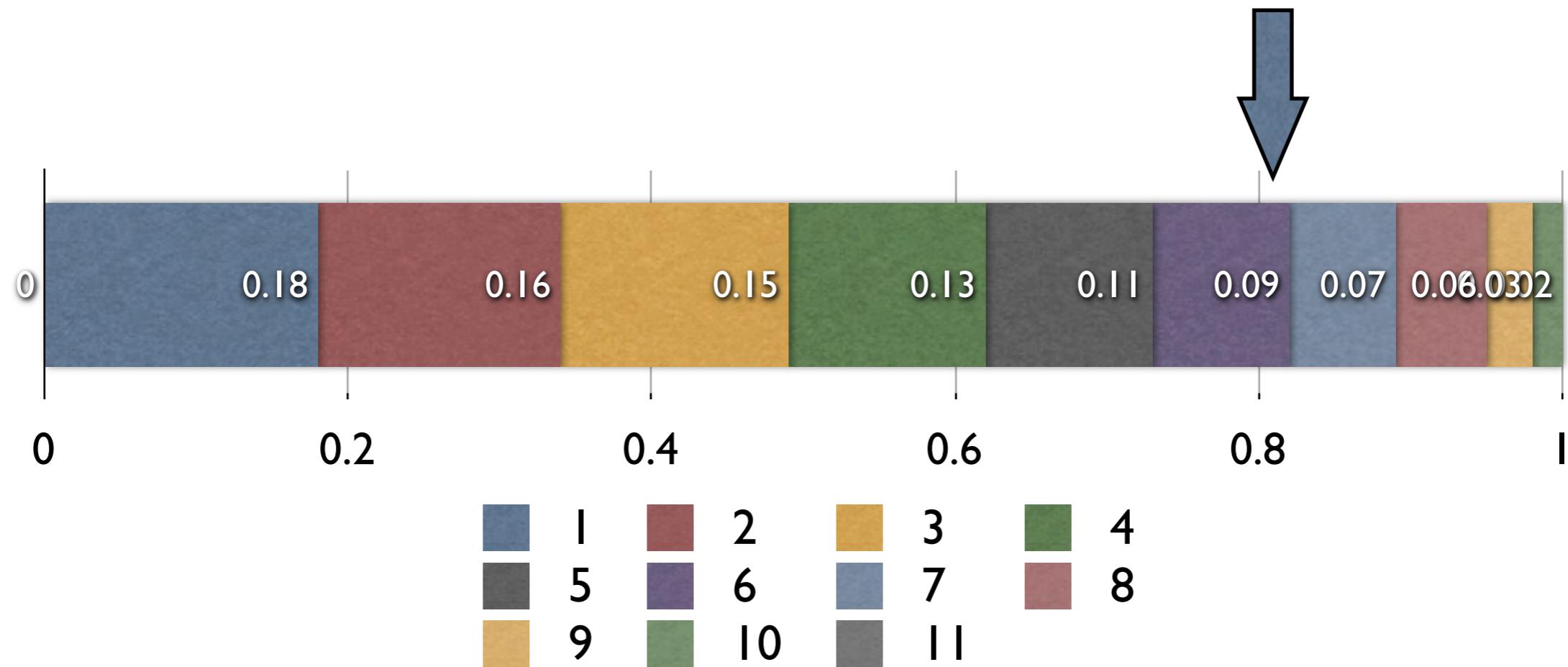
6



- Chosen value: 0,8 |

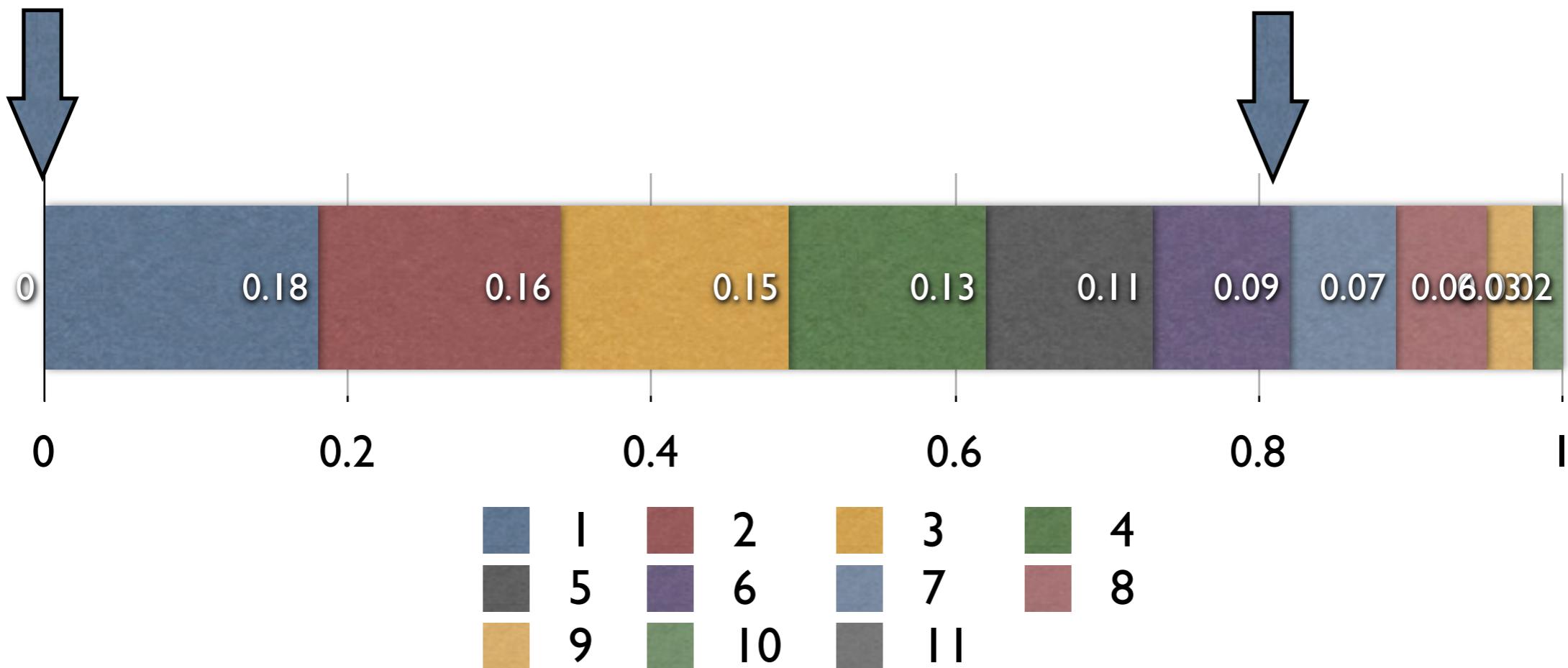
6

- Chosen value: 0,32



6

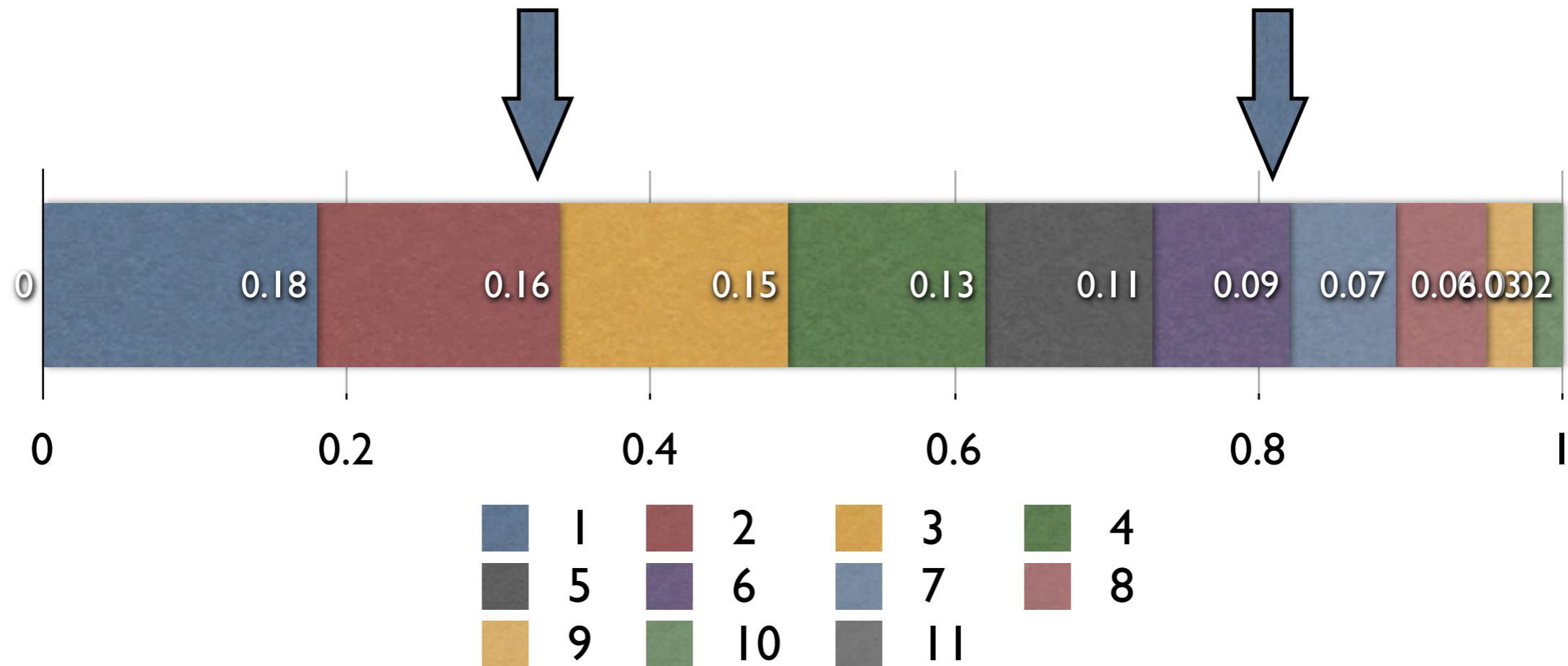
- Chosen value: 0,8 |
- Chosen value: 0,32



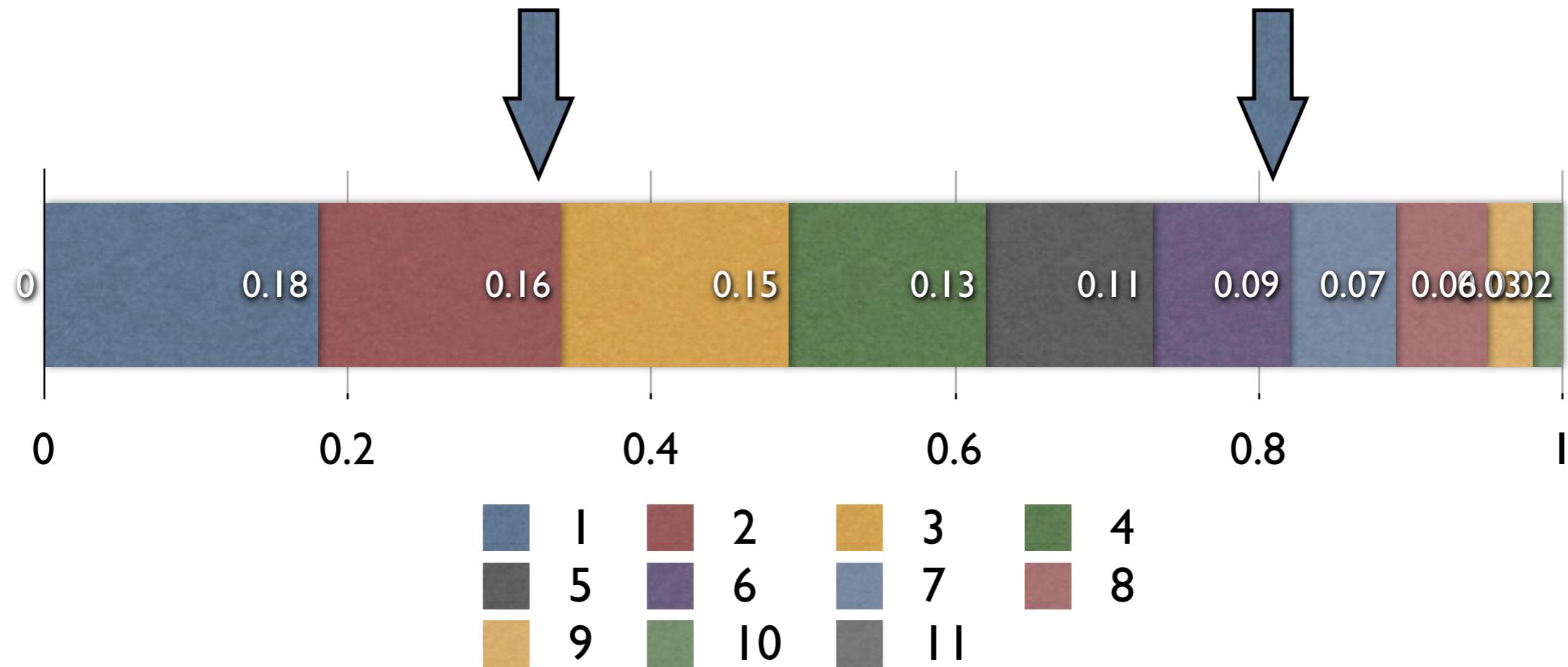
6

- Chosen value: 0,8 |

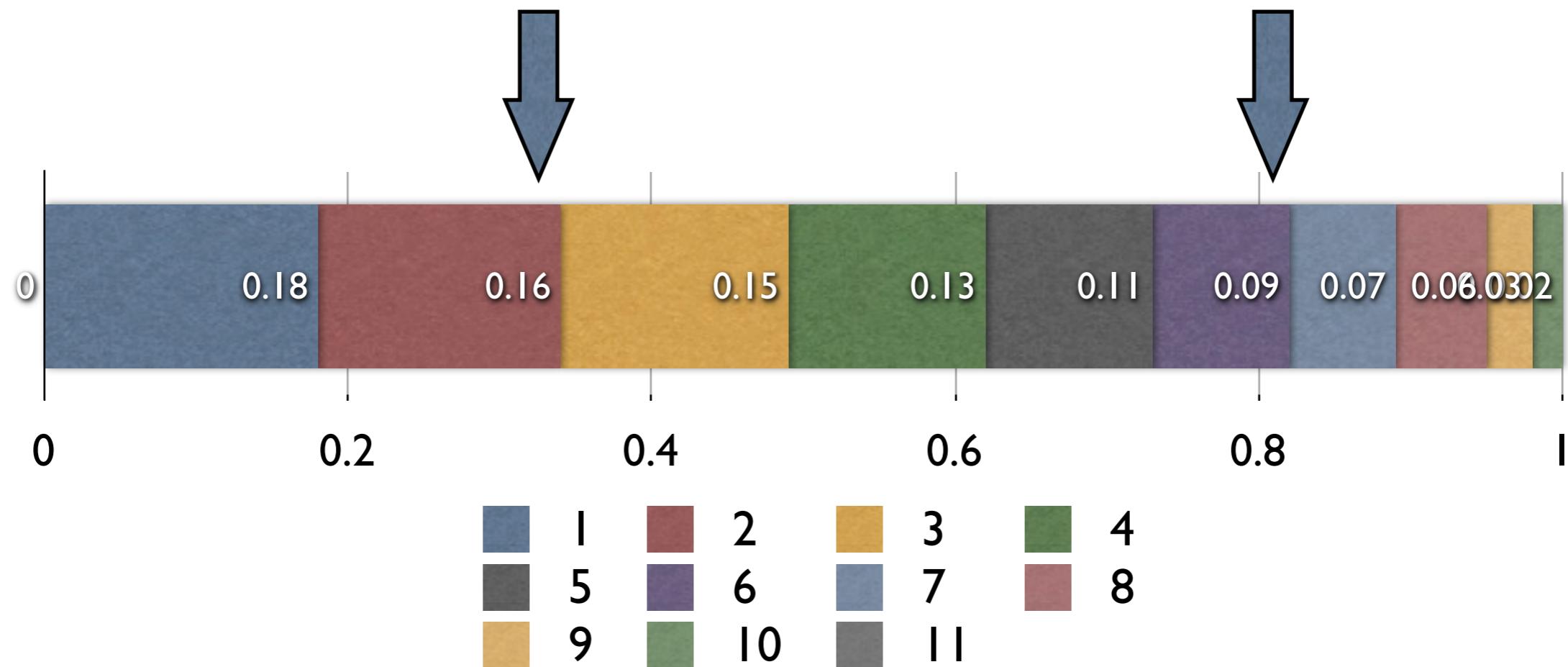
- Chosen value: 0,32



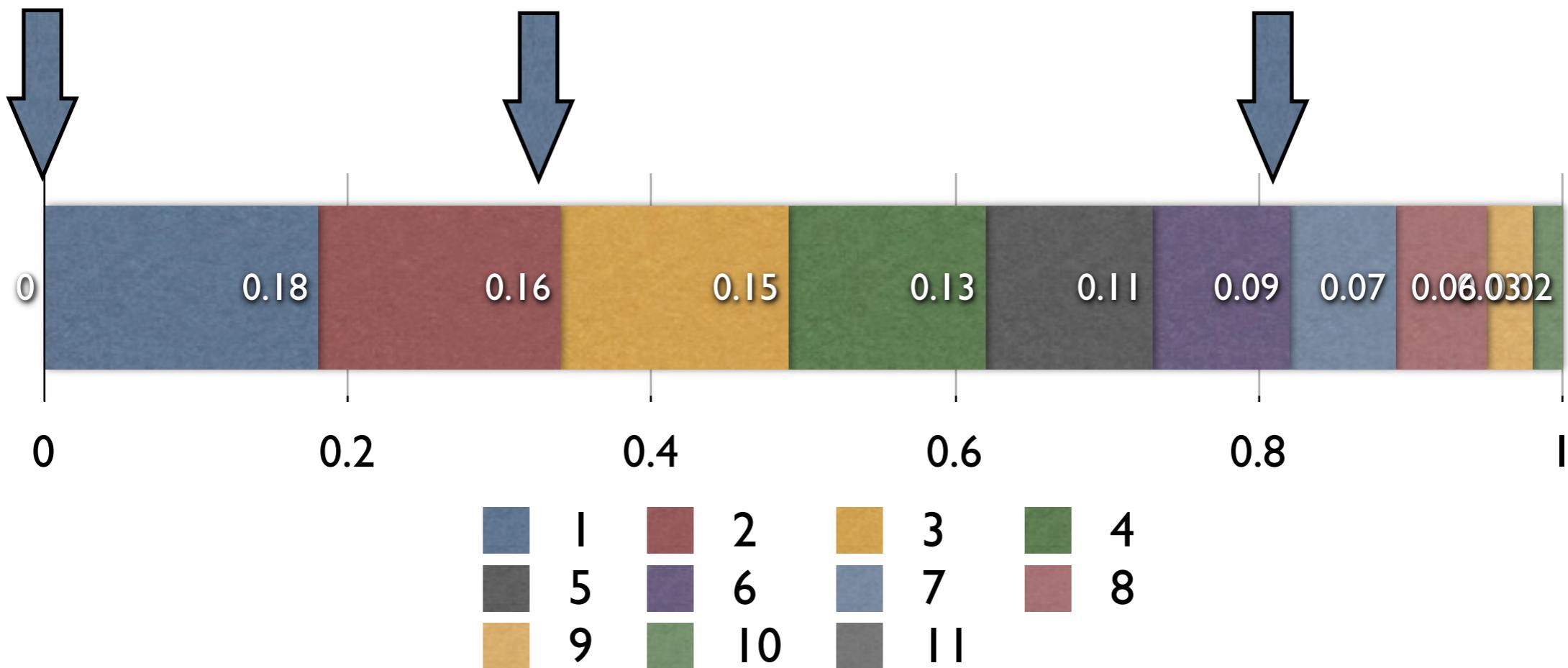
- Chosen value: 0,81
- Chosen value: 0,32



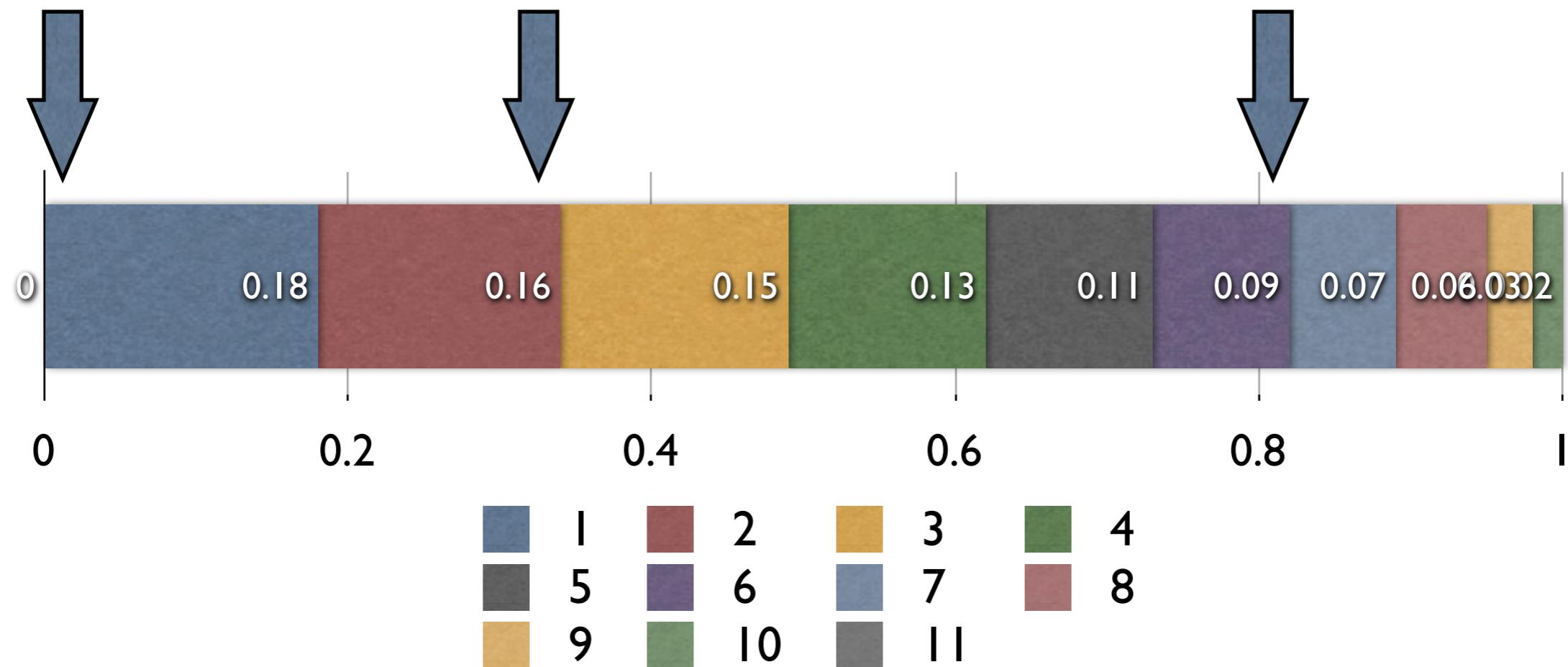
- Chosen value: 0,8 |
- Chosen value: 0,32 |
- Chosen value: 0,0 |



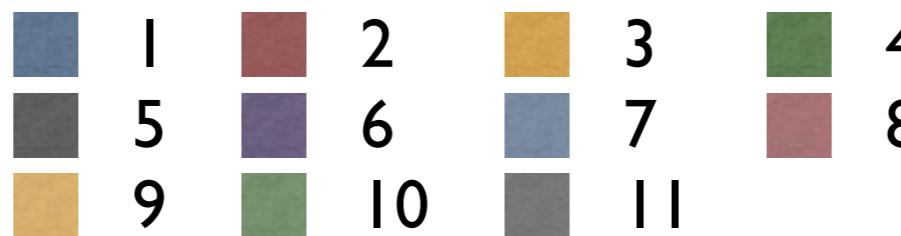
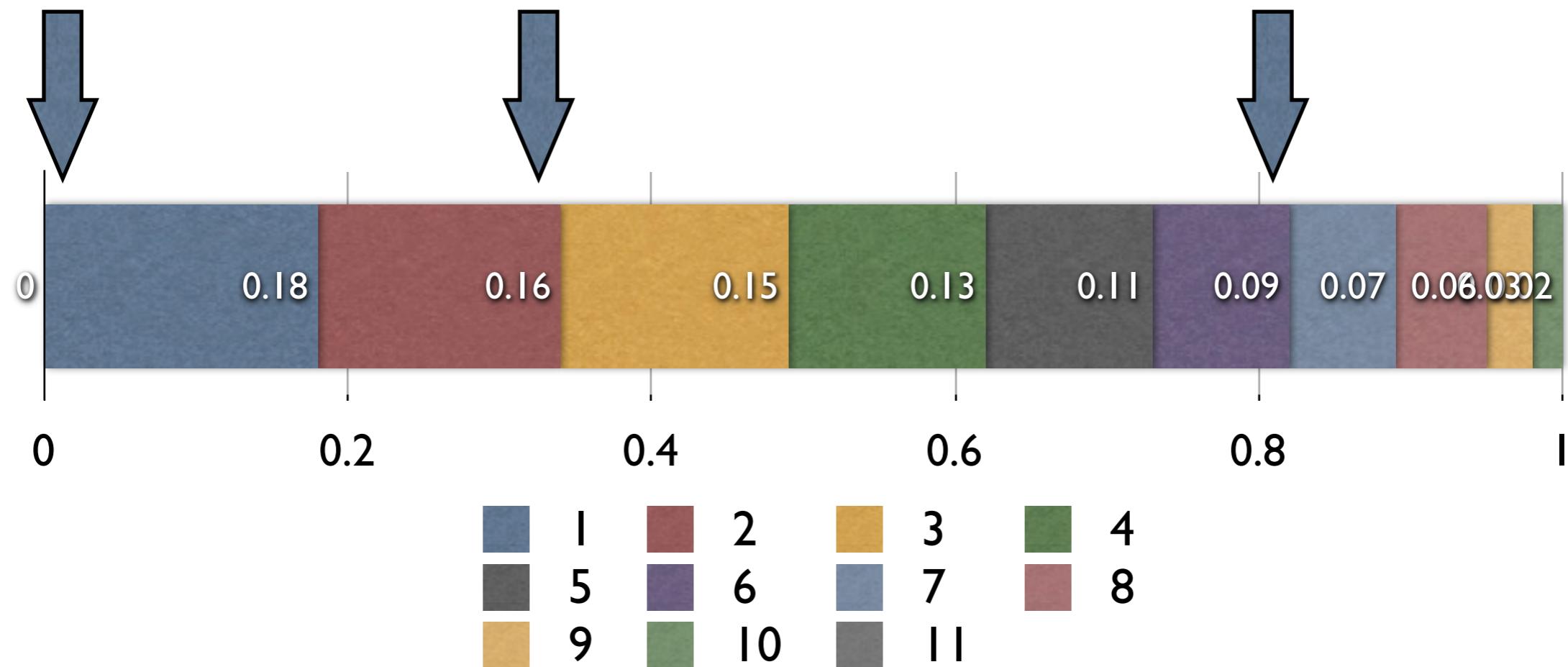
- Chosen value: 0,81
- Chosen value: 0,32
- Chosen value: 0,01

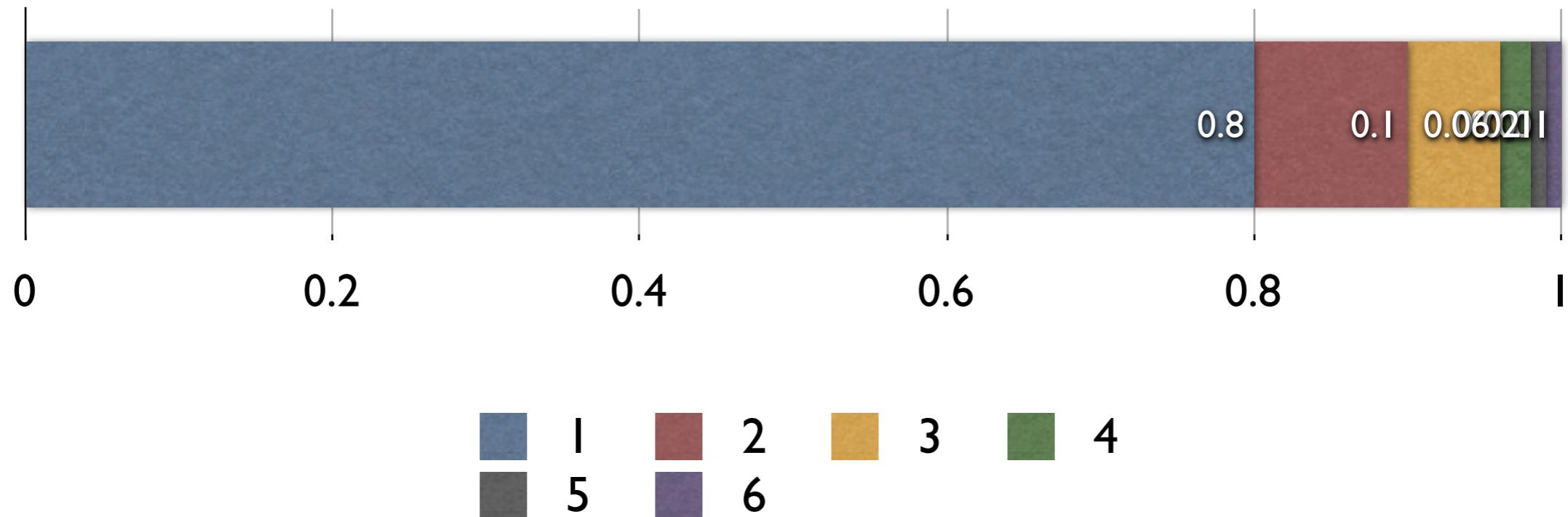


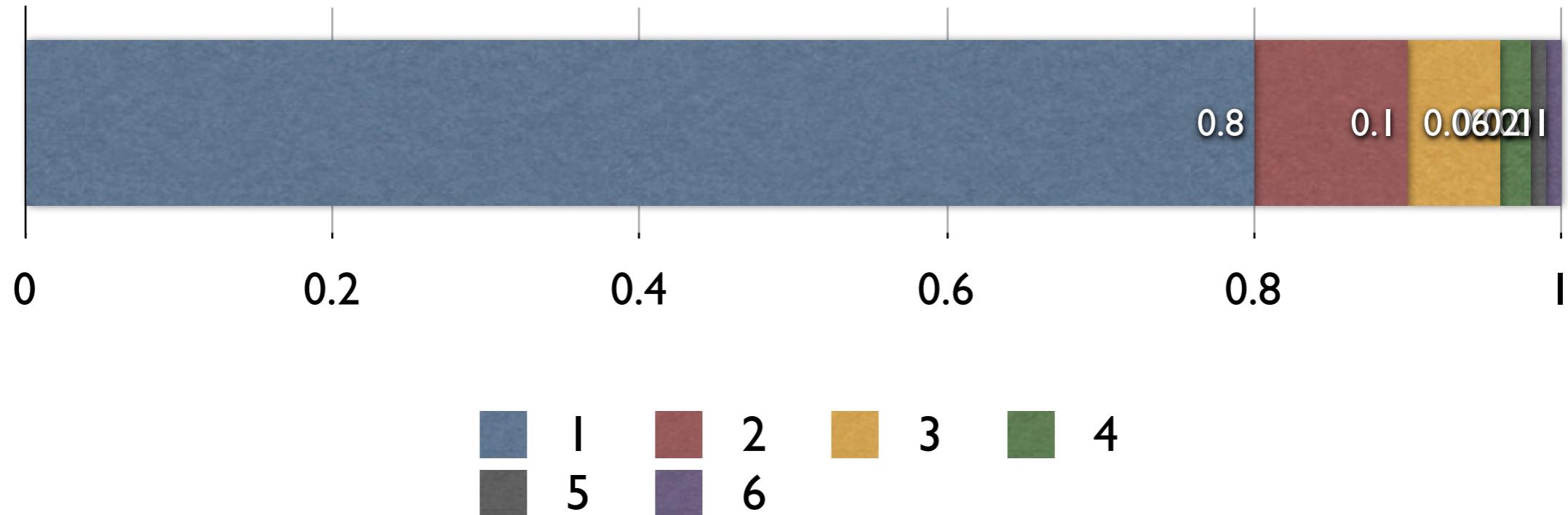
- Chosen value: 0,81
- Chosen value: 0,32
- Chosen value: 0,01



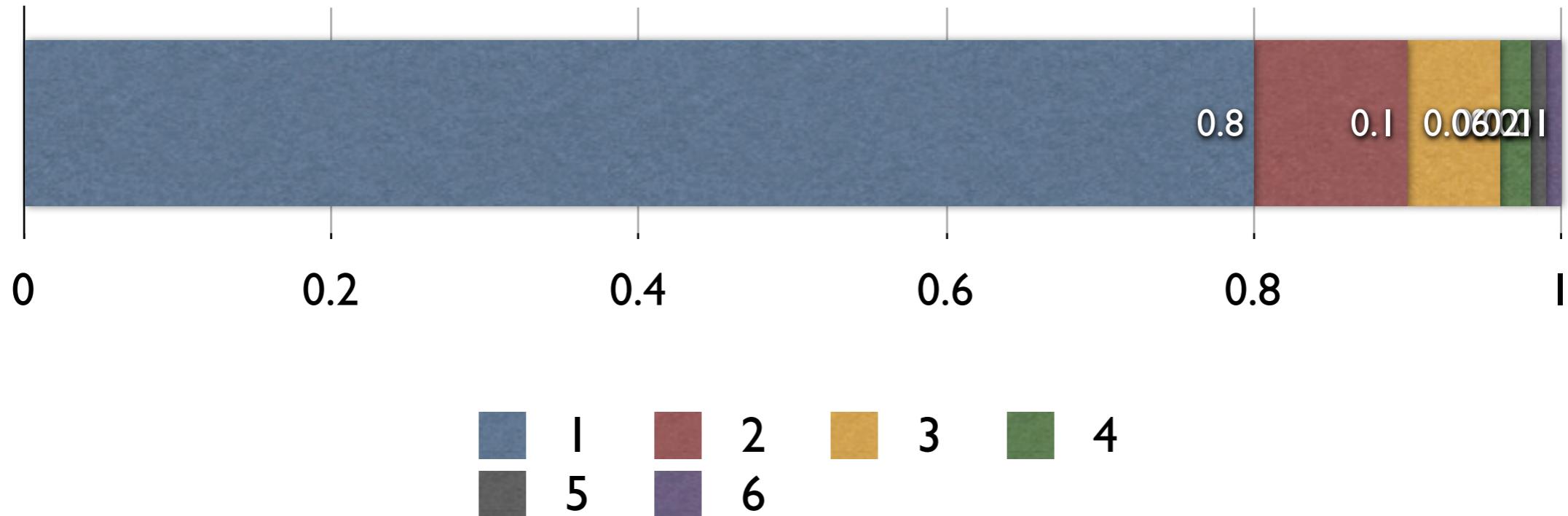
- Chosen value: 0,81
- Chosen value: 0,32
- Chosen value: 0,01







- Individual 1 will dominate selection



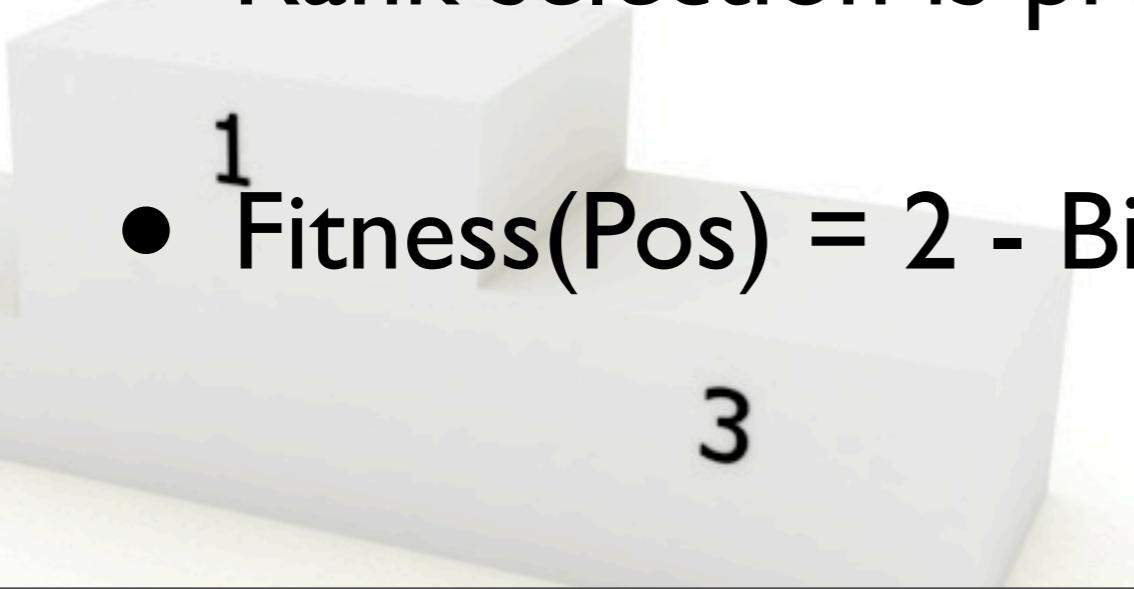
- Individual I will dominate selection
- **Don't use roulette wheel selection!**

# Problems with Selection

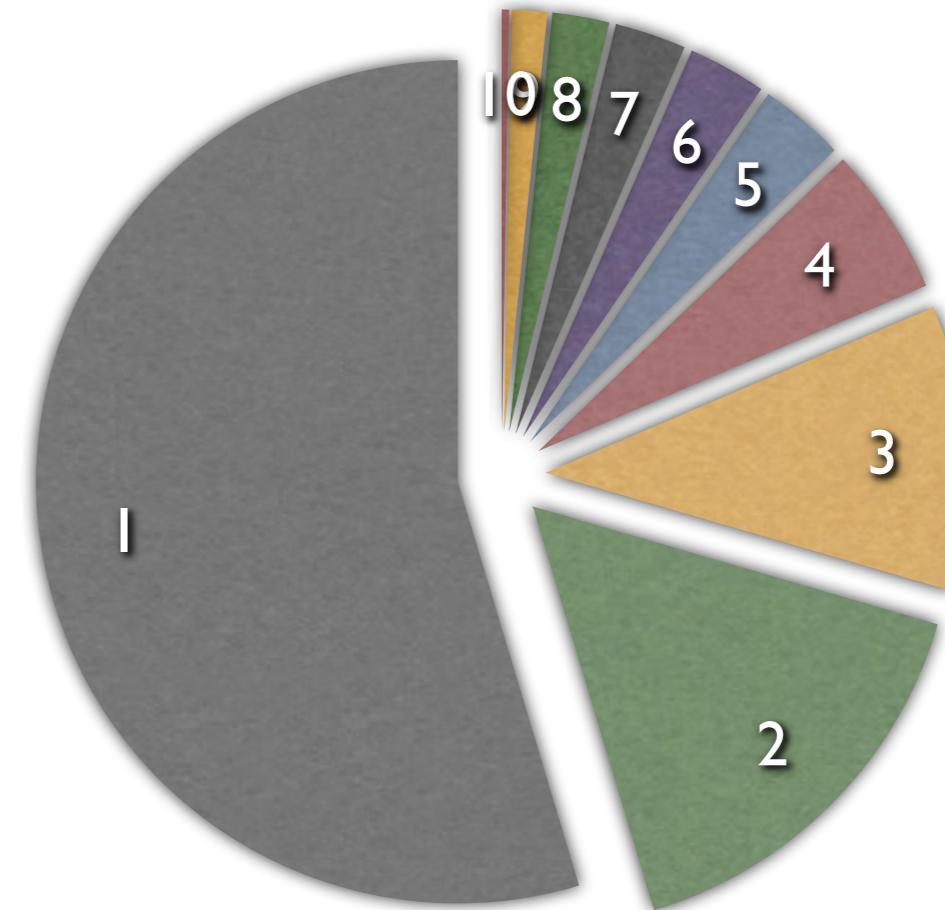
- Selective pressure:  
The higher, the more likely the fittest are chosen
- Stagnation:  
Selective pressure too small
- Premature convergence:  
Selective pressure too high

# Rank Selection

- Rank individuals according to their fitness
- No difference whether fittest candidate is ten times fitter than the next fittest or 0.001% fitter
- Rank selection is preferable in practice


$$\text{Fitness}(\text{Pos}) = 2 - \text{Bias} + 2 * (\text{Bias} - 1) * \frac{\text{Pos} - 1}{\text{Num} - 1}$$

Individual	Fitness
1	2.0
2	0.58
3	0.4
4	0.21
5	0.12
6	0.11
7	0.1
8	0.08
9	0.05
10	0.01
11	0.0



Individual	Fitness
1	2.0
2	0.58
3	0.4
4	0.21
5	0.12
6	0.11
7	0.1
8	0.08
9	0.05
10	0.01
11	0.0



# Tournament Selection

- $N$  = Tournament size
- Select  $N$  individuals randomly
- Best of the  $N$  individuals is selected
- Tournament size defines selective pressure
- A worse individual can win with a given probability



Individual	Fitness
1	2.0
2	1.8
3	1.6
4	1.4
5	1.2
6	1.0
7	0.8
8	0.6
9	0.4
10	0.2
11	0.0

## Tournament size: 4

Individual	Fitness
1	2.0
2	1.8
3	1.6
4	1.4
5	1.2
6	1.0
7	0.8
8	0.6
9	0.4
10	0.2
11	0.0

## Tournament size: 4

4  
6  
2  
8

Individual	Fitness
1	2.0
2	1.8
3	1.6
4	1.4
5	1.2
6	1.0
7	0.8
8	0.6
9	0.4
10	0.2
11	0.0

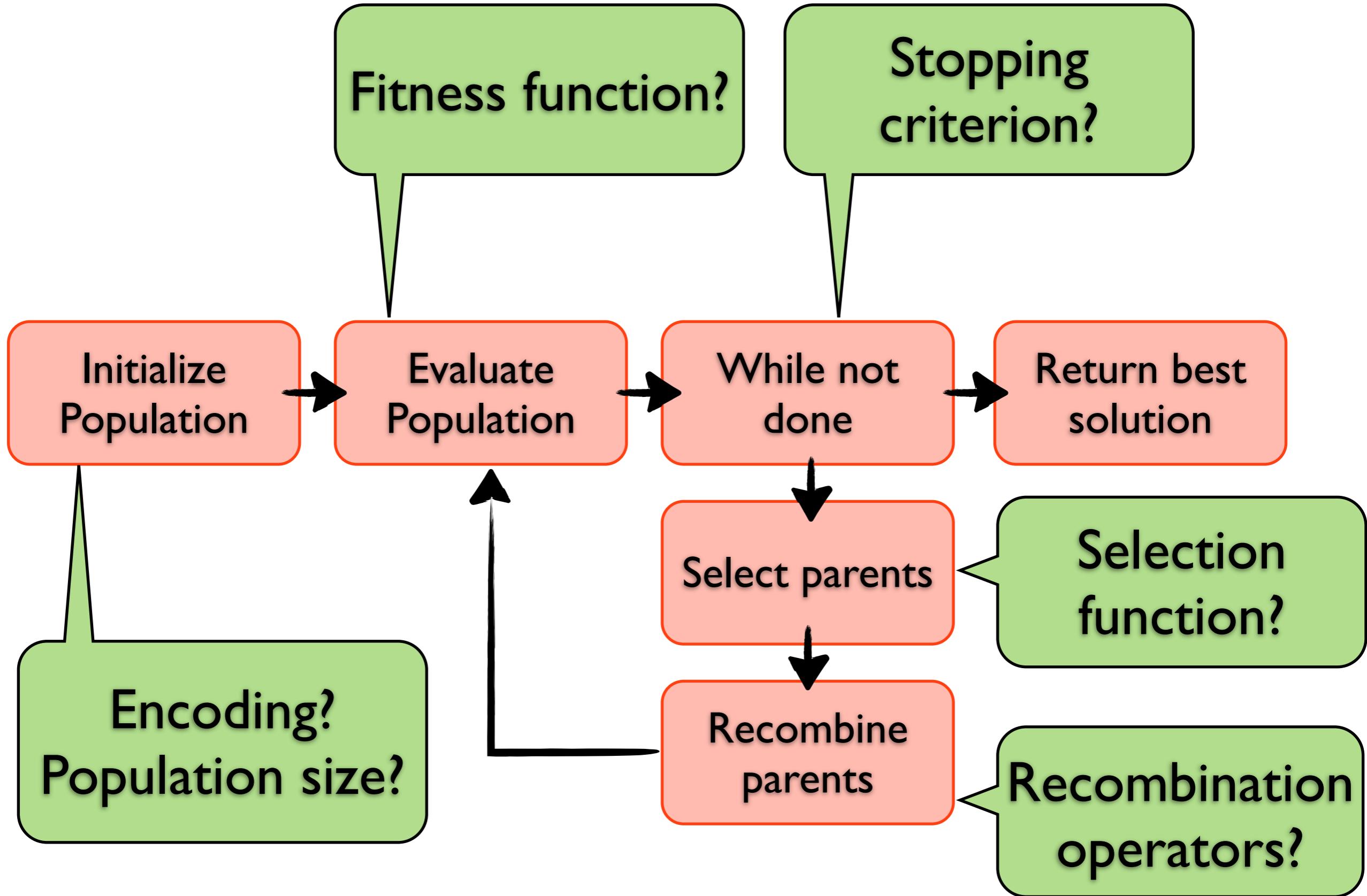
Individual	Fitness
1	2.0
2	1.8
3	1.6
4	1.4
5	1.2
6	1.0
7	0.8
8	0.6
9	0.4
10	0.2
11	0.0

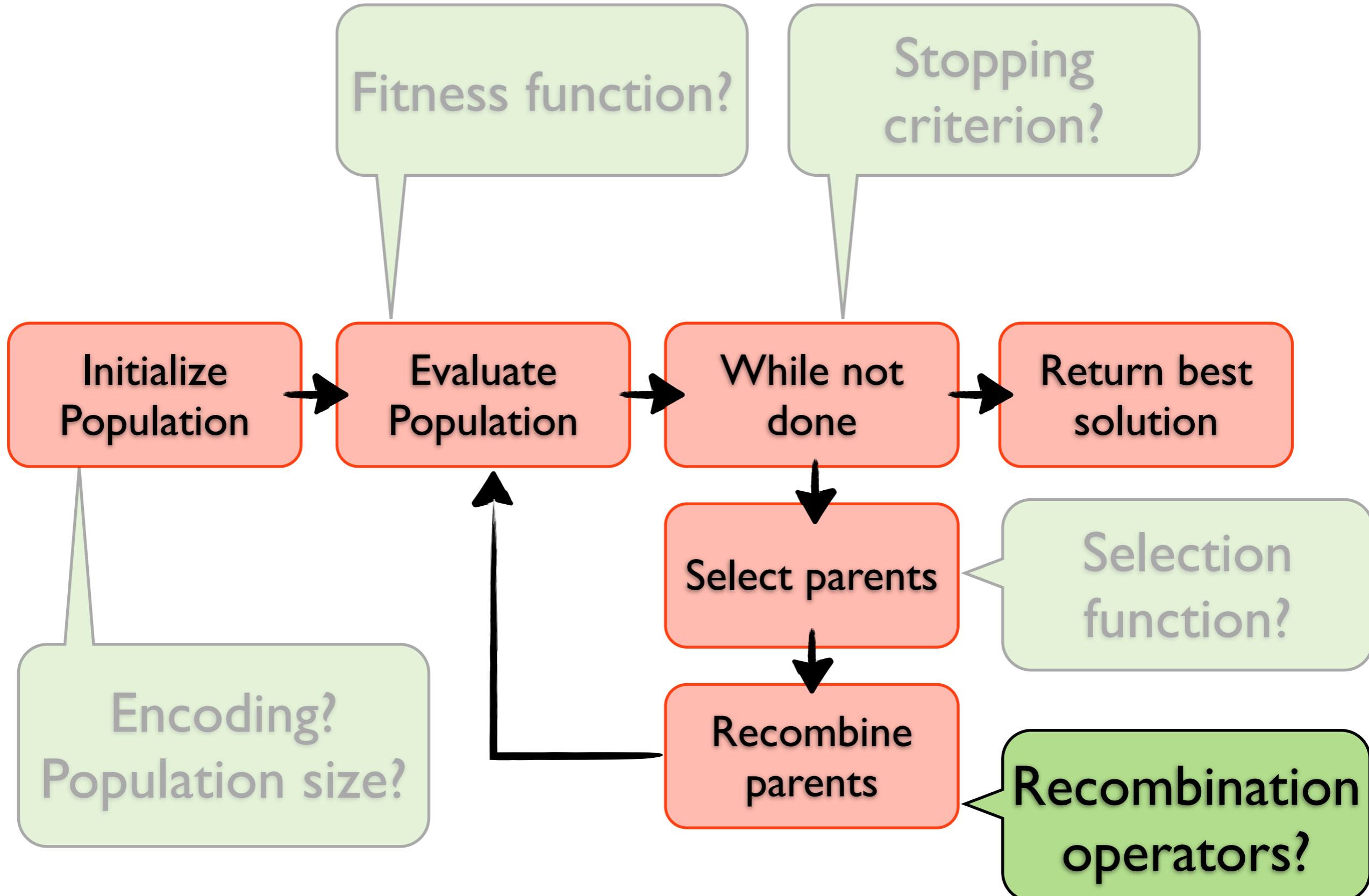
Tournament size: 4

4  
6  
2  
8

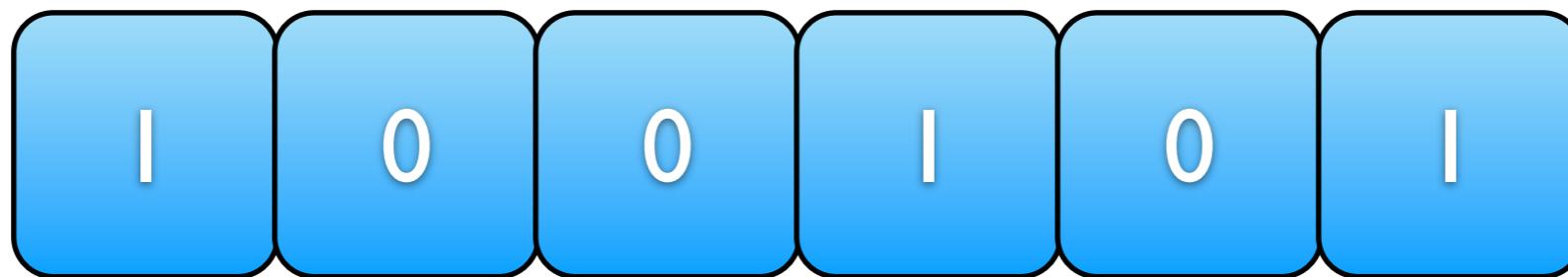
---

**2 (1,8)**

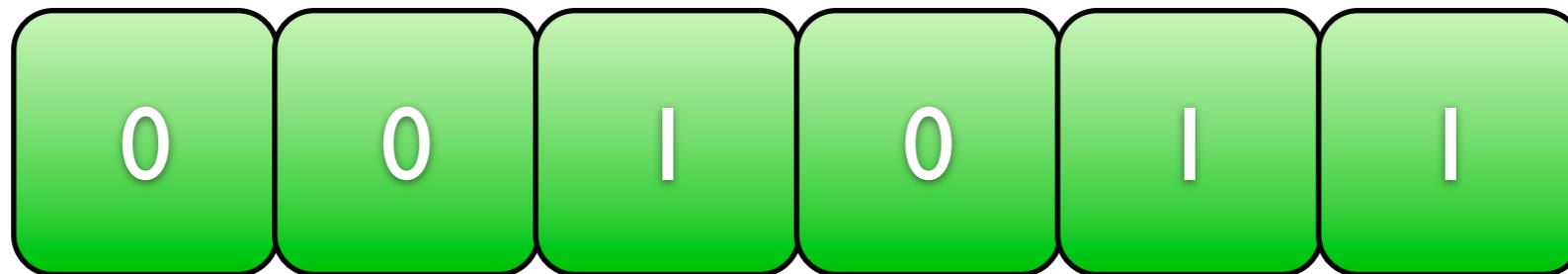




# Crossover

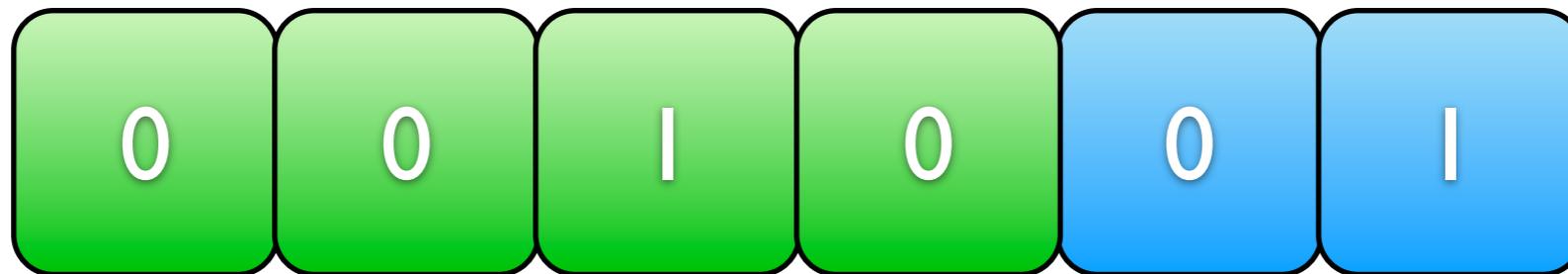
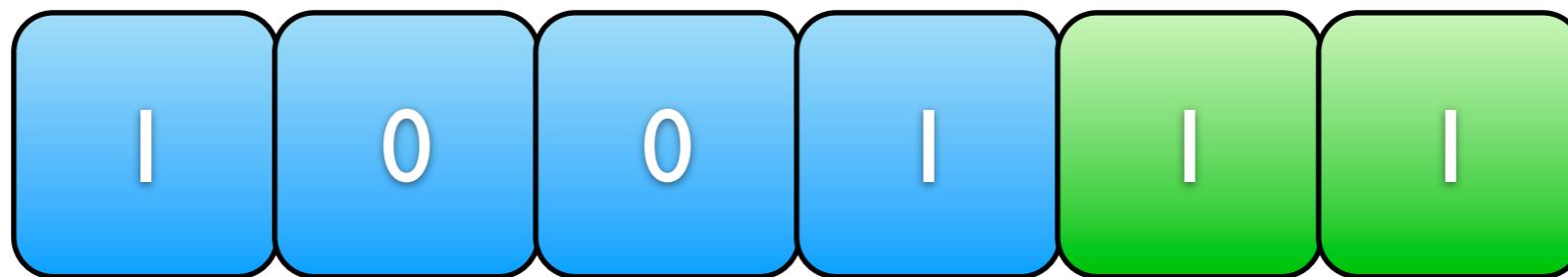


I 0 0 I 0 I



0 0 I 0 I I

# Crossover



# Crossover

- **Single point crossover**

Choose one point in parents and split/merge at that point

- **Two point crossover**

Choose two points in parents and exchange middle parts

- **Fixed vs. variable length**

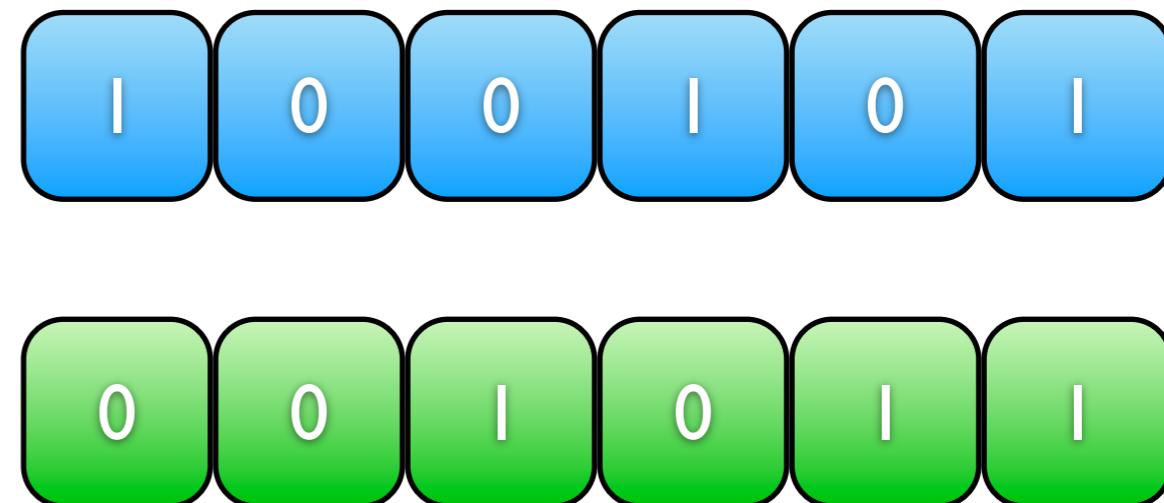
Same crossover point in both parents - constant length

- **Uniform crossover**

Genes are randomly chosen from either parent chromosome

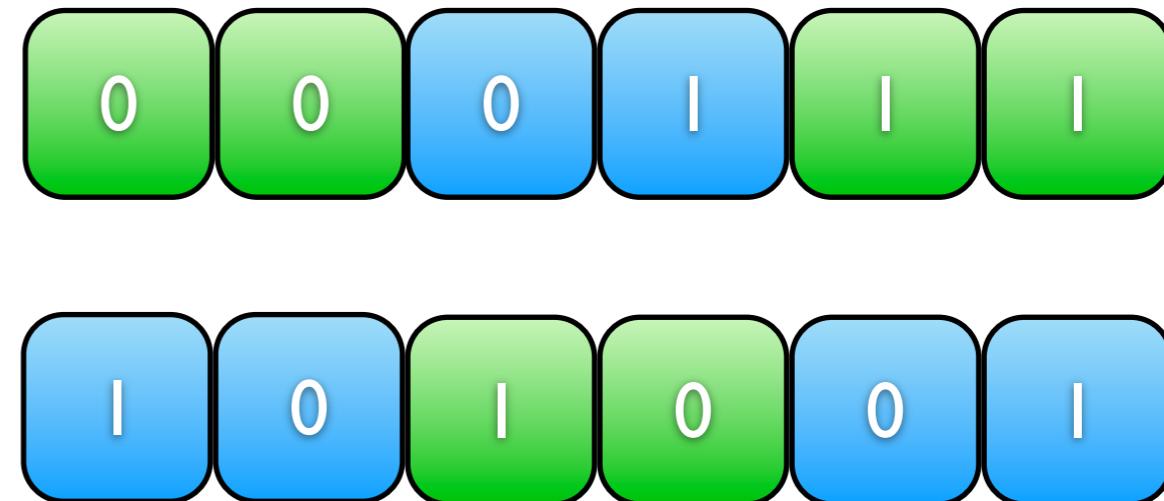
# Crossover

Two-point  
crossover



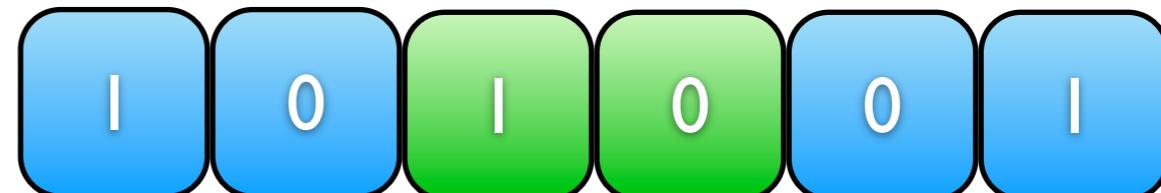
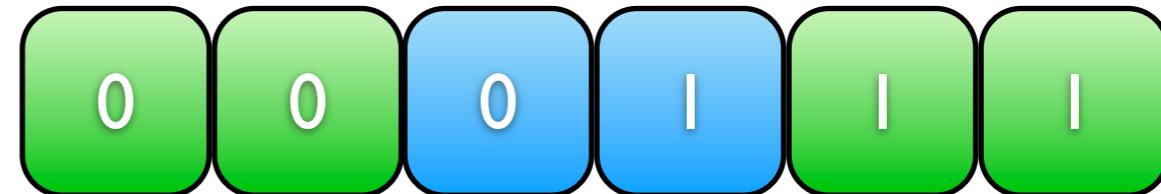
# Crossover

Two-point  
crossover

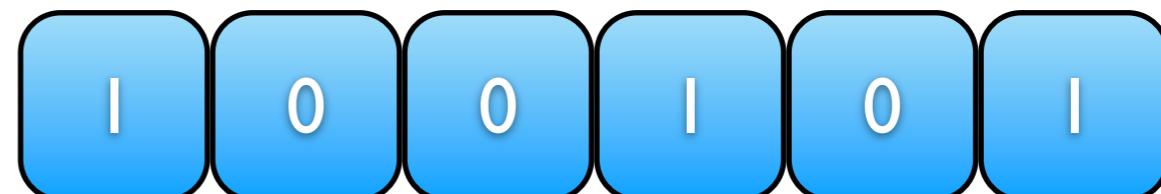


# Crossover

Two-point  
crossover

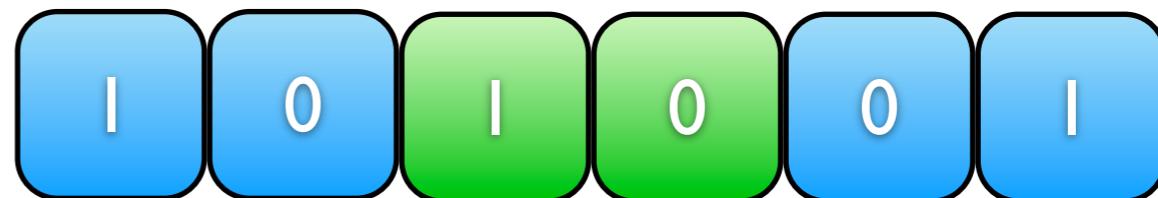
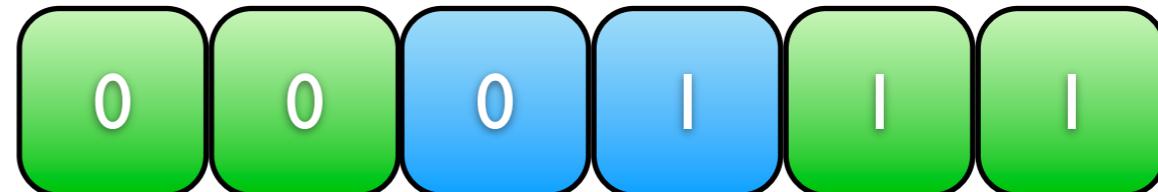


Variable length  
crossover

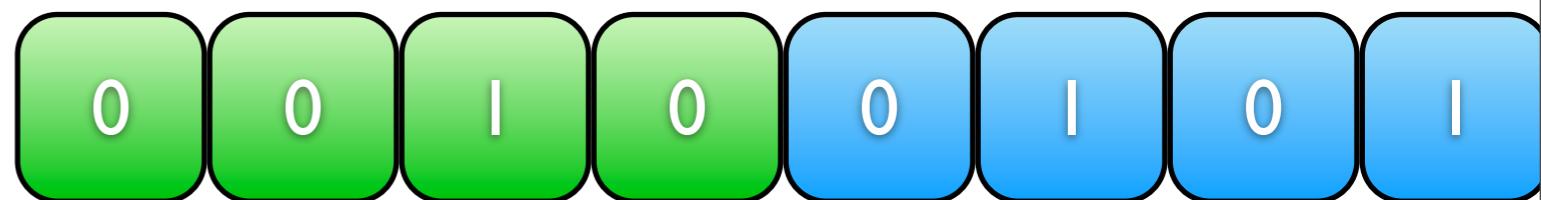
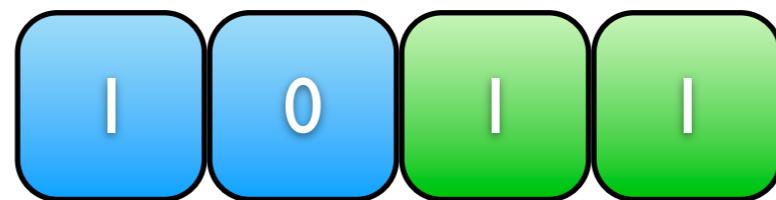


# Crossover

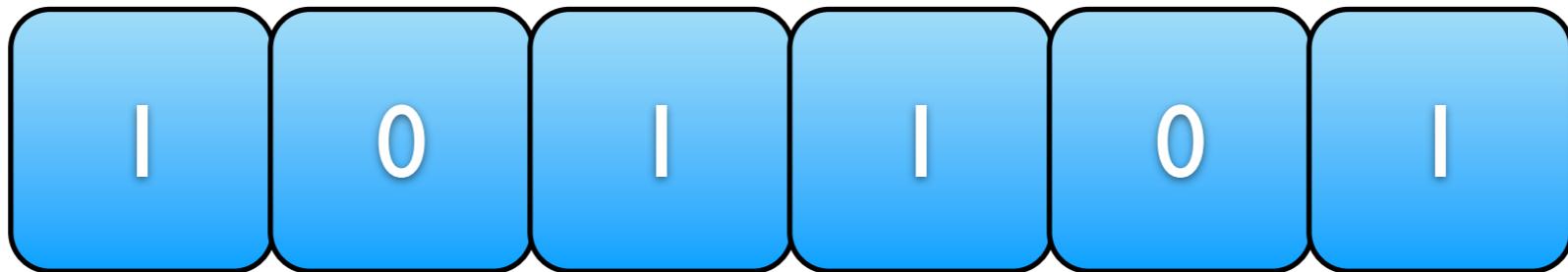
Two-point  
crossover



Variable length  
crossover

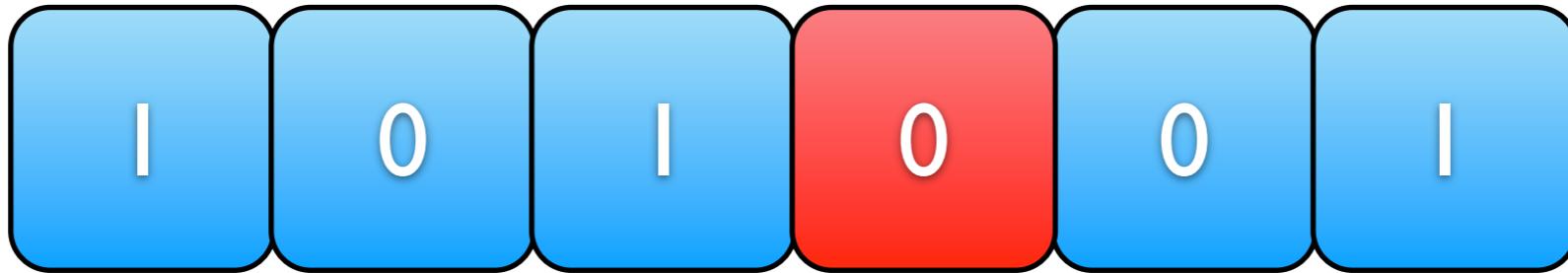


# Mutation



- Change of genes
- Mutation depends on genetic operators
- Binary representation - bit flip
- For binary encoding - use Gray code  
Guarantees Hamming distance of 1 for bit flips

# Mutation



- Change of genes
- Mutation depends on genetic operators
- Binary representation - bit flip
- For binary encoding - use Gray code  
Guarantees Hamming distance of 1 for bit flips

# Gray Code

# Gray Code

- Binary code: 0111

# Gray Code

- Binary code: 0111
- Right shift inverted code: 0011

# Gray Code

- Binary code: 0111
- Right shift inverted code: 0011
- 0111 xor 0011

# Gray Code

- Binary code: 0111
- Right shift inverted code: 0011
- 0111 xor 0011
- 0100

# Gray Code

- Binary code: 0111
- Right shift inverted code: 0011
- 0111 xor 0011
- 0100

0	000	000
1	001	001
2	010	011
3	011	010
4	100	110
5	101	111
6	110	101
7	111	100

# Determining Parameters

- Which cross over function?
- Which mutation operators?
- Which selection function?
- Selection bias?
- Tournament size?
- Encoding?
- How often does crossover occur?
- How often does mutation occur?
- How many generations?
- Population size?
- Elite size?

# No Free Lunch Theorem

- In the entire domain of search problem, all search algorithms perform equally on average
- Each algorithm makes assumptions on the underlying problem
- Choose / adapt algorithms to specific domain

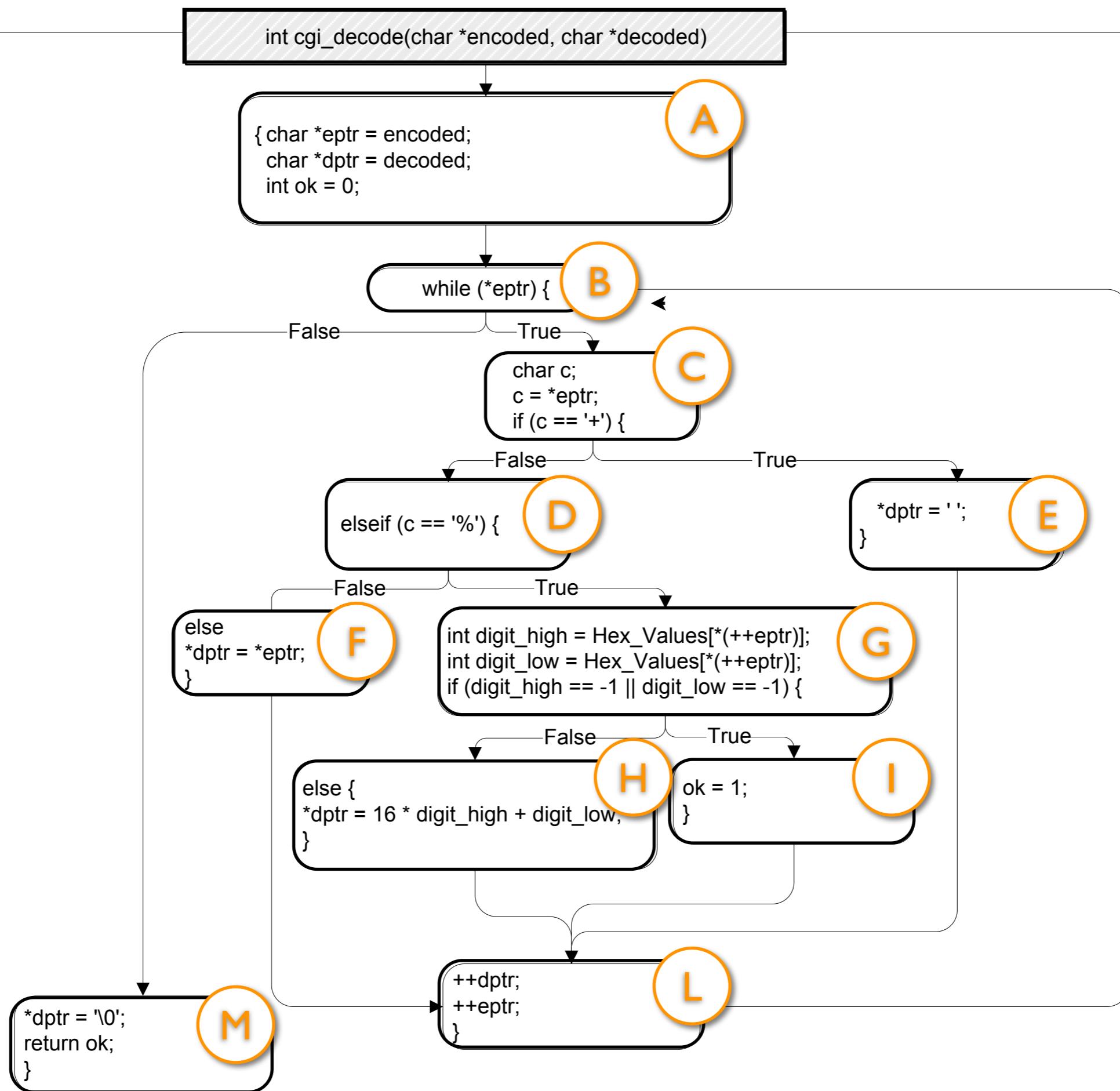


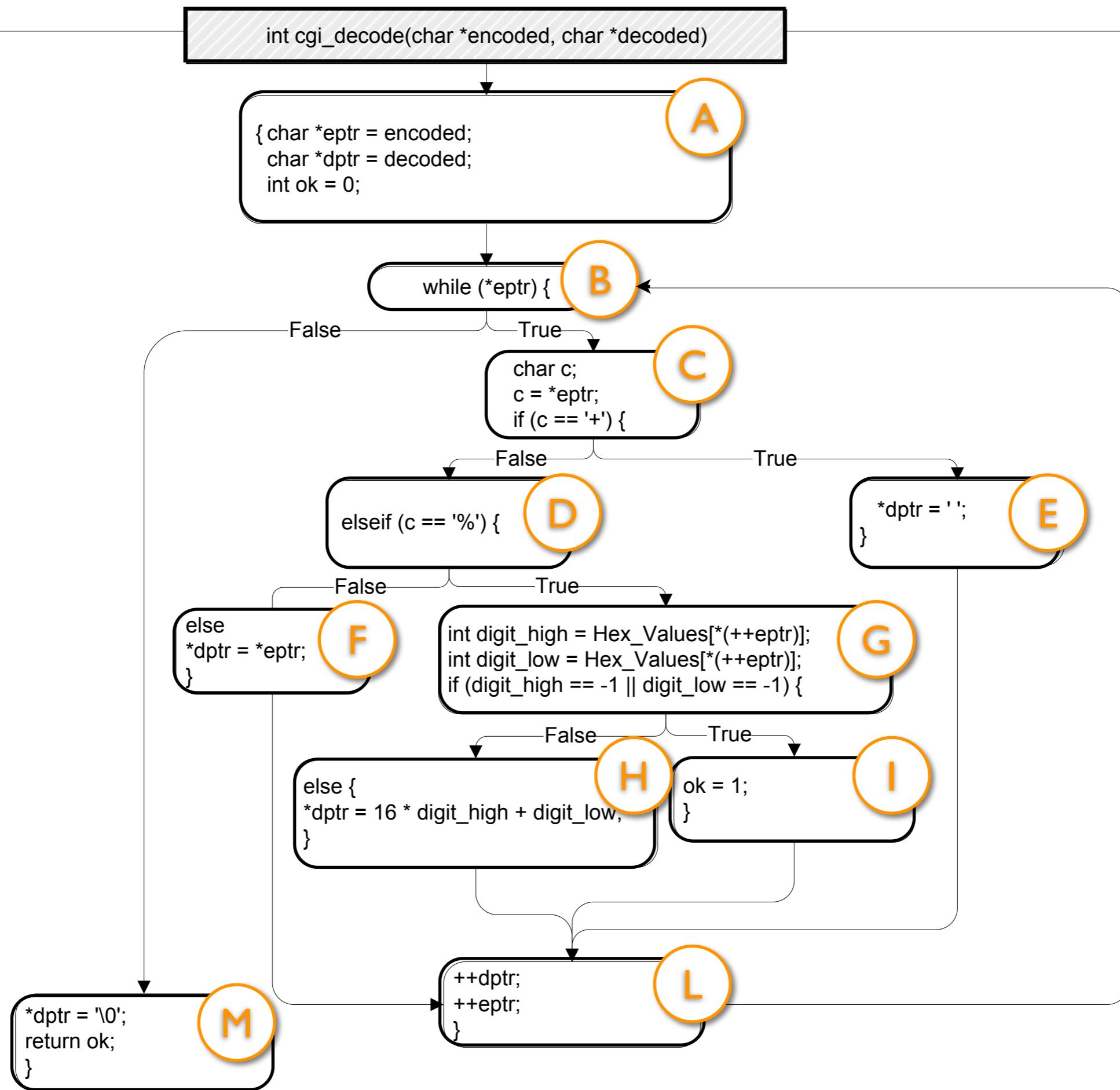
# Test Data Generation

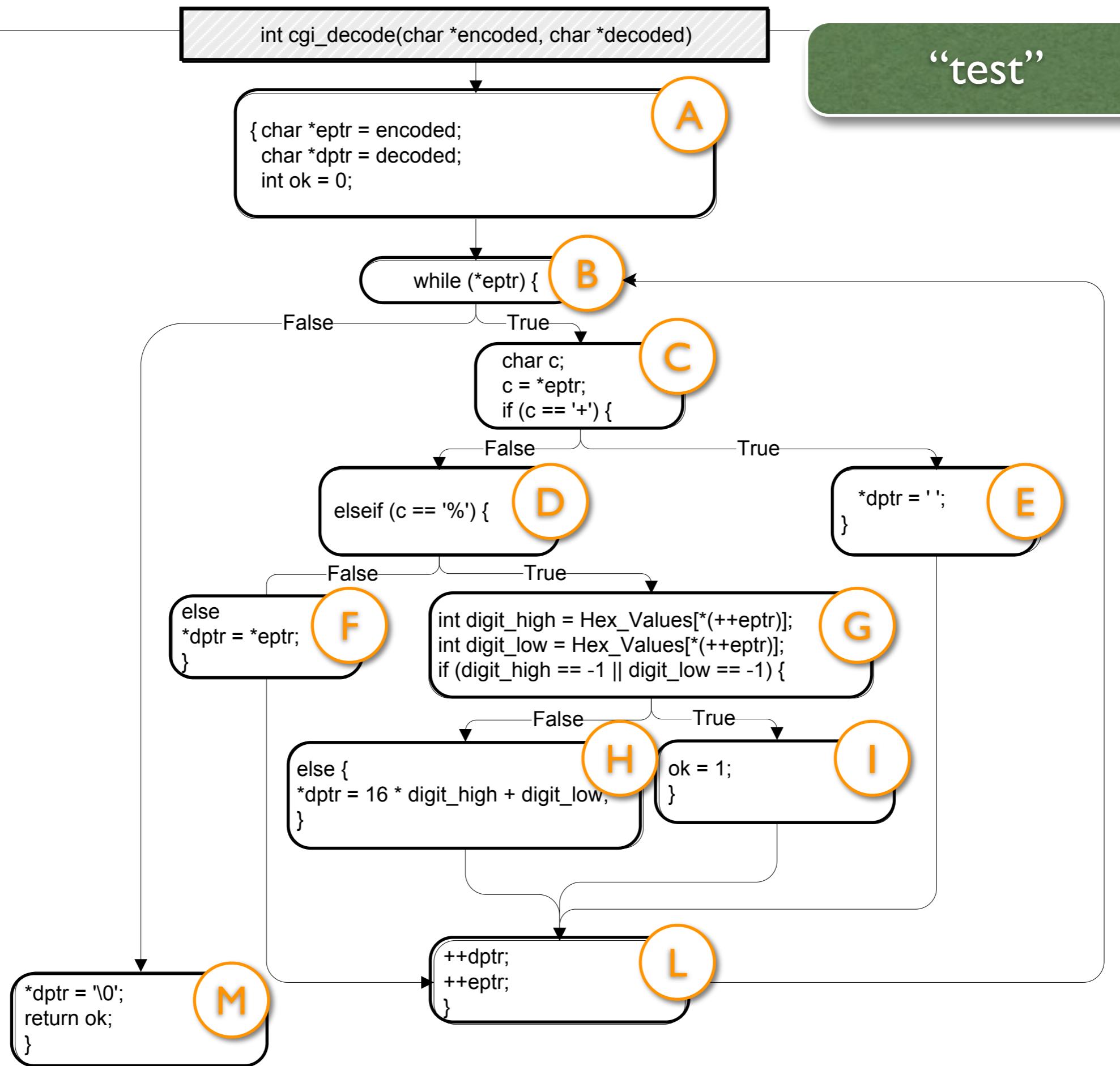


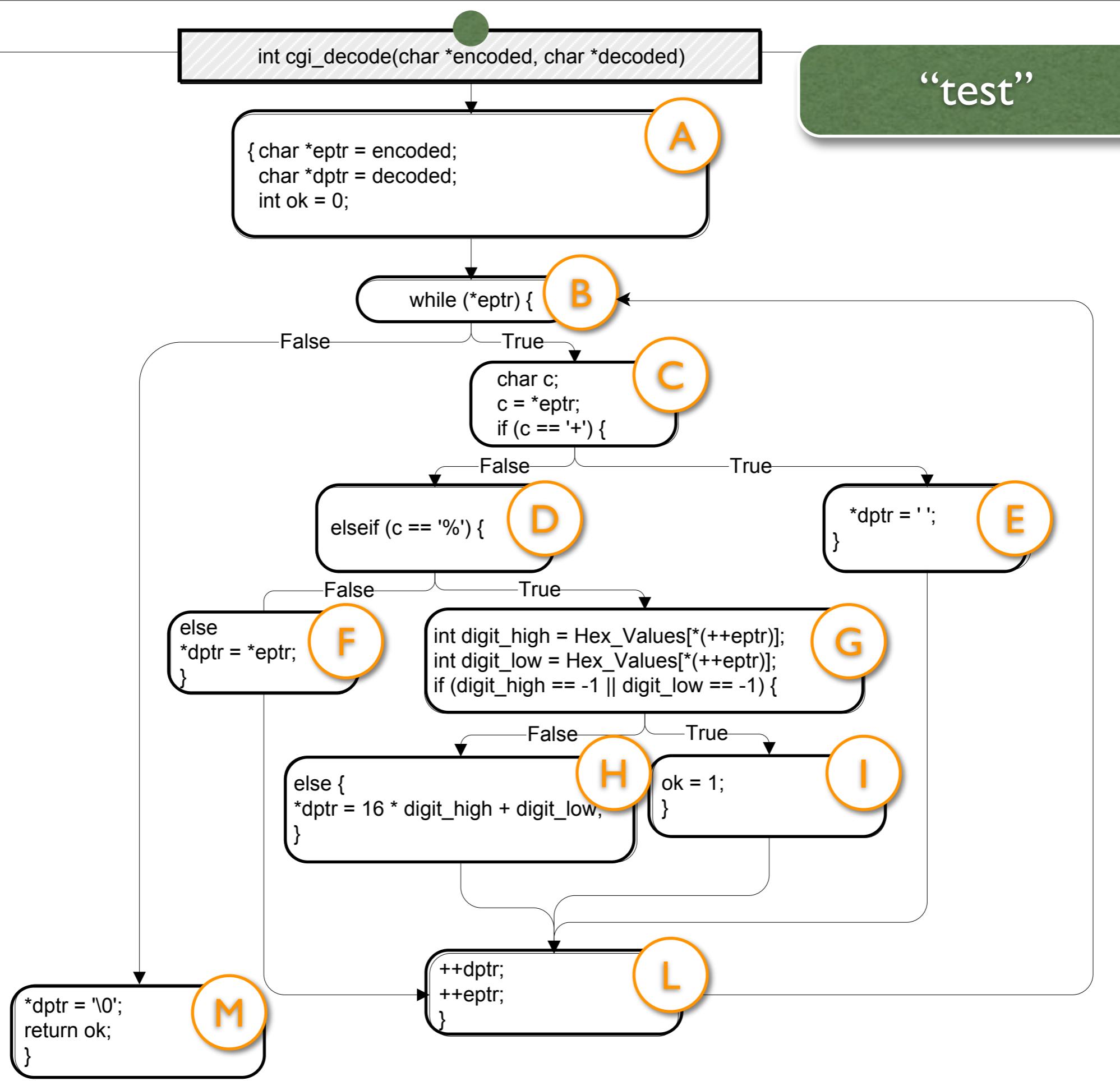
# Objectives

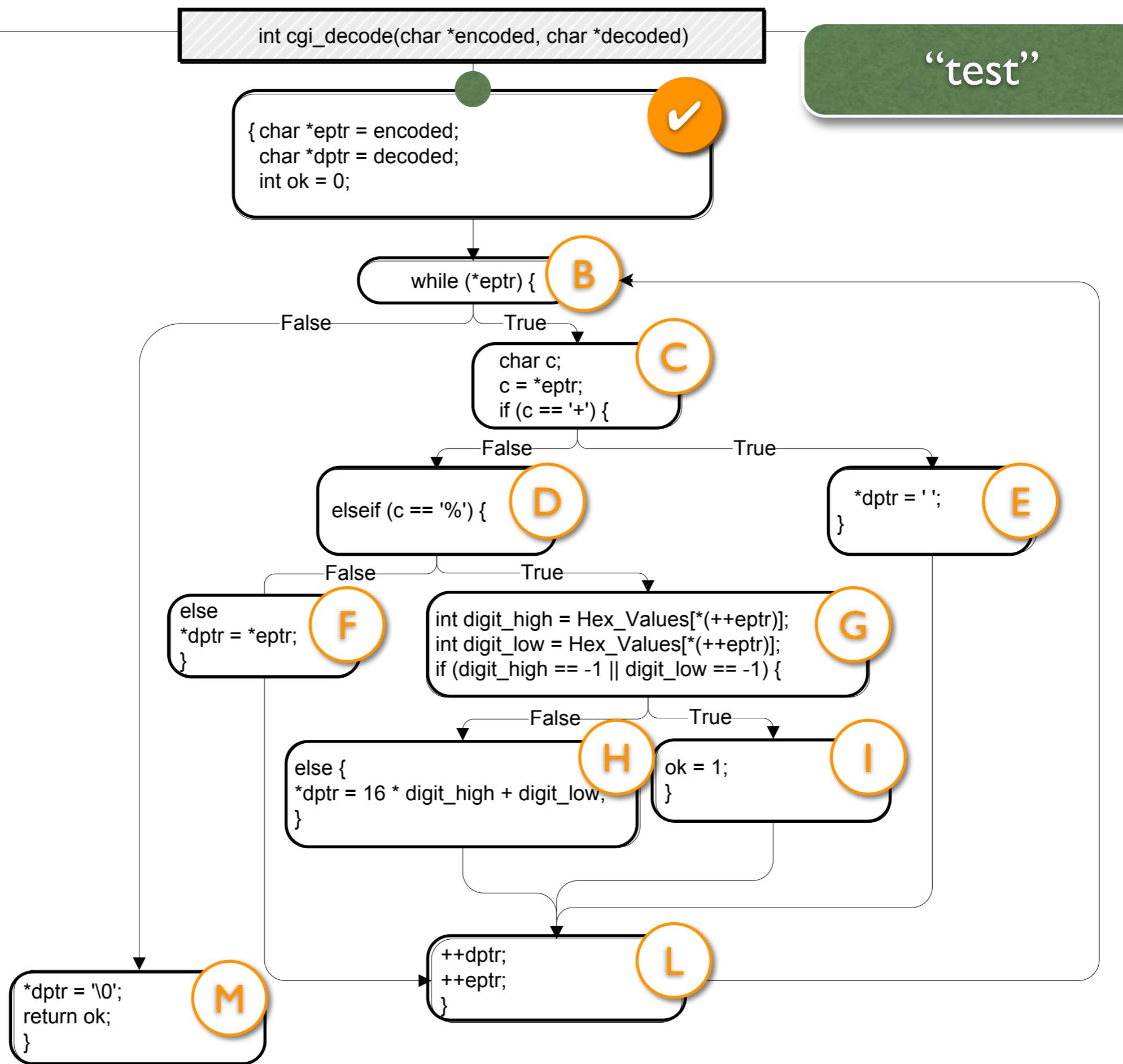
- Structural testing
- Given: Entry function / function to test
- Test function has parameters
- Determine values for parameters to satisfy a chosen test requirement
- Search space: Values of input parameters

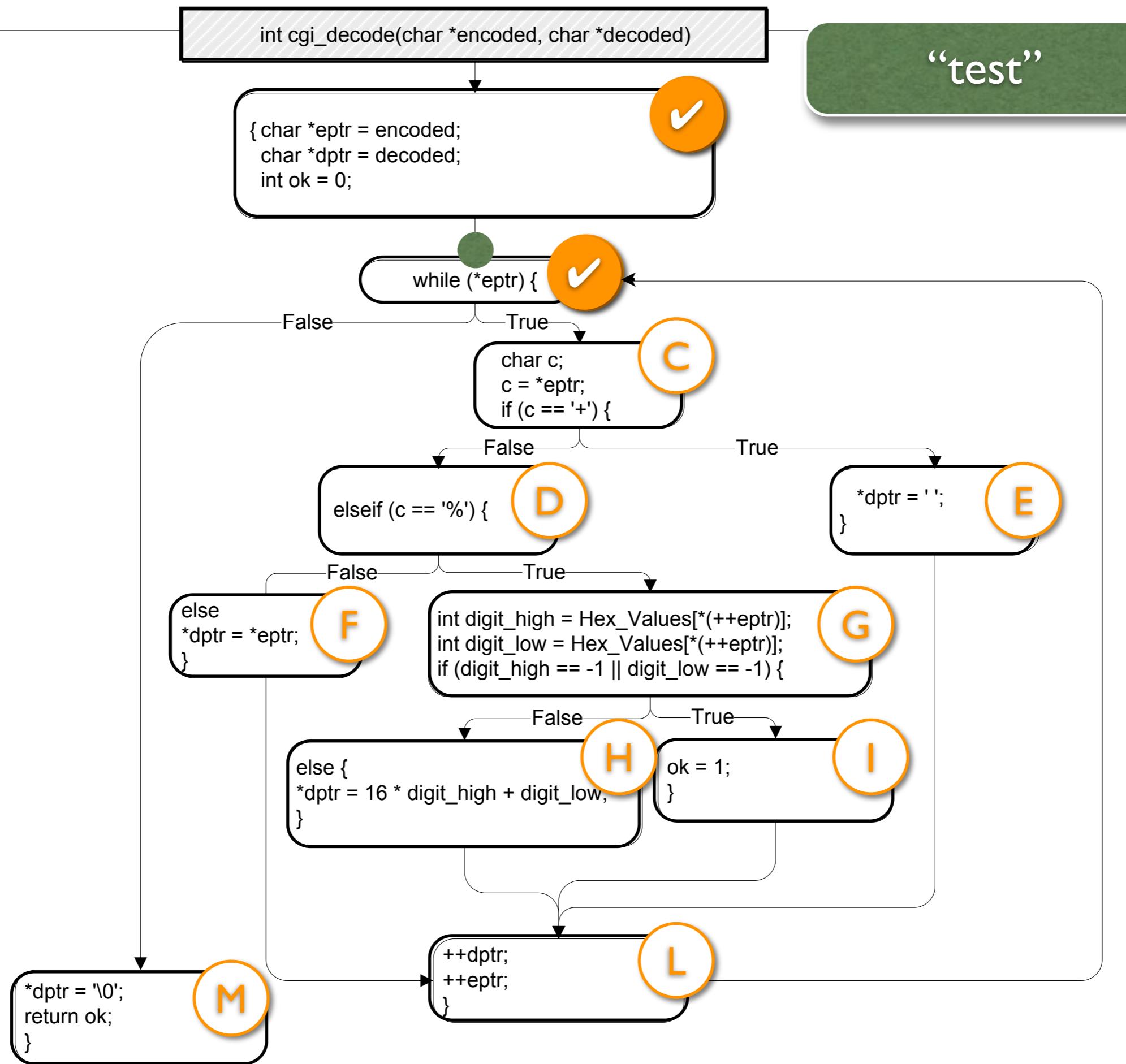


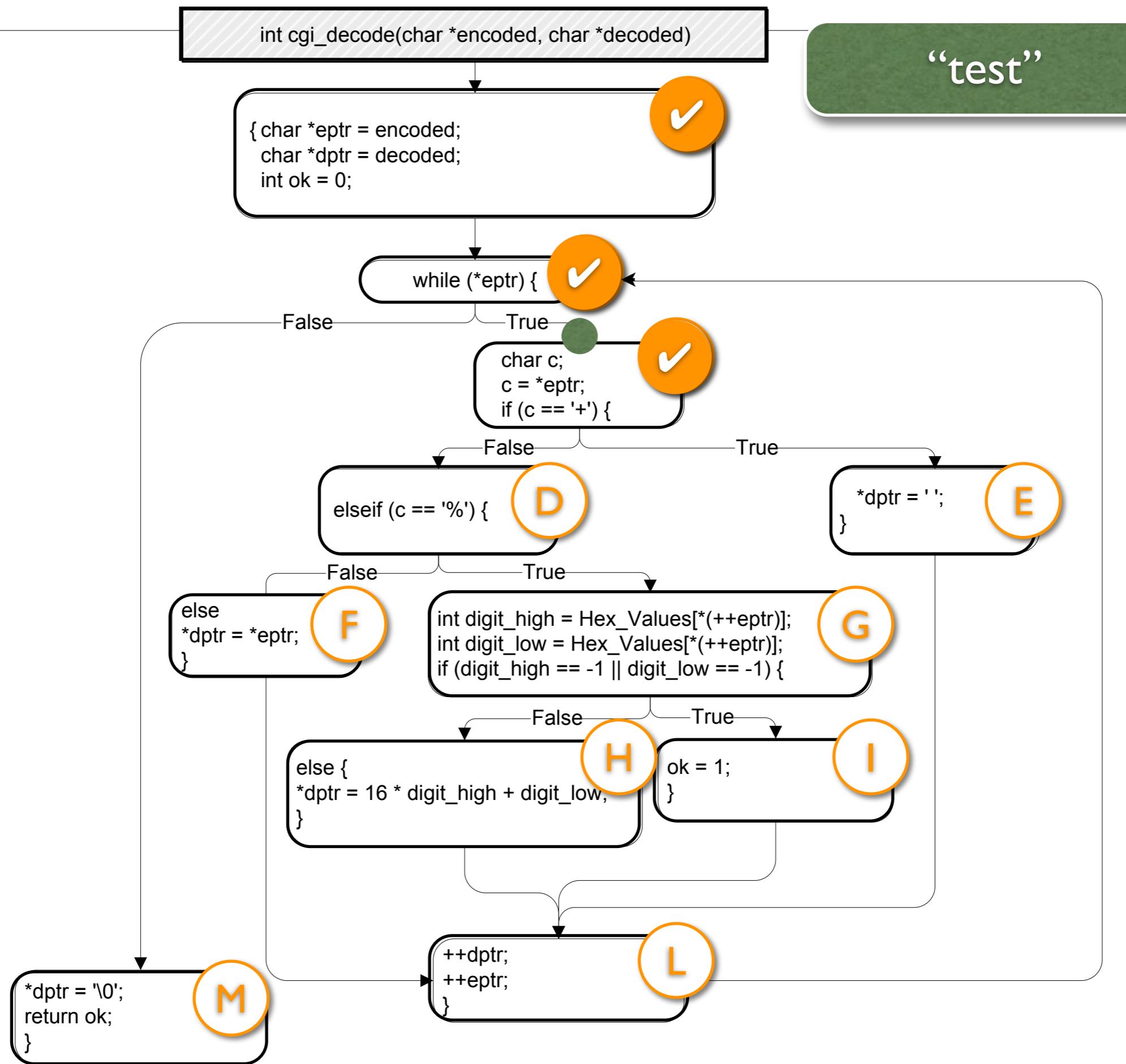


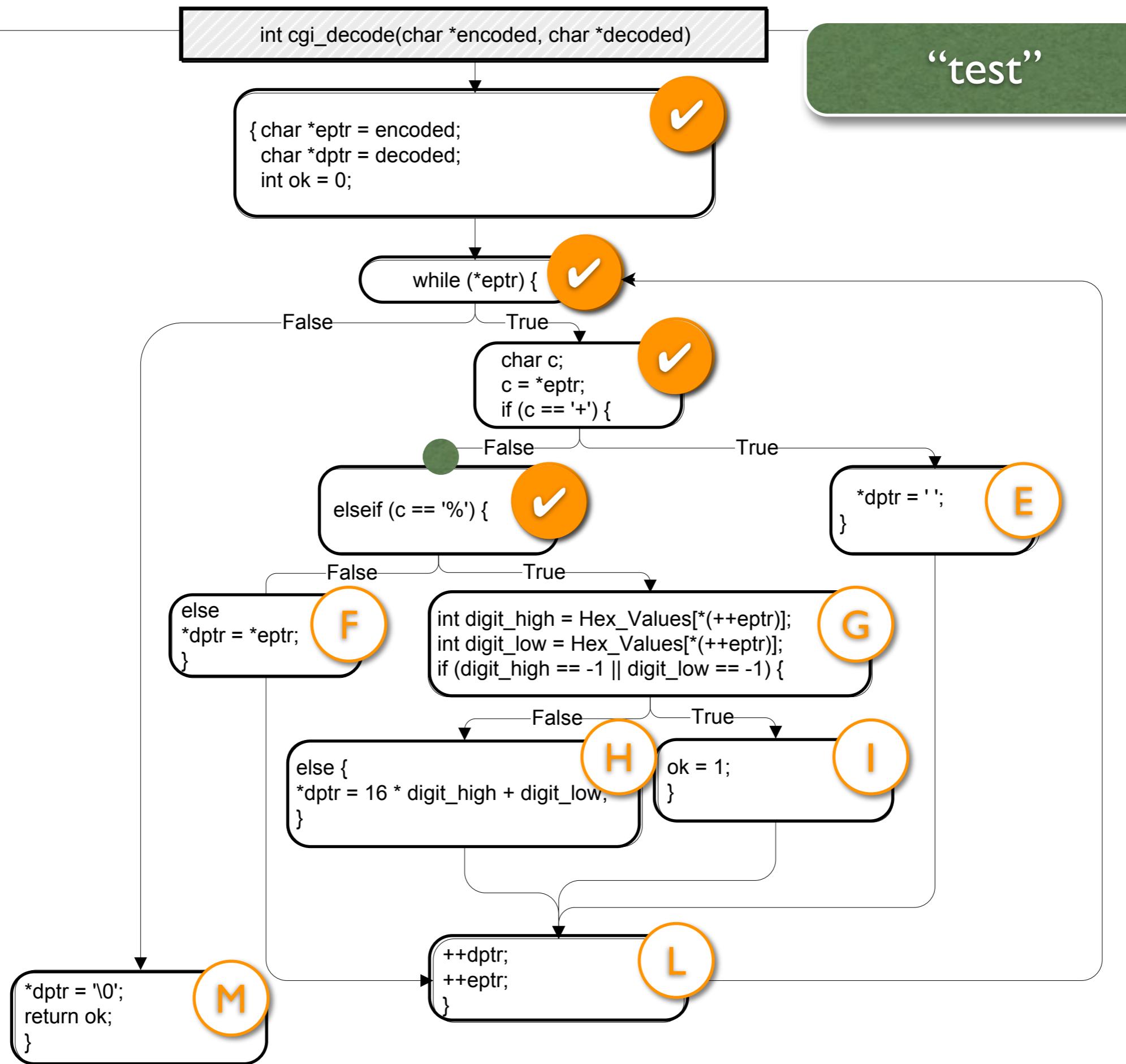


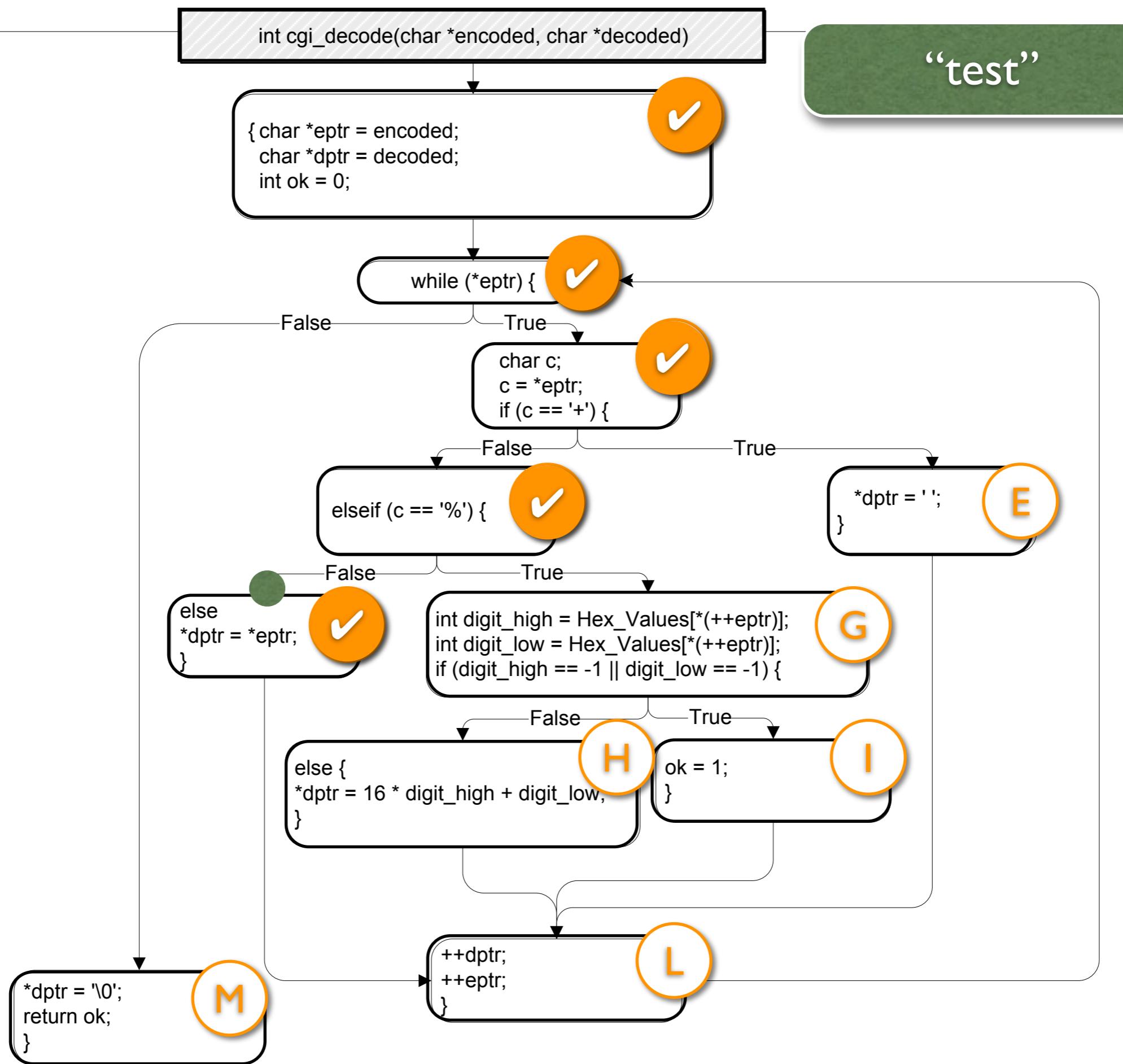


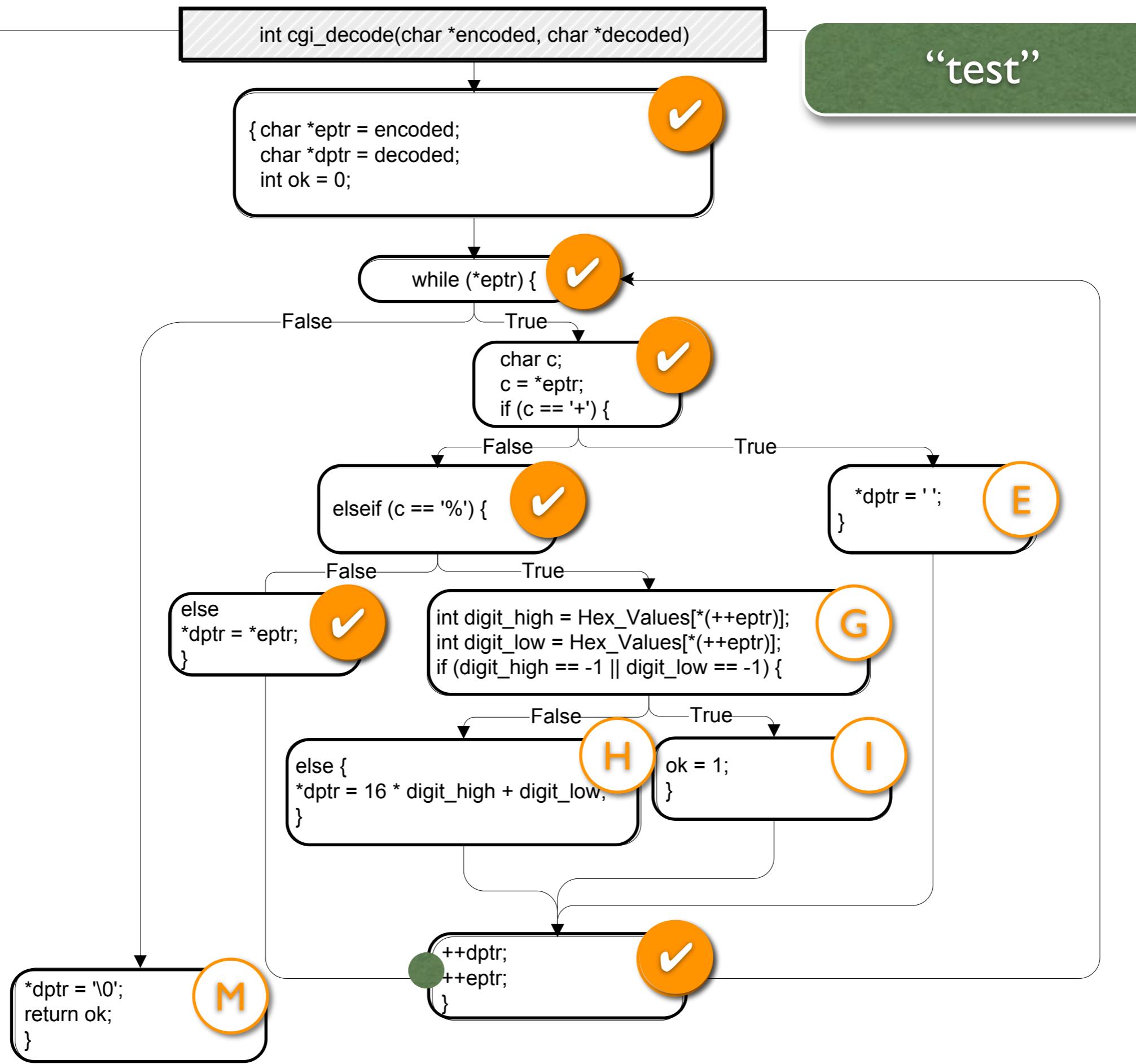


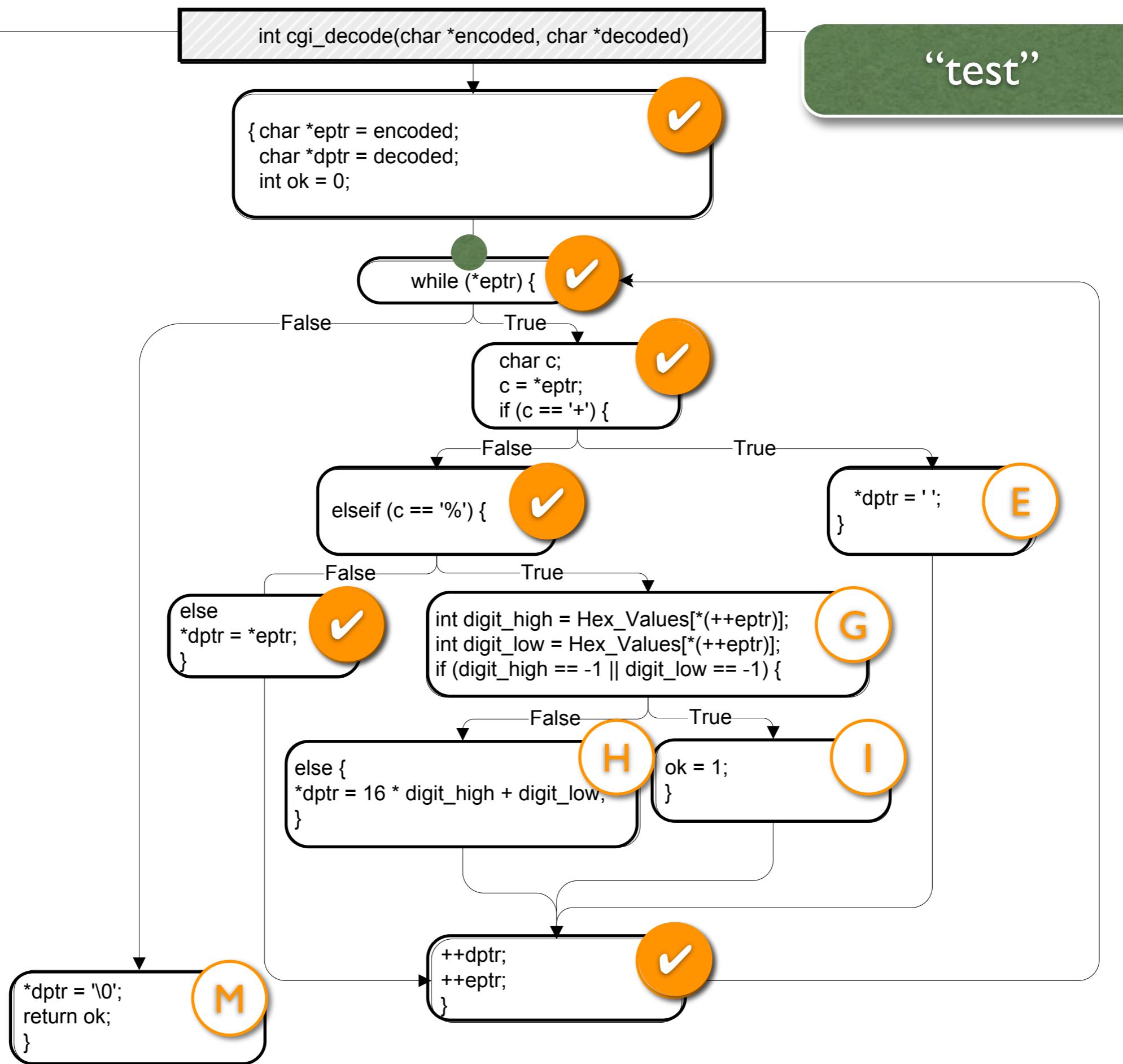


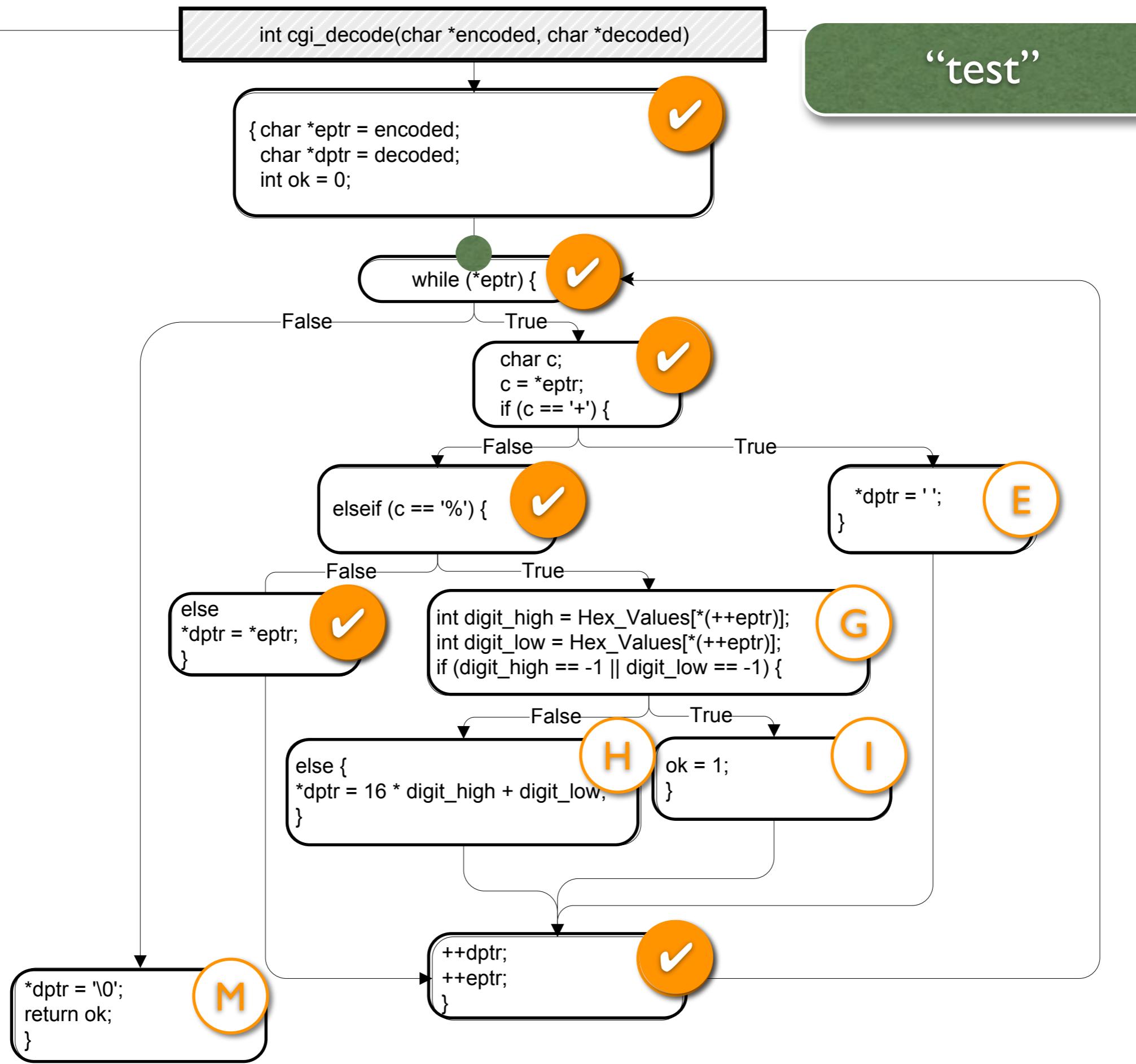


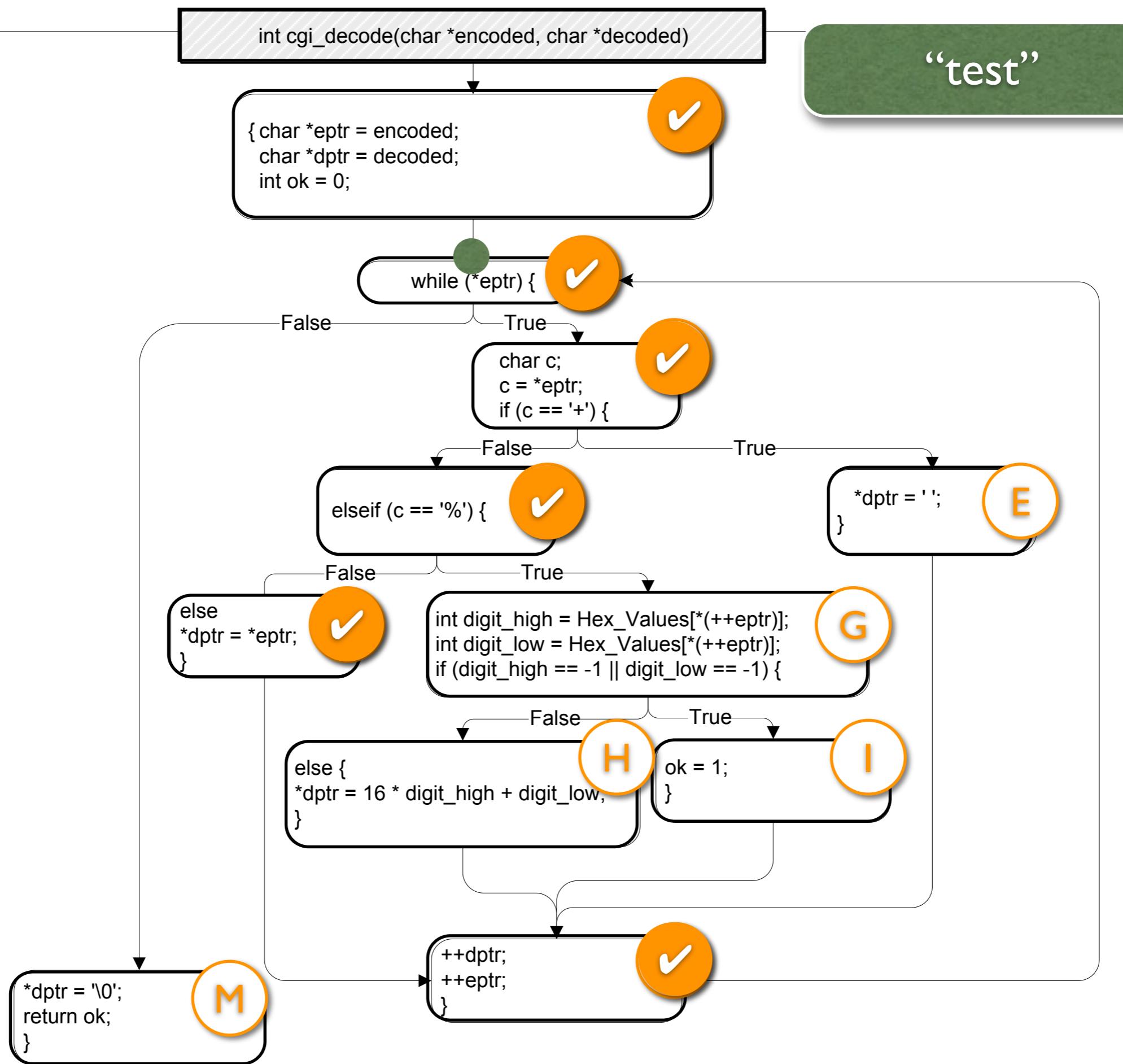


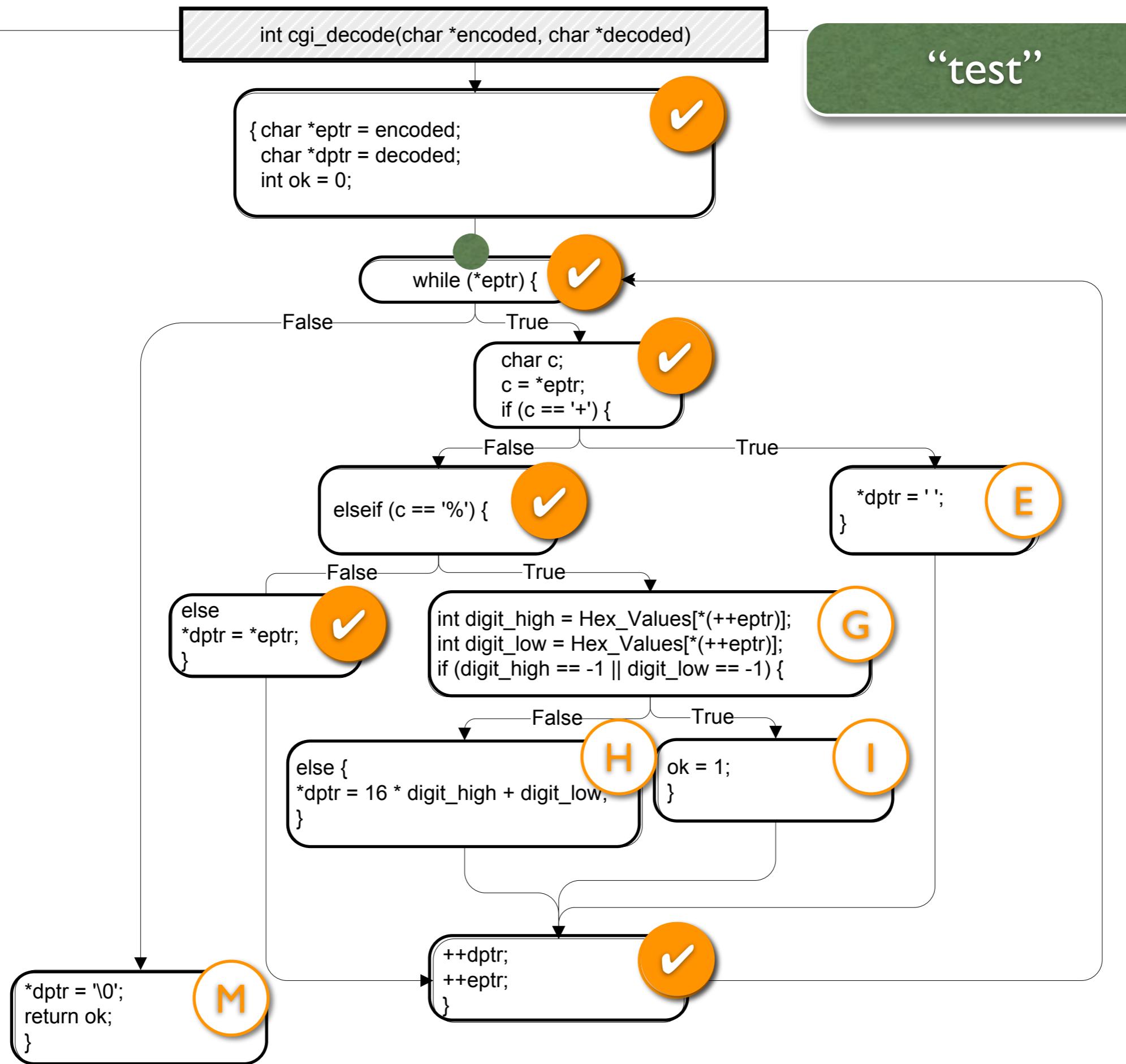


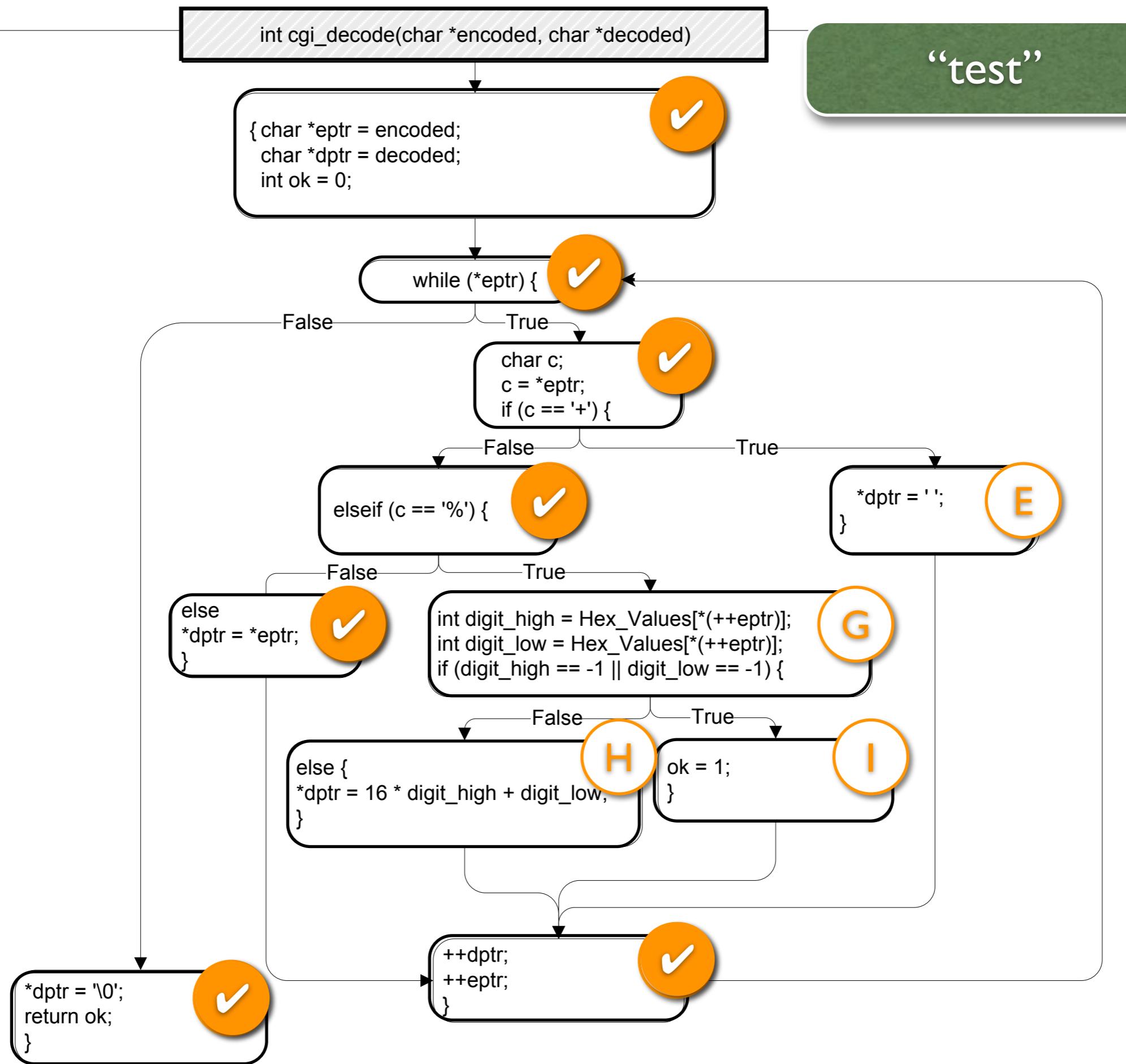


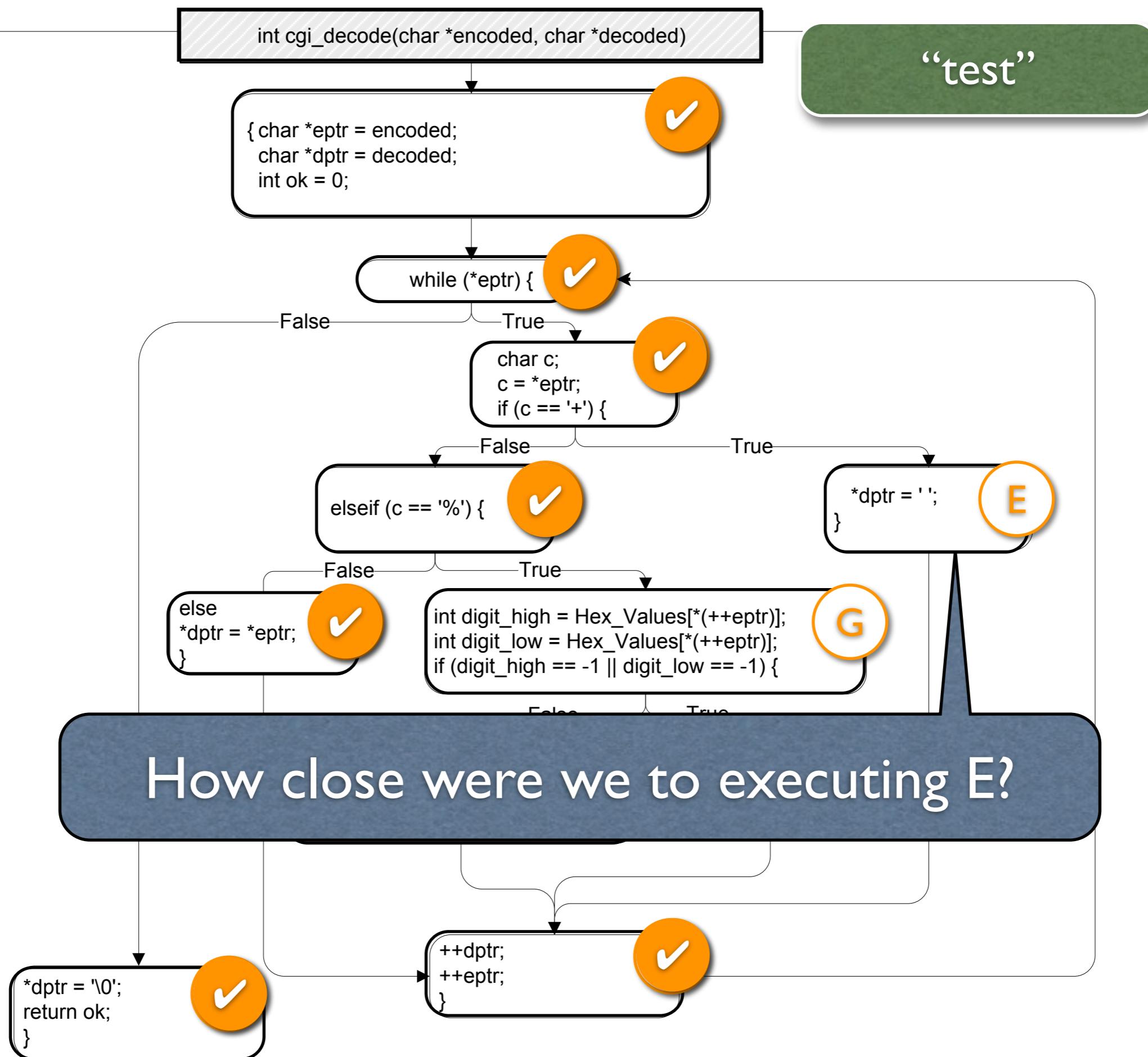












## How close were we to executing E?

```
public int gcd(int x, int y) {  
    int tmp;  
    while (y != 0) {  
        tmp = x % y;  
        x = y;  
        y = tmp;  
    }  
    return x;  
}
```

```
public int gcd
```

```
public int gcd(int x, int y) {  
    int tmp;
```

```
    while(y != 0) {
```

```
        tmp = x % y;
```

```
        x = y;
```

```
        y = tmp;
```

```
    return x; }
```

A

B

C

D

E

F

# Dominators

- Node A dominates B if every path to B goes through A
- Immediate dominator: Closest dominator on any path from the root
- Root node has no immediate dominator

```
public int gcd
```

```
public int gcd(int x, int y) {  
    int tmp;
```

```
    while(y != 0) {
```

```
        tmp = x % y;
```

```
        x = y;
```

```
        y = tmp;
```

```
    return x; }
```

A

B

C

D

E

F

A

```
public int gcd
```

```
public int gcd(int x, int y) {  
    int tmp;
```

```
    while(y != 0) {
```

```
        tmp = x % y;
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```
        x = y;
```

```
        y = tmp;
```

```
    return x; }
```

A

B

C

D

E

F

```
public int gcd
```

A

```
public int gcd(int x, int y) {  
    int tmp;
```

A,B

```
while(y != 0) {
```

A

B

```
    tmp = x % y;
```

C

```
    x = y;
```

D

```
    y = tmp;
```

E

```
return x; }
```

F

```
public int gcd
```

A

```
public int gcd(int x, int y) {  
    int tmp;
```

A,B

```
while(y != 0) {
```

A,B,C

```
    tmp = x % y;
```

```
    x = y;
```

```
    y = tmp;
```

```
return x; }
```

A

B

C

D

E

F

```
public int gcd
```

A

```
public int gcd(int x, int y) {  
    int tmp;
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A,B

```
while(y != 0) {
```

A,B,C

```
    tmp = x % y;
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A,B,C,D

```
    x = y;
```

```
    y = tmp;
```

```
return x; }
```

A

B

C

D

E

F

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A,B,C

```
    tmp = x % y;
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A,B,C,D

```
    x = y;
```

A,B,C,D,E

```
    y = tmp;
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A

B

C

D

E

F

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return x; }
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A

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A,B

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A,B,C

```
    tmp = x % y;
```

A,B,C,D

```
    x = y;
```

A,B,C,D,E

```
    y = tmp;
```

A,B,F

```
return x; }
```

A

B

C

D

E

F

# Post Dominators

- Dominators viewed in reverse (paths from exit node)
- Node A post-dominates B if all paths from B to exit must go through A
- Immediate post dominator: Closest dominator on any path to the exit node

```
public int gcd
```

```
public int gcd(int x, int y) {  
    int tmp;
```

```
    while(y != 0) {
```

```
        tmp = x % y;
```

```
        x = y;
```

```
        y = tmp;
```

```
    return x; }
```

A

B

C

D

E

F

```
public int gcd
```

```
public int gcd(int x, int y) {  
    int tmp;
```

```
    while(y != 0) {
```

```
        tmp = x % y;
```

```
        x = y;
```

```
        y = tmp;
```

```
    return x; }
```

A

B

C

D

E

F

F

```
public int gcd
```

```
public int gcd(int x, int y) {  
    int tmp;
```

```
    while(y != 0) {
```

```
        tmp = x % y;
```

```
        x = y;
```

```
        y = tmp;
```

```
    return x; }
```

A

B

C

D

E

F

E,B,F

F

```
public int gcd
```

```
public int gcd(int x, int y) {  
    int tmp;
```

```
while(y != 0) {
```

```
    tmp = x % y;
```

```
    x = y;
```

```
    y = tmp;
```

```
return x; }
```

A

B

C

D

E

F

D,E,B,F

E,B,F

F

```
public int gcd
```

```
public int gcd(int x, int y) {  
    int tmp;
```

```
while(y != 0) {
```

```
    tmp = x % y;
```

```
    x = y;
```

```
    y = tmp;
```

```
return x; }
```

A

B

C

D

E

F

C,D,E,B,F

D,E,B,F

E,B,F

F

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public int gcd(int x, int y) {  
    int tmp;
```

```
while(y != 0) {
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```
    tmp = x % y;
```

```
    x = y;
```

```
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```

```
return x; }
```

A

B

C

D

E

F

B,F

C,D,E,B,F

D,E,B,F

E,B,F

F

```
public int gcd
```

```
public int gcd(int x, int y) {  
    int tmp;
```

```
while(y != 0) {
```

```
    tmp = x % y;
```

```
    x = y;
```

```
    y = tmp;
```

```
return x; }
```

A

A,B,F

B

B,F

C

C,D,E,B,F

D

D,E,B,F

E

E,B,F

F

F

```
public int gcd
```

A

```
public int gcd(int x, int y) {  
    int tmp;
```

A

A,B,F

```
while(y != 0) {
```

B

B,F

```
    tmp = x % y;
```

C

C,D,E,B,F

```
    x = y;
```

D

D,E,B,F

```
    y = tmp;
```

E

E,B,F

```
return x; }
```

F

F

```
public int gcd
```

A

```
public int gcd(int x, int y) {  
    int tmp;
```

A,B

```
while(y != 0) {
```

A

```
    tmp = x % y;
```

B

```
    x = y;
```

C

```
    y = tmp;
```

D

```
return x; }
```

E

F

A,B,F

B,F

C,D,E,B,F

D,E,B,F

E,B,F

F

```
public int gcd
```

A

```
public int gcd(int x, int y) {  
    int tmp;
```

A,B

```
while(y != 0) {
```

A,B,C

```
    tmp = x % y;
```

```
    x = y;
```

```
    y = tmp;
```

```
return x; }
```

A

B

C

D

E

F

A,B,F

B,F

C,D,E,B,F

D,E,B,F

E,B,F

F

```
public int gcd
```

A

```
public int gcd(int x, int y) {  
    int tmp;
```

A,B

A,B,C

A,B,C,D

```
while(y != 0) {
```

A

B

C

D

E

F

```
    tmp = x % y;
```

```
    x = y;
```

```
    y = tmp;
```

```
return x; }
```

A,B,F

B,F

C,D,E,B,F

D,E,B,F

E,B,F

F

```
public int gcd
```

A

```
public int gcd(int x, int y) {  
    int tmp;
```

A,B

```
while(y != 0) {
```

A,B,C

```
    tmp = x % y;
```

A,B,C,D

```
    x = y;
```

A,B,C,D,E

```
    y = tmp;
```

A

B

C

D

E

F

A,B,F

B,F

C,D,E,B,F

D,E,B,F

E,B,F

F

```
return x; }
```

```
public int gcd
```

```
public int gcd(int x, int y) {  
    int tmp;
```

A

A,B,F

A,B

B,F

A,B,C

C,D,E,B,F

A,B,C,D

D,E,B,F

A,B,C,D,E

E,B,F

A,B,F

F

```
return x; }
```

A

B

C

D

E

F

# Calculating Post Dominance

$$PDom(n_e) = \{n_e\}$$

$$PDom(n) = (\bigcap_{s \in succ(n)} PDom(s)) \cup \{n\}$$

- Initialize  $PDom(n)$  with set of all nodes for all  $n$  except exit node  $n_e$
- while some  $PDom(n)$  changed
  - recalculate  $PDom(n)$  for all  $n$  with above formula

# Control Dependence

- A is control dependent on B if:
- B has at least two successors in the CFG
- B dominates A
- B is not post-dominated by A
- There is a successor of B that is post-dominated by A

```
public int gcd
```

```
public int gcd(int x, int y) {  
    int tmp;
```

```
    while(y != 0) {
```

```
        tmp = x % y;
```

```
        x = y;
```

```
        y = tmp;
```

```
    return x; }
```

A

B

C

D

E

F

A

```
public int gcd
```

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public int gcd(int x, int y) {  
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```
    while(y != 0) {
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        tmp = x % y;
```

```
        x = y;
```

```
        y = tmp;
```

```
    return x; }
```

A

B

C

D

E

F

```
public int gcd
```

A

```
public int gcd(int x, int y) {  
    int tmp;
```

A,B

```
while(y != 0) {
```

A

B

C

D

E

F

```
    tmp = x % y;
```

```
    x = y;
```

```
    y = tmp;
```

```
return x; }
```

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public int gcd
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A,B,C

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A,B,C

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A,B,C

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A,B,C,D,E

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A,B,C

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    tmp = x % y;
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A,B,C,D

```
    x = y;
```

A,B,C,D,E

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A,B,F

```
return x; }
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A,B,C

```
    tmp = x % y;
```

B

A,B,C,D

```
    x = y;
```

C

A,B,C,D,E

```
    y = tmp;
```

D

A,B,F

```
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A,B,C

```
    tmp = x % y;
```

A,B,C,D

```
    x = y;
```

A,B,C,D,E

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    y = tmp;
```

A,B,F

```
return x; }
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A,B,C

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A,B,C,D

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A,B,C,D,E

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    y = tmp;
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A,B,F

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return x; }
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A

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F

D,E,B,F

E,B,F

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A,B,C

```
    tmp = x % y;
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A,B,C,D

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    x = y;
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A,B,C,D,E

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    y = tmp;
```

A,B,F

```
return x; }
```

A

B

C

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E

F

C,D,E,B,F

D,E,B,F

E,B,F

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A,B

A,B,C

A,B,C,D

A,B,C,D,E

A,B,F

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while(y != 0) {
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B,F

C,D,E,B,F

D,E,B,F

E,B,F

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A,B

A,B,C

A,B,C,D

A,B,C,D,E

A,B,F

A

```
while(y != 0) {
```

B

```
    tmp = x % y;
```

C

```
    x = y;
```

D

```
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```

E

```
return x; }
```

F

A,B,F

B,F

C,D,E,B,F

D,E,B,F

E,B,F

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    int tmp;
```

```
while(y != 0) {
```

```
    return x; }
```

```
tmp = x % y;
```

```
y = tmp;
```

```
x = y;
```

A

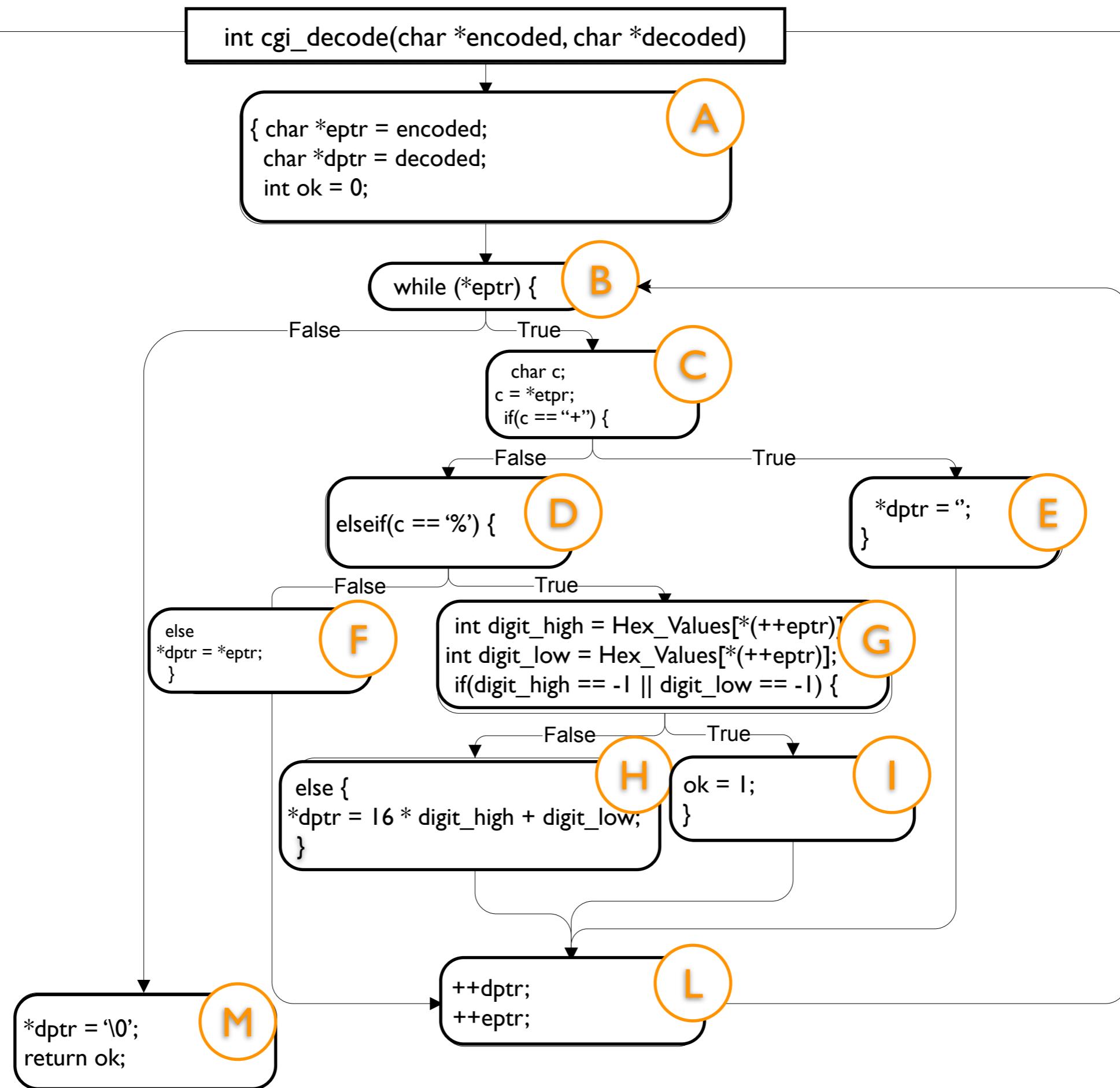
B

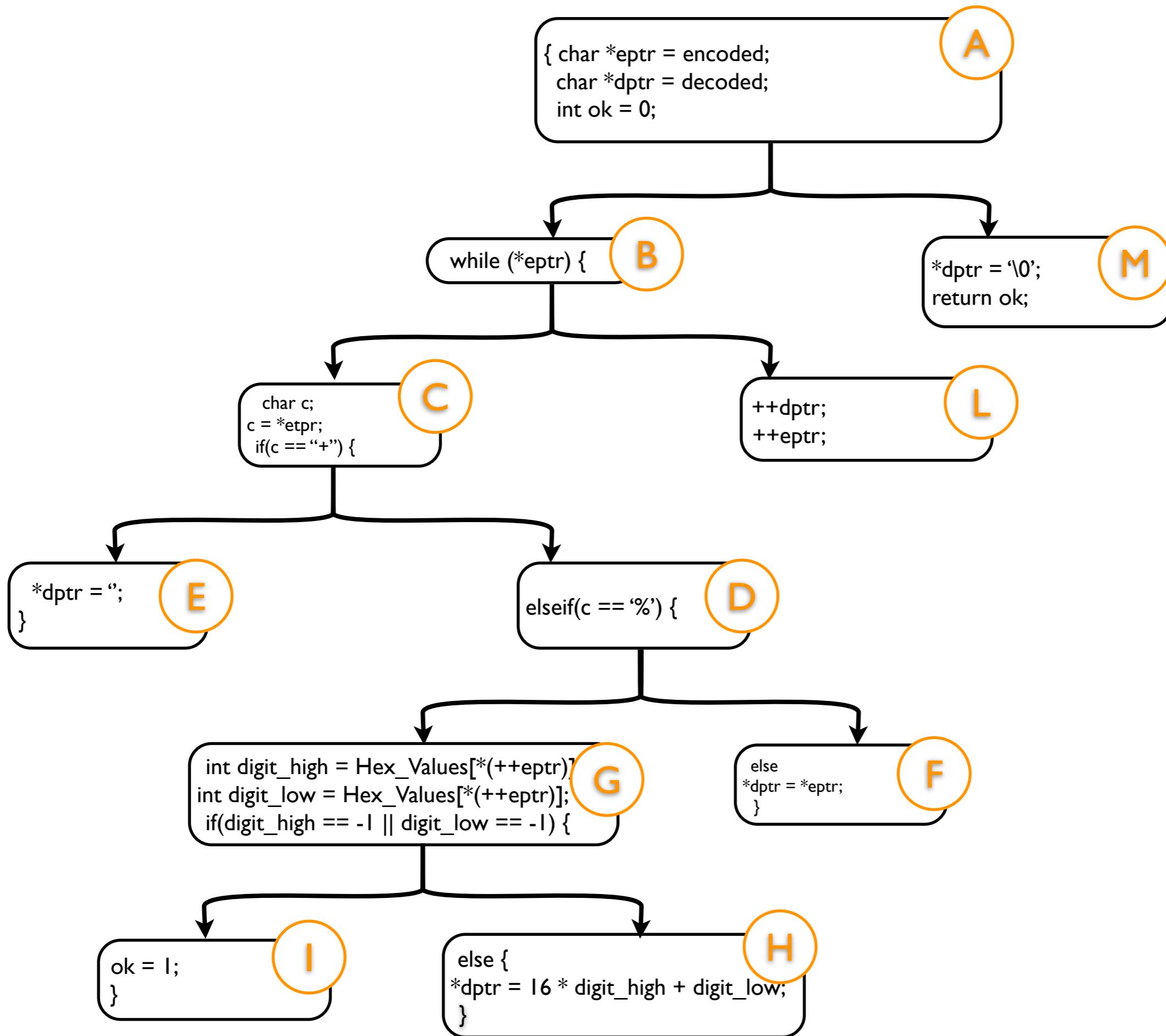
F

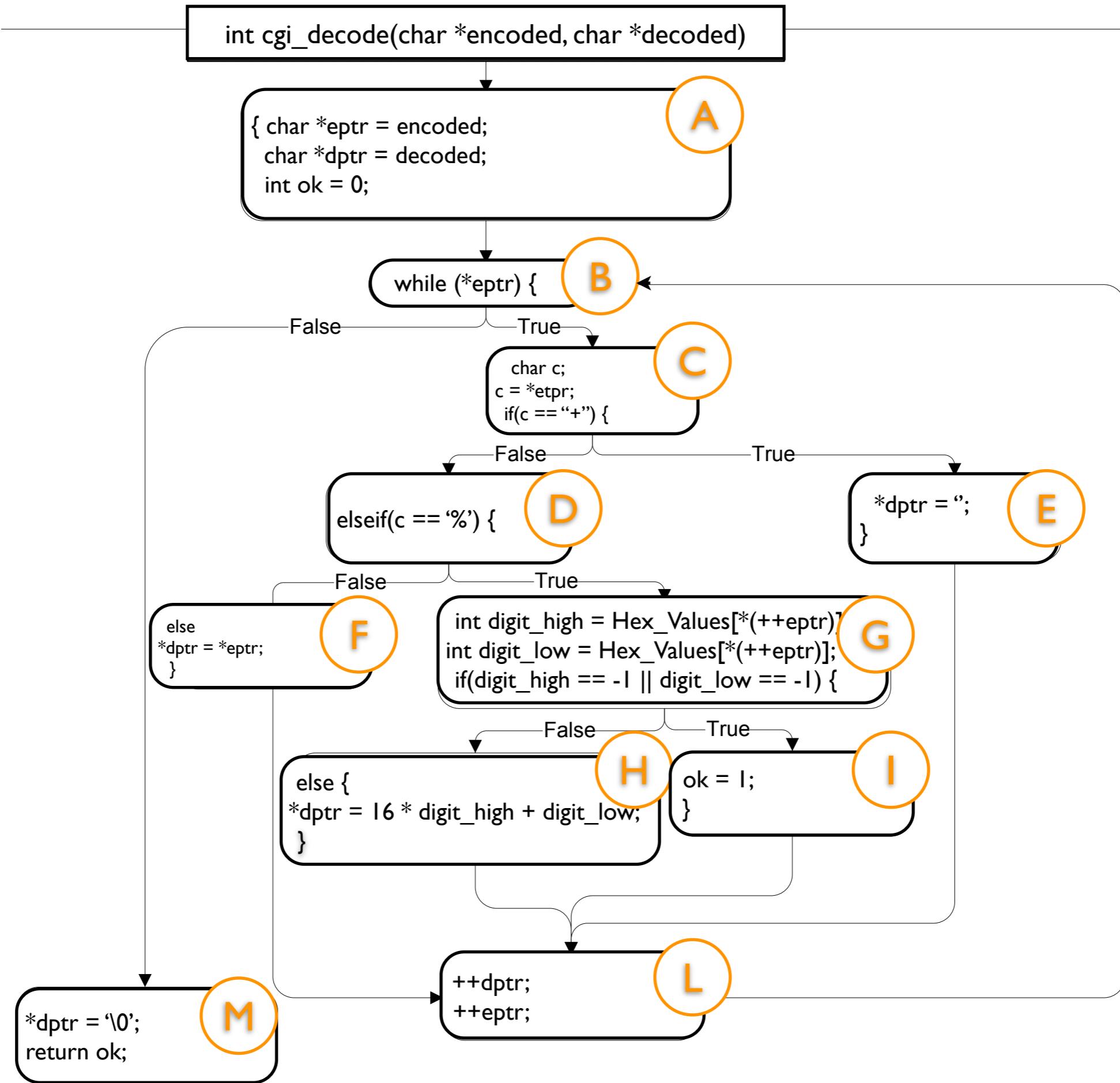
C

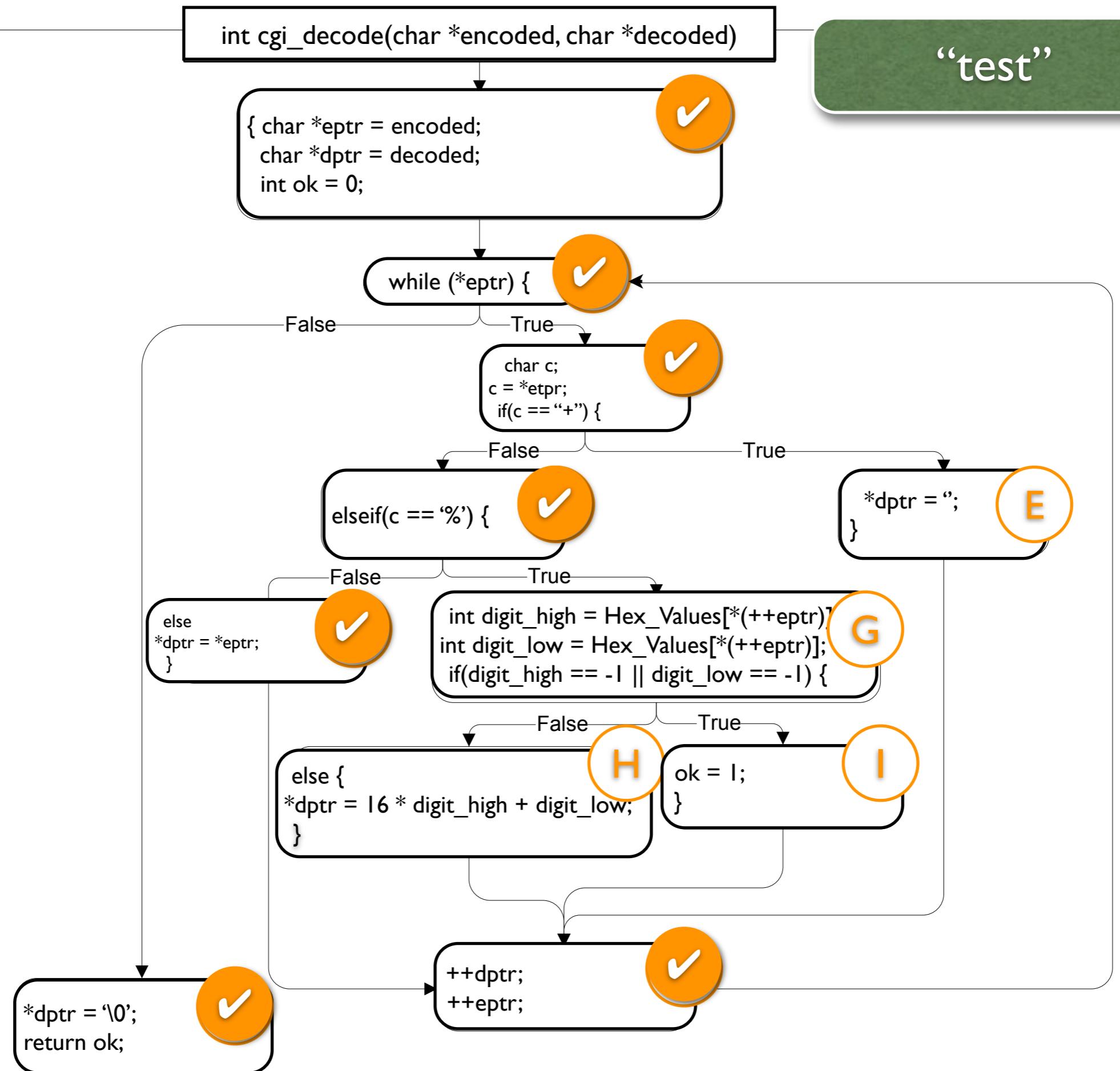
E

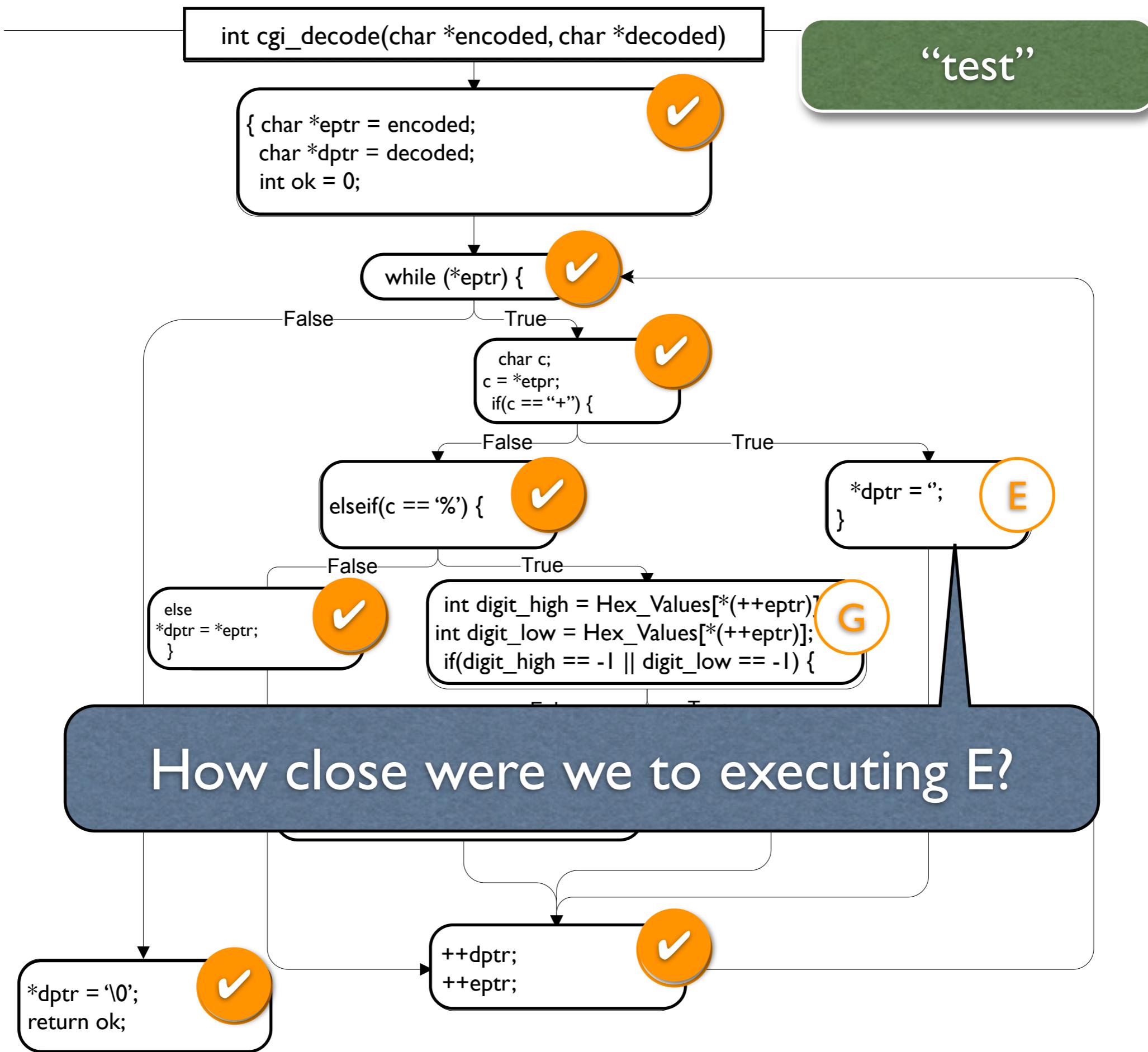
D



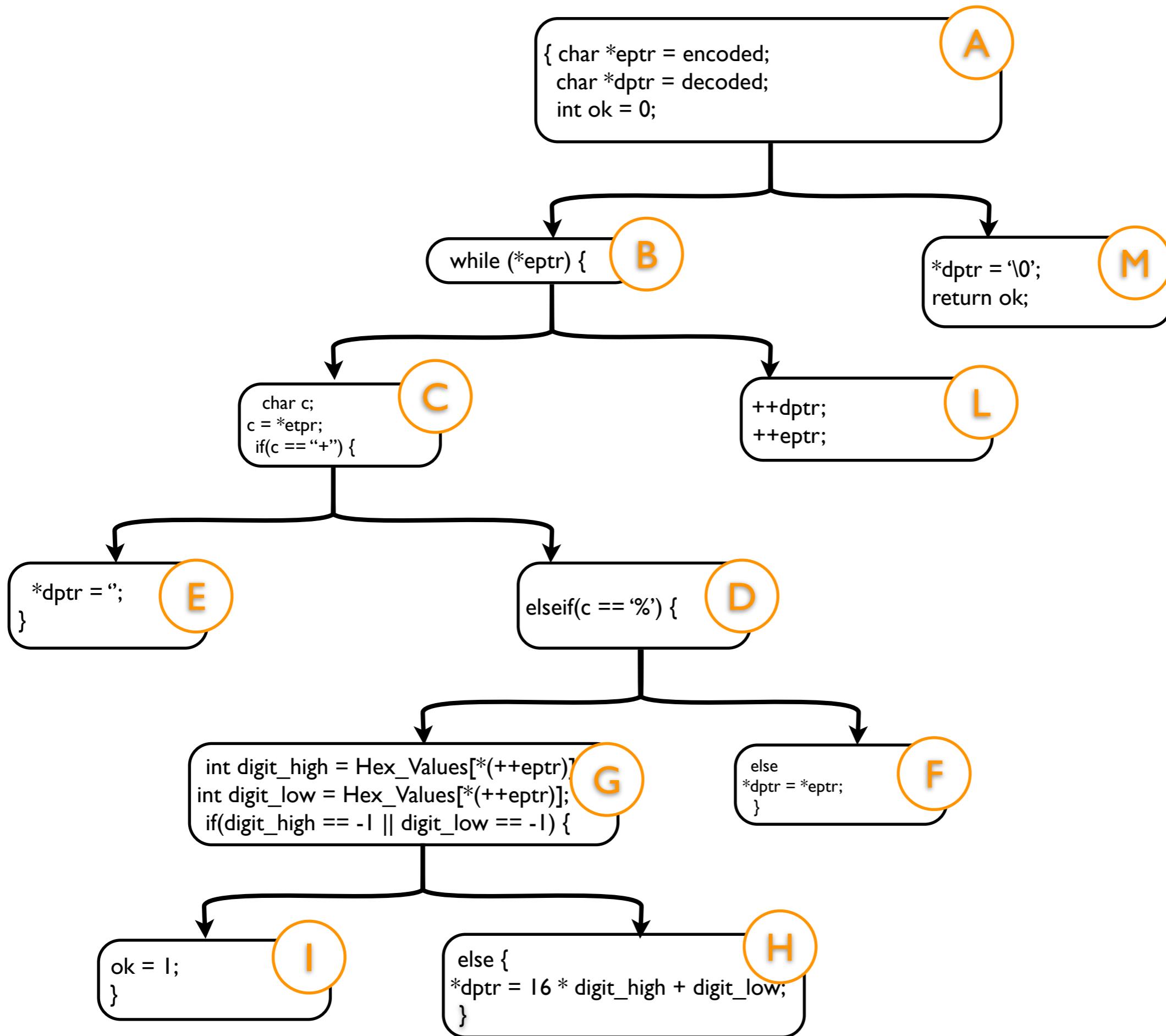




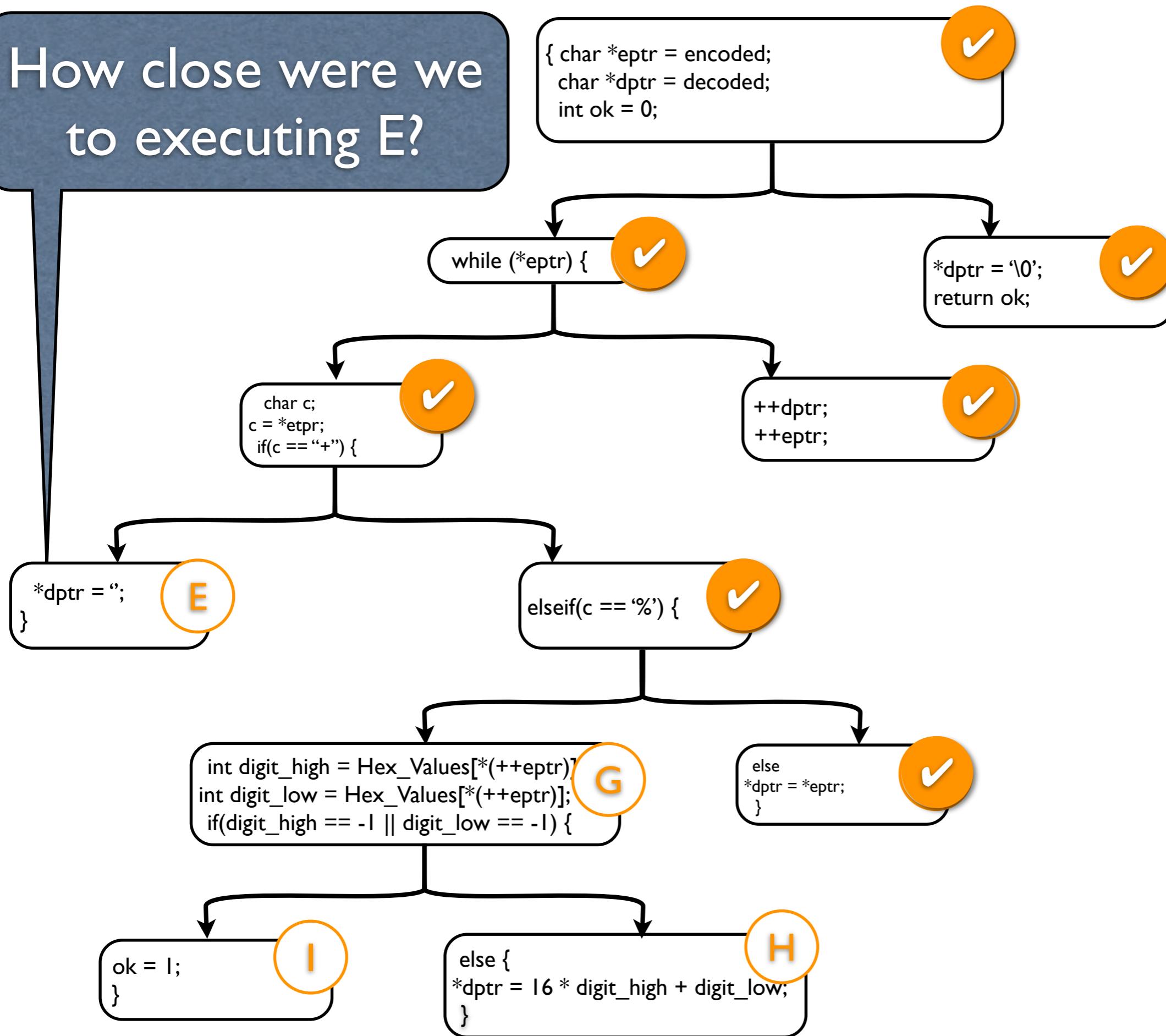




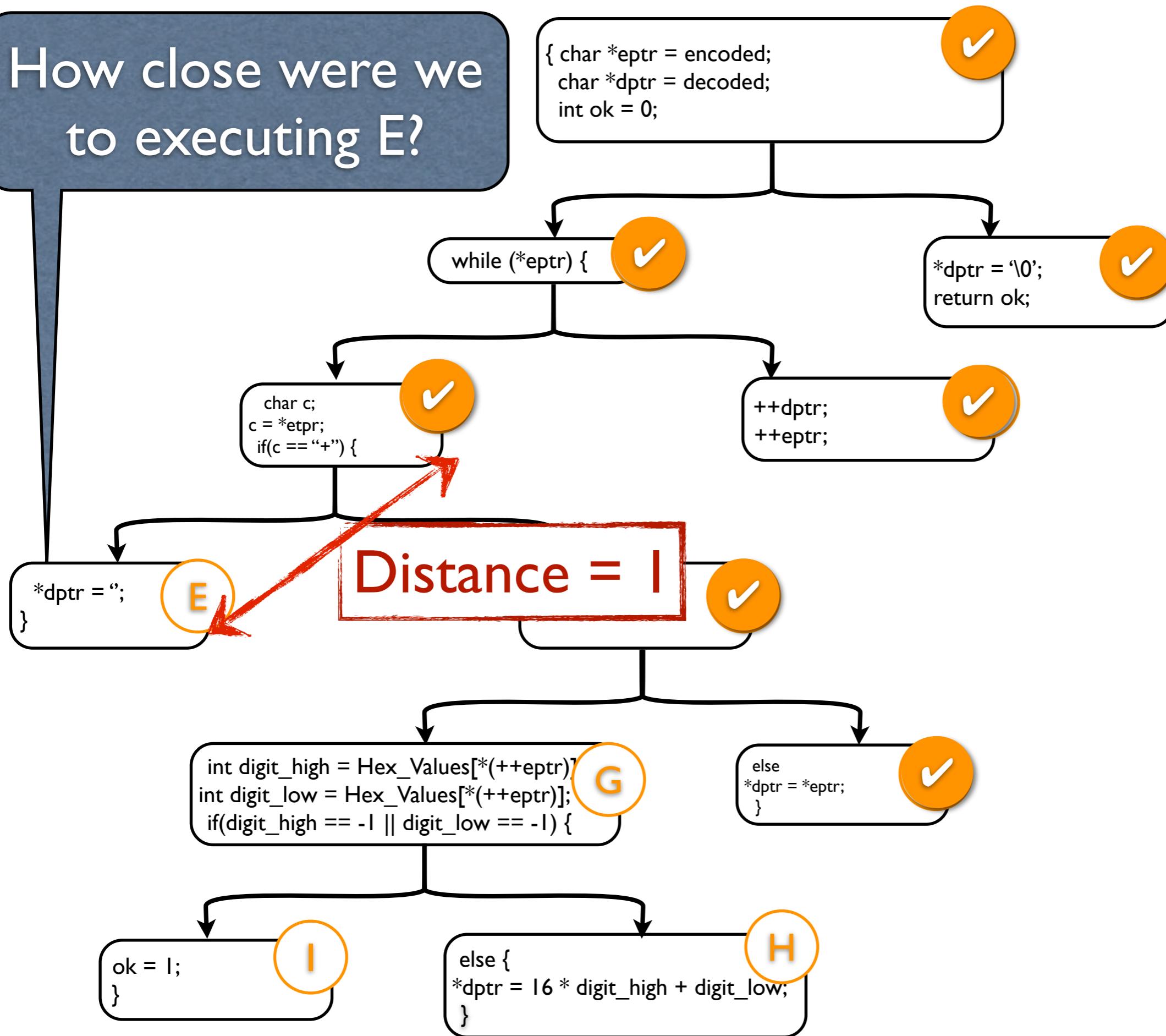
How close were we to executing E?



# How close were we to executing E?



# How close were we to executing E?

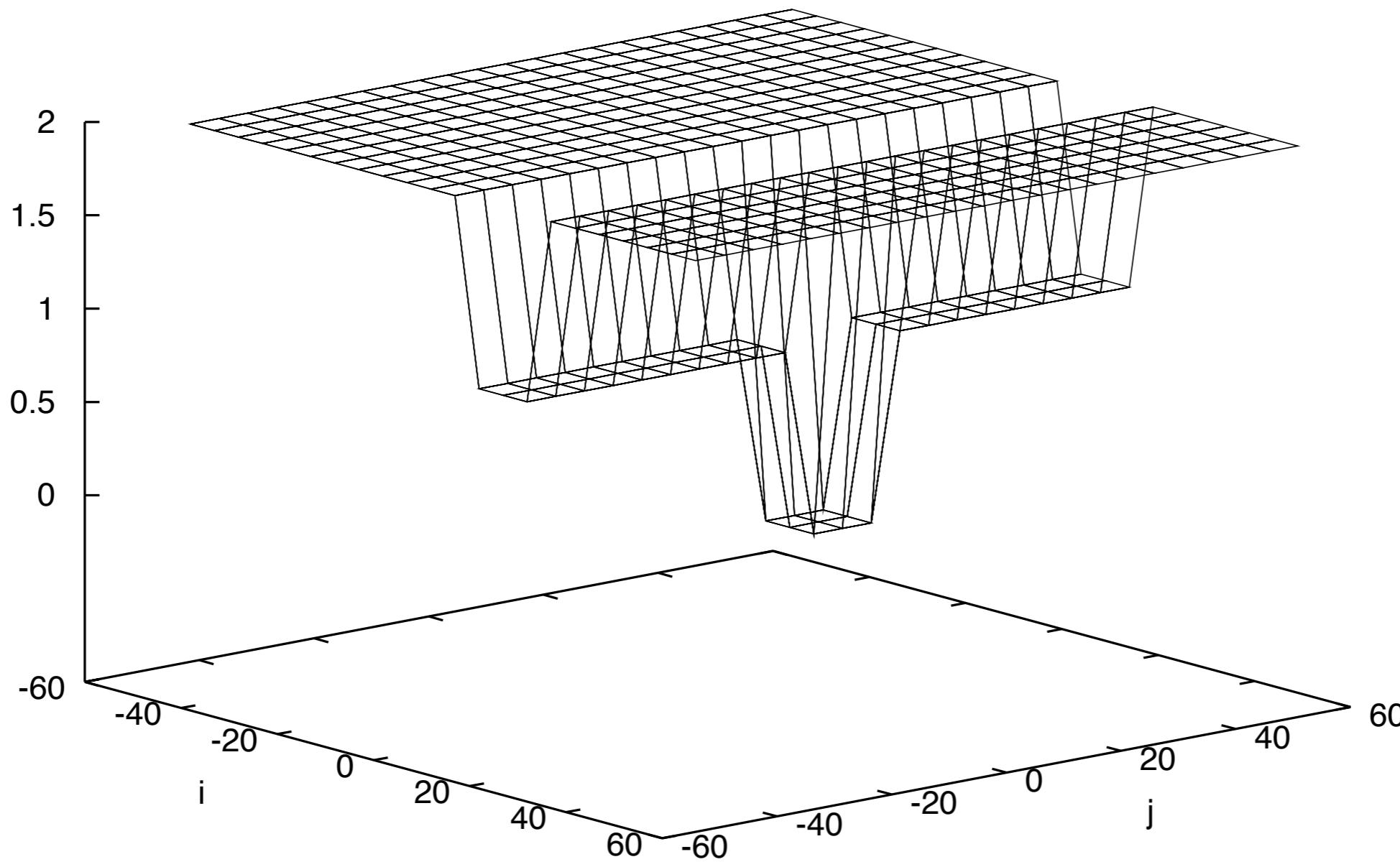


# Approach Level

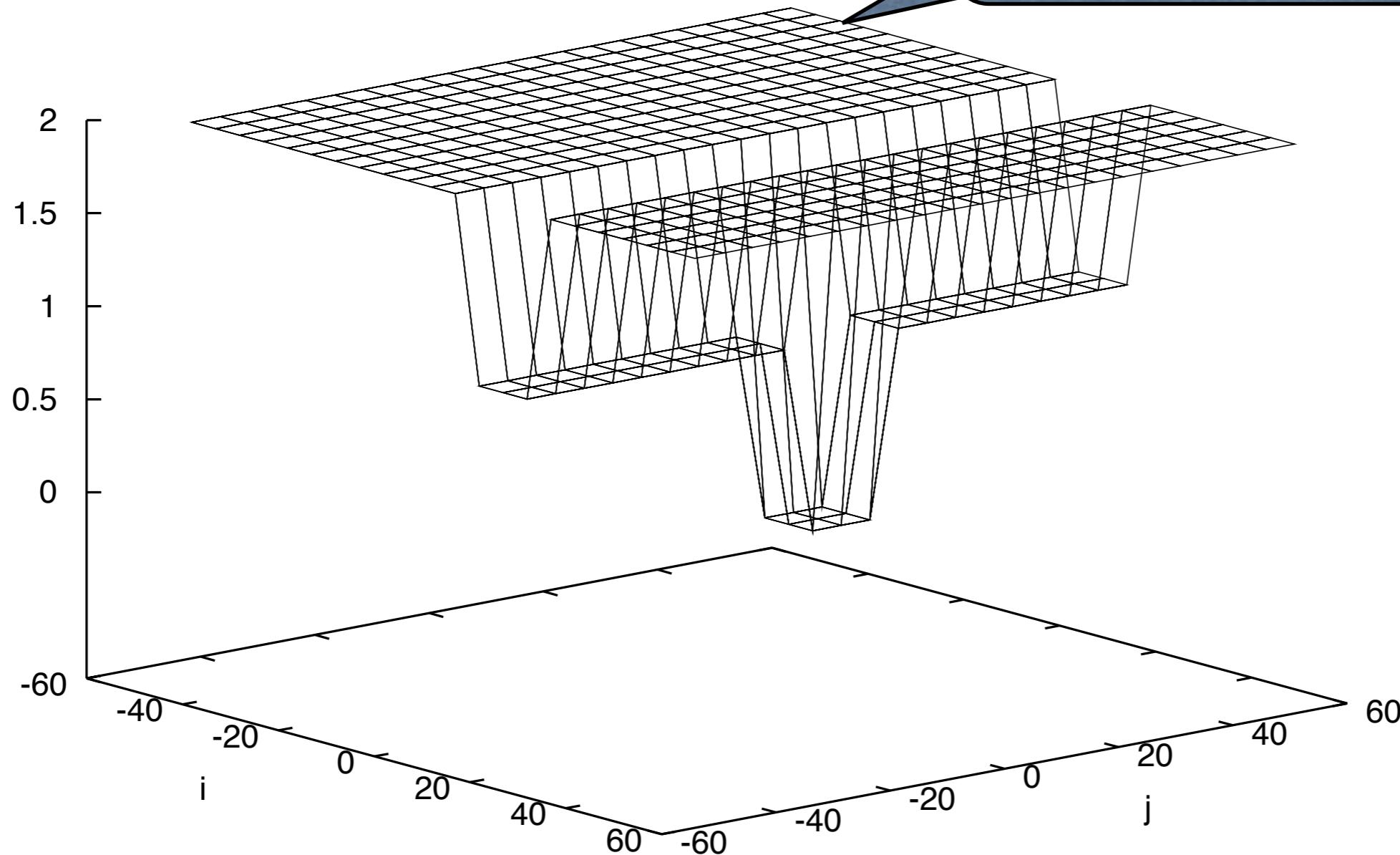


- Number of control dependent edges between goal and chosen path
- Approach = Number of dependent nodes - number of executed nodes

```
void landscape_example(int i, int j) {  
    if (10 <= i && i <= 20) {  
        if (0 <= j && j <= 10) {  
            // target statement  
            // ...  
        }  
    }  
}
```



Plateaux are  
problematic for search



```
void landscape_example(int i, int j) {  
    if (10 <= i && i <= 20) {  
        if (0 <= j && j <= 10) {  
            // target statement  
            // ...  
        }  
    }  
}
```

```
void landscape_example(int i, int j) {  
    if (10 <= i && i <= 20) {  
        if (0 <= j && j <= 10) {  
            // target statement  
            // ...  
        }  
    }  
}
```

How close is this predicate  
to being true?

# Branch Distance

- Critical branch = branch where control flow diverged from reaching target
- Distance to branch = distance to predicate being true / false
- Distance metric for logical formulas
- E.g. distance from true - false = 1

# Branch distance

$a = b$

$\text{abs}(a-b) = 0 ? 0 : \text{abs}(a-b) + K$

$a \neq b$

$\text{abs}(a-b) \neq 0 ? 0 \text{ else } K$

$a < b$

$a - b < 0 ? 0 : (a - b) + K$

$a \leq b$

$a - b \leq 0 ? 0 : (a - b) + K$

$a > b$

$b - a > 0 ? 0 : (b - a) + K$

$a \geq b$

$b - a \geq 0 ? 0 : (b - a) + K$

# Branch distance

Boolean

`true ? 0 : K`

`A && B`

`distance(A) + distance(B)`

`A || B`

`min(distance(A), distance(B))`

`!a`

Move inward and propagate

# Example

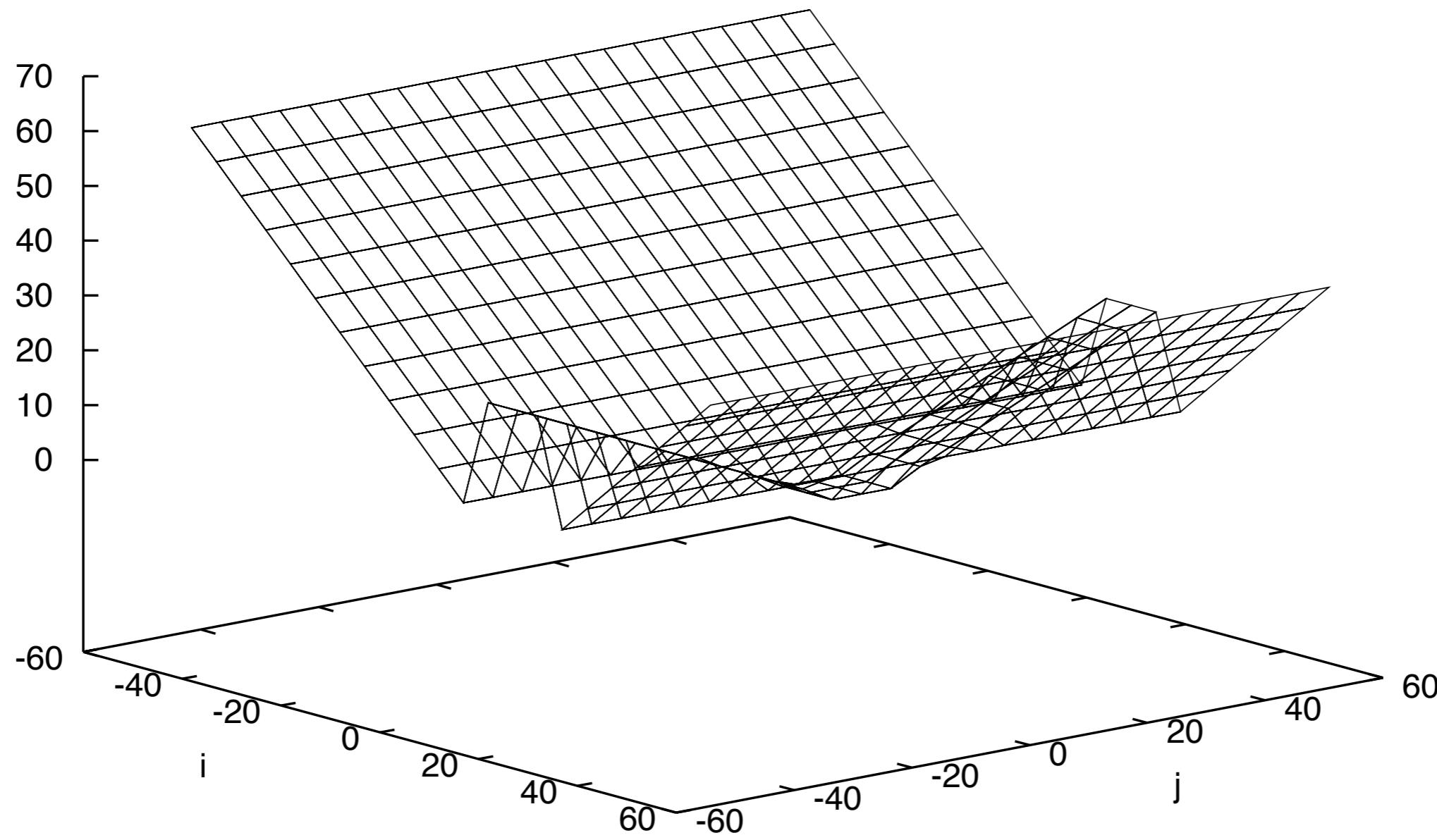
j = 5

- $j \geq 0$
- $j \geq 10$
- $j \geq 10 \text{ && } j \leq 20$

# Branch Distance

- How close is the last branch that would lead closer to the goal to being taken?
- How do we make use of the branch distance predicates?

```
void landscape_example(int i, int j) {  
    if (10 <= i && i <= 20) {  
        if (0 <= j && j <= 10) {  
            // target statement  
            // ...  
        }  
    }  
}
```

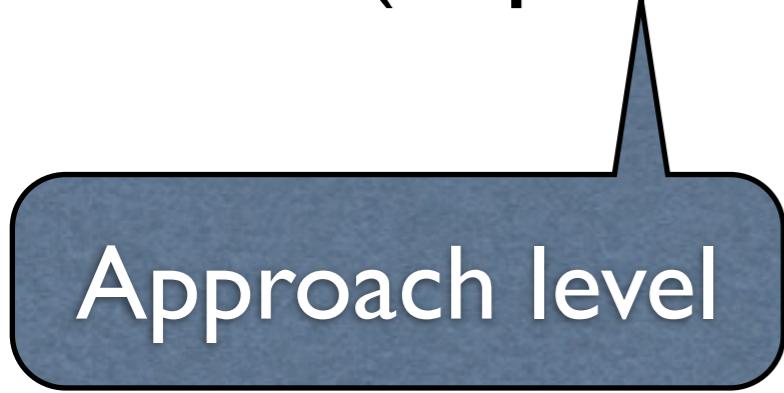


# Control+Branch Distance

- Control distance results in plateaux
- Branch distance results in local optima
- Combination!
- $(\text{dependent} - \text{executed}) + \text{branch\_distance}$

# Control+Branch Distance

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Approach level

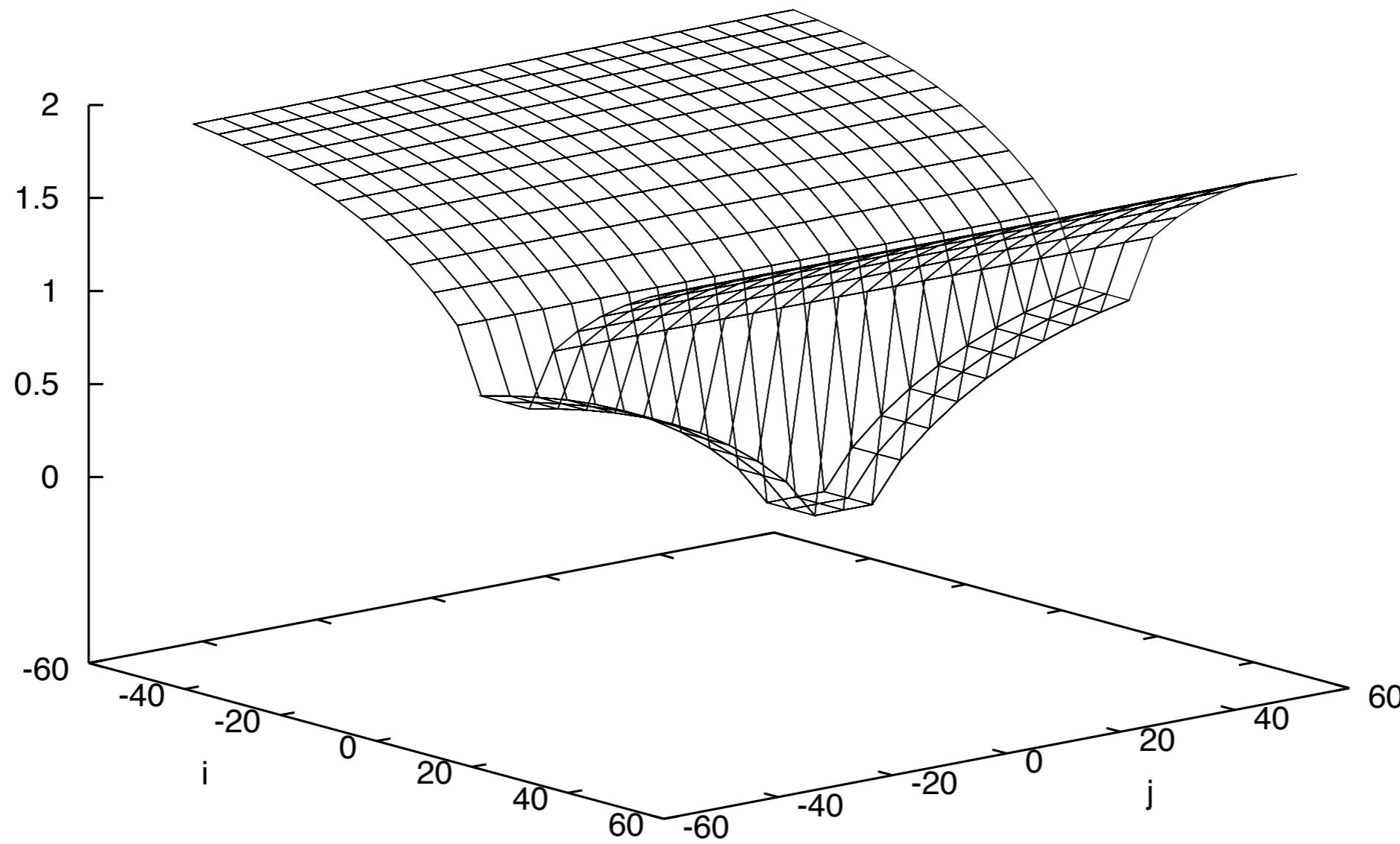
# Control+Branch Distance

- Control distance results in plateaux
- Branch distance results in local optima
- Combination!
- $(\text{dependent} - \text{executed}) + \text{branch\_distance}$

Approach level

Branch distance at node of diversion

```
void landscape_example(int i, int j) {  
    if (10 <= i && i <= 20) {  
        if (0 <= j && j <= 10) {  
            // target statement  
            // ...  
        }  
    }  
}
```



```
void landscape_example(int i, int j) {  
    if (10 <= i && i <= 20) {  
        if (0 <= j && j <= 10) {  
            // target statement  
            // ...  
        }  
    }  
}
```

Which one is closer?

```
void landscape_example(int i, int j) {  
    if (10 <= i && i <= 20) {  
        if (0 <= j && j <= 10) {  
            // target statement  
            // ...  
        }  
    }  
}
```

- $i = 0, j = 0$

Which one is closer?

```
void landscape_example(int i, int j) {  
    if (10 <= i && i <= 20) {  
        if (0 <= j && j <= 10) {  
            // target statement  
            // ...  
        }  
    }  
}
```

- $i = 0, j = 0$
- Approach level: 2

Which one is closer?

```
void landscape_example(int i, int j) {  
    if (10 <= i && i <= 20) {  
        if (0 <= j && j <= 10) {  
            // target statement  
            // ...  
        }  
    }  
}
```

- $i = 0, j = 0$
- Approach level: 2
- Branch distance: 10

Which one is closer?

```
void landscape_example(int i, int j) {  
    if (10 <= i && i <= 20) {  
        if (0 <= j && j <= 10) {  
            // target statement  
            // ...  
        }  
    }  
}
```

- $i = 0, j = 0$

- $i = 10, j = 25$

- Approach level: 2

- Branch distance: 10

Which one is closer?

```
void landscape_example(int i, int j) {  
    if (10 <= i && i <= 20) {  
        if (0 <= j && j <= 10) {  
            // target statement  
            // ...  
        }  
    }  
}
```

- $i = 0, j = 0$
- Approach level: 2
- Branch distance: 10
- $i = 10, j = 25$
- Approach level: 1

Which one is closer?

```
void landscape_example(int i, int j) {  
    if (10 <= i && i <= 20) {  
        if (0 <= j && j <= 10) {  
            // target statement  
            // ...  
        }  
    }  
}
```

- $i = 0, j = 0$

- Approach level: 2

- Branch distance: 10

- $i = 10, j = 25$

- Approach level: 1

- Branch distance: 15

Which one is closer?

# Normalization Functions

- Approach level should be dominant in fitness
- The branch distance has to be normalized

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$$1 - \alpha^{-x}$$

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$$\frac{x}{x + 1}$$

# Normalization Functions

- Approach level should be dominant in fitness
- The branch distance has to be normalized

$$1 - \alpha^{-x}$$

$$\frac{x}{x + 1}$$

Use this!

# Measuring Fitness

To measure the fitness of an individual, we need to

- Execute the test case
- Record the control flow path taken
- Record the branch distance at every branch
- Instrumentation is necessary for this

```
void landscape_example(int i, int j) {  
    if (i >= 10 && i <= 20) {  
        if (j >= 0 && j <= 10) {  
            // target statement  
            // ...  
        }  
    }  
}
```

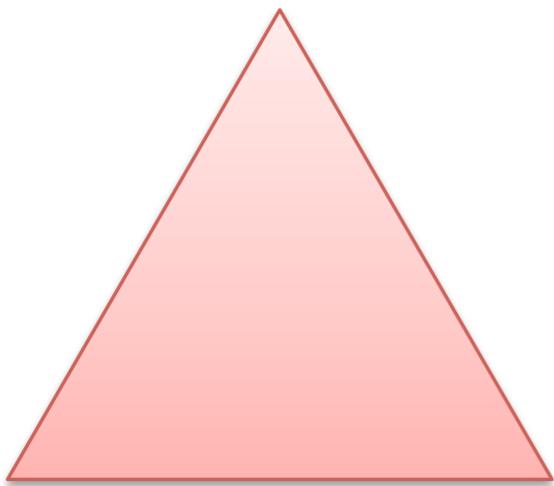
```
void landscape_example(int i, int j) {  
    Trace(Entry);  
    if (i >= 10 && i <= 20) {  
        Trace(Branch1);  
        if (j >= 0 && j <= 10) {  
            Trace(Branch2);  
            // target statement  
            // ...  
        }  
    }  
}
```

```
void landscape_example(int i, int j) {  
    Trace(Entry);  
    if (i >= 10 && i <= 20) {  
        Trace(Branch1);  
        if (j >= 0 && j <= 10) {  
            Trace(Branch2);  
            // target statement  
            // ...  
        }  
    }  
}
```

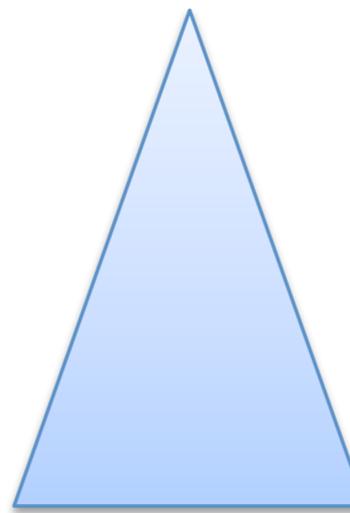
```
void landscape_example(int i, int j) {  
    Trace(Entry);  
    Distance(Branch1, (10 - i <= 0 ? 0 : 10 - i)  
              + (i - 20 <= 0 ? 0 : i - 20));  
    if (i >= 10 && i <= 20) {  
        Trace(Branch1);  
        Distance(Branch2, (0-j <= 0 ? 0 : 0 - j)  
                  + (j - 10 <= 0 ? 0 : j - 10))  
        if (j >= 0 && j <= 10) {  
            Trace(Branch2);  
            // target statement  
            // ...  
        }  
    }  
}
```

# Example

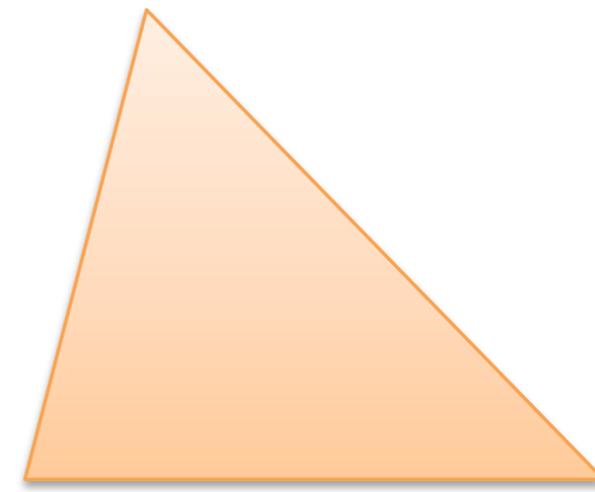
Classify triangle by the length of the sides



Equilateral



Isosceles

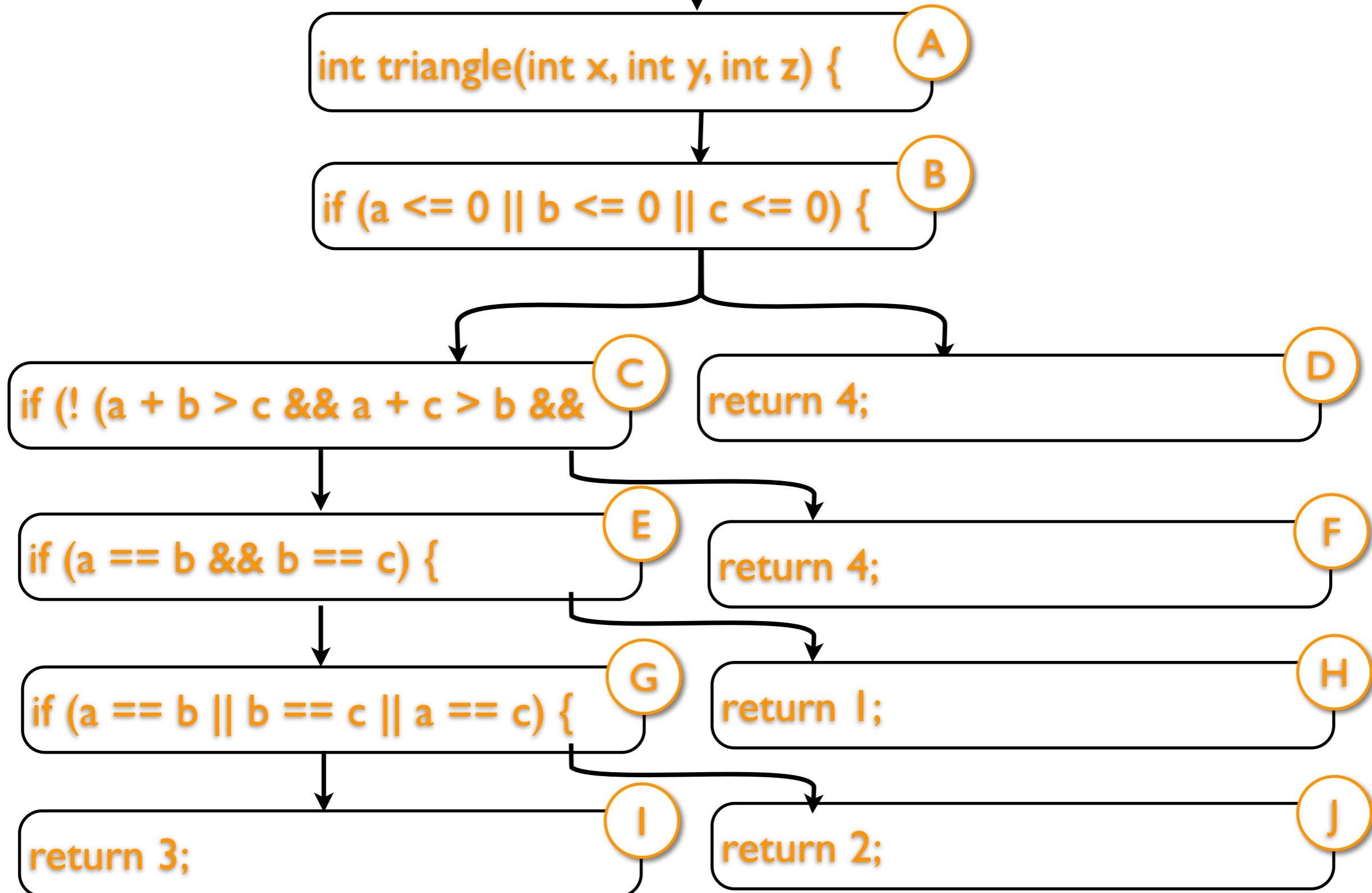


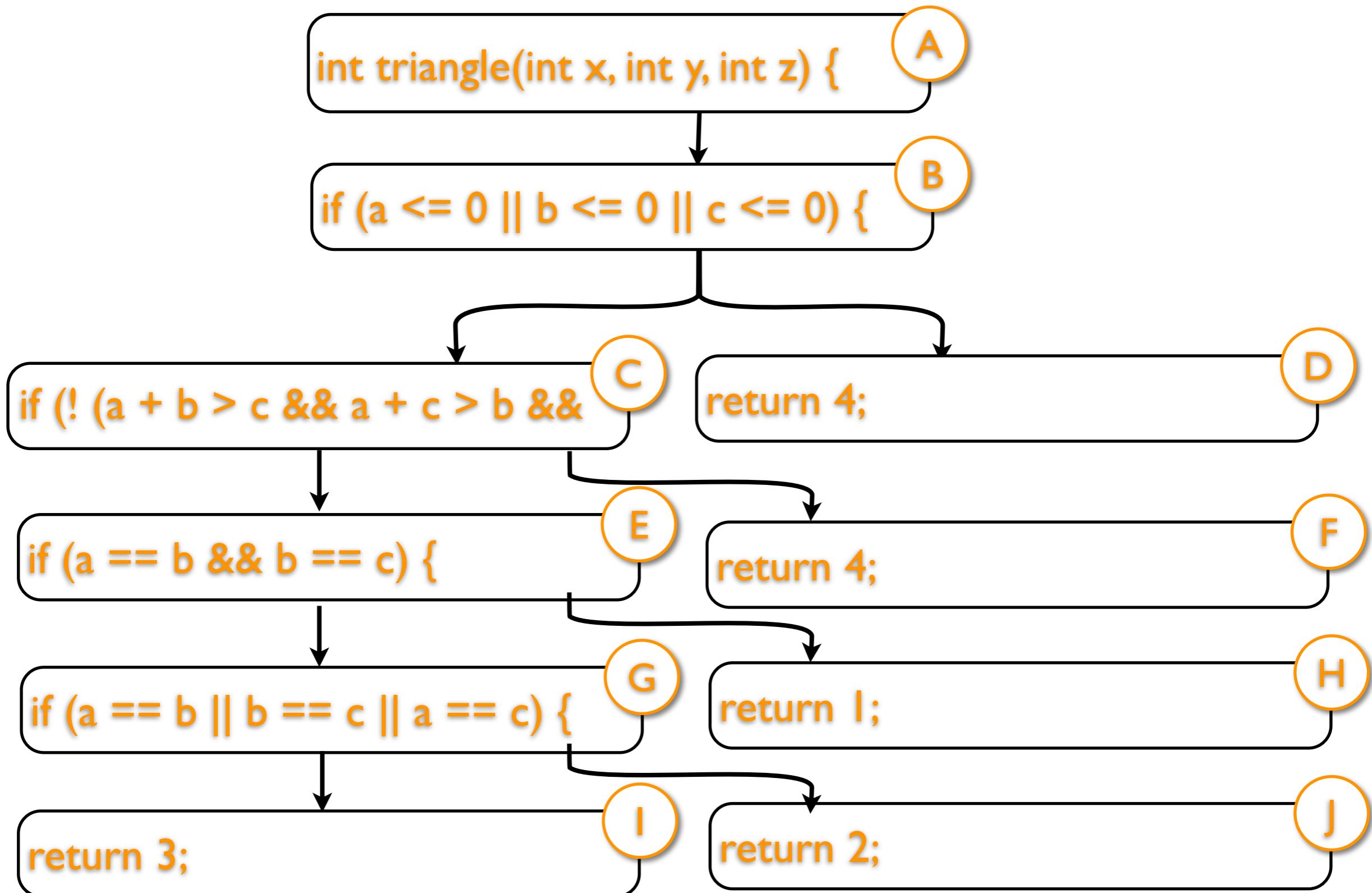
Scalene

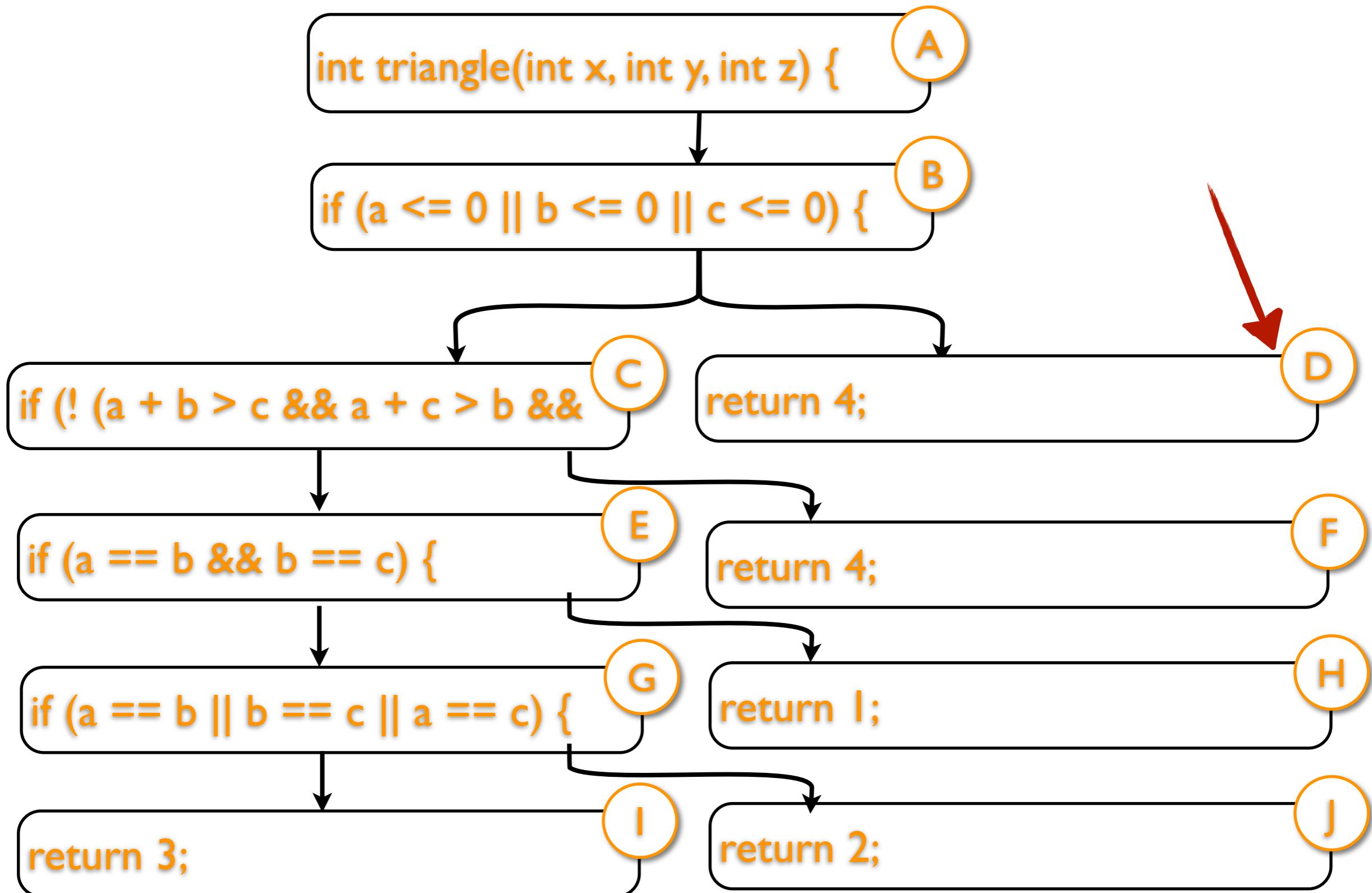
```
int triangle(int a, int b, int c) {  
  
    if (a <= 0 || b <= 0 || c <= 0) {  
        return 4; // invalid  
    }  
    if (!(a + b > c && a + c > b && b + c > a)) {  
        return 4; // invalid  
    }  
    if (a == b && b == c) {  
        return 1; // equilateral  
    }  
    if (a == b || b == c || a == c) {  
        return 2; // isosceles  
    }  
  
    return 3; // scalene  
}
```

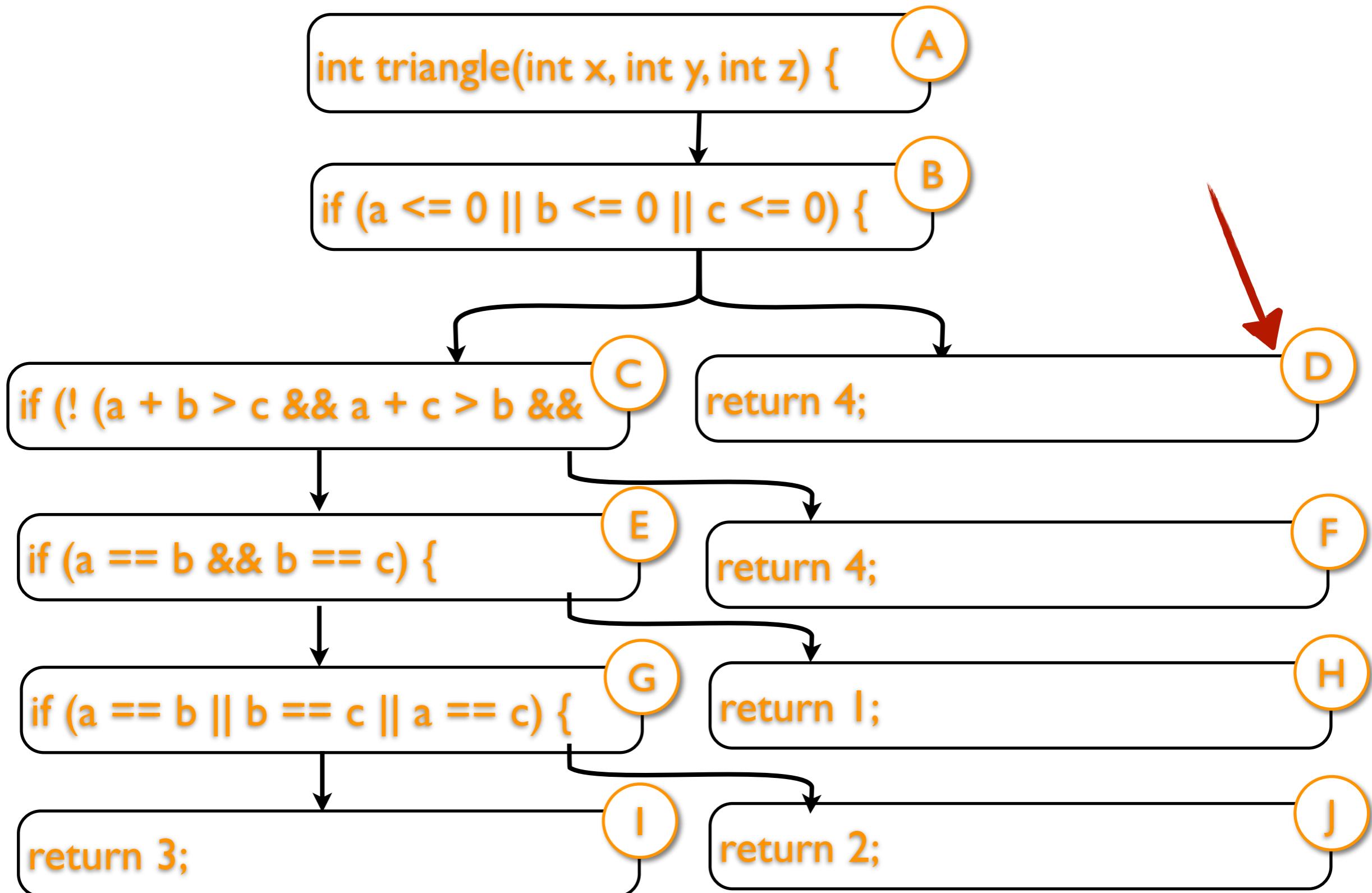
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int triangle(int a, int b, int c) {  
  
    if (a <= 0 || b <= 0 || c <= 0) {  
        return 4; // invalid ←  
    }  
    if (!(a + b > c && a + c > b && b + c > a)) {  
        return 4; // invalid  
    }  
    if (a == b && b == c) {  
        return 1; // equilateral  
    }  
    if (a == b || b == c || a == c) {  
        return 2; // isosceles  
    }  
  
    return 3; // scalene  
}
```

# int triangle

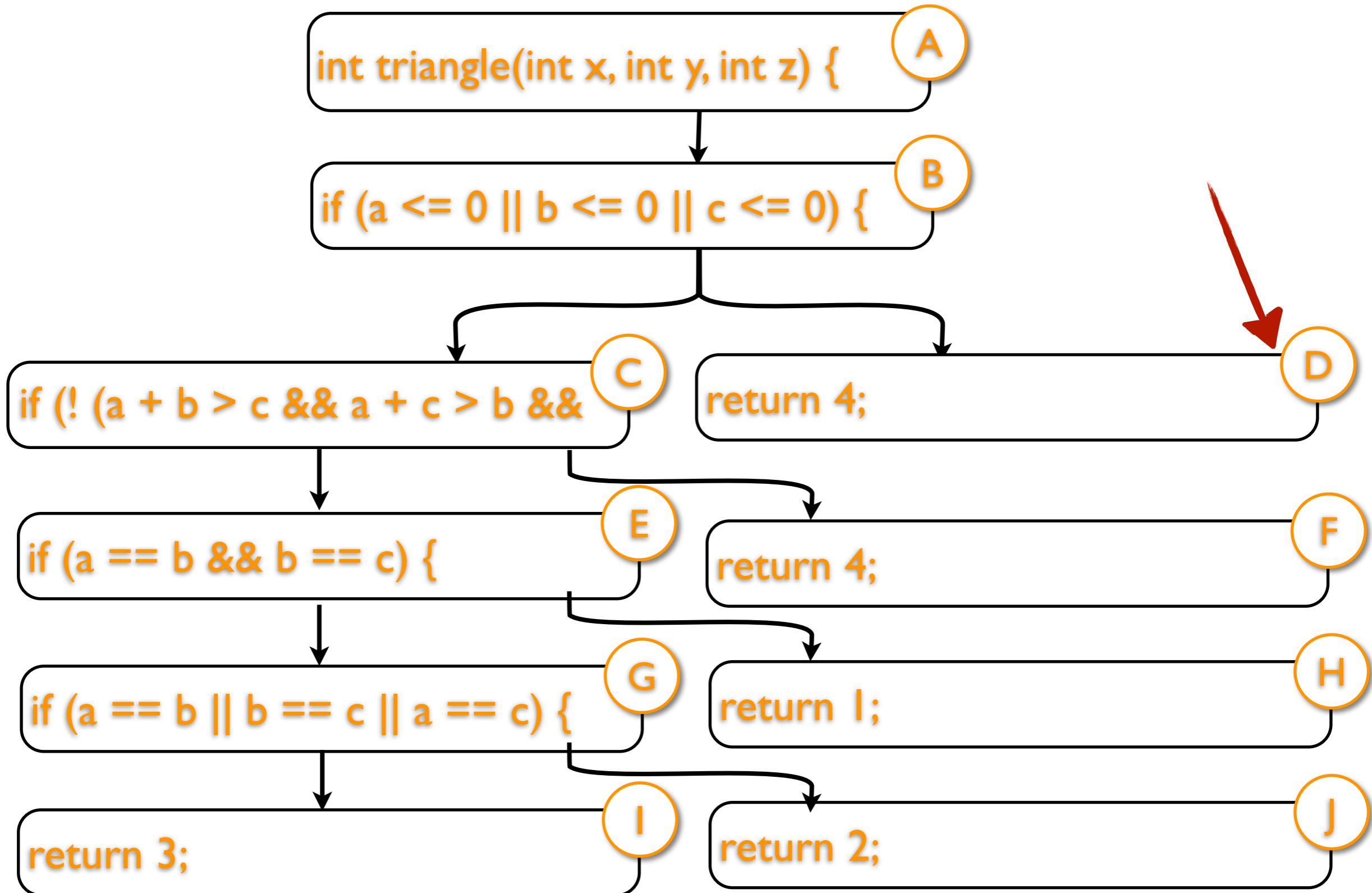




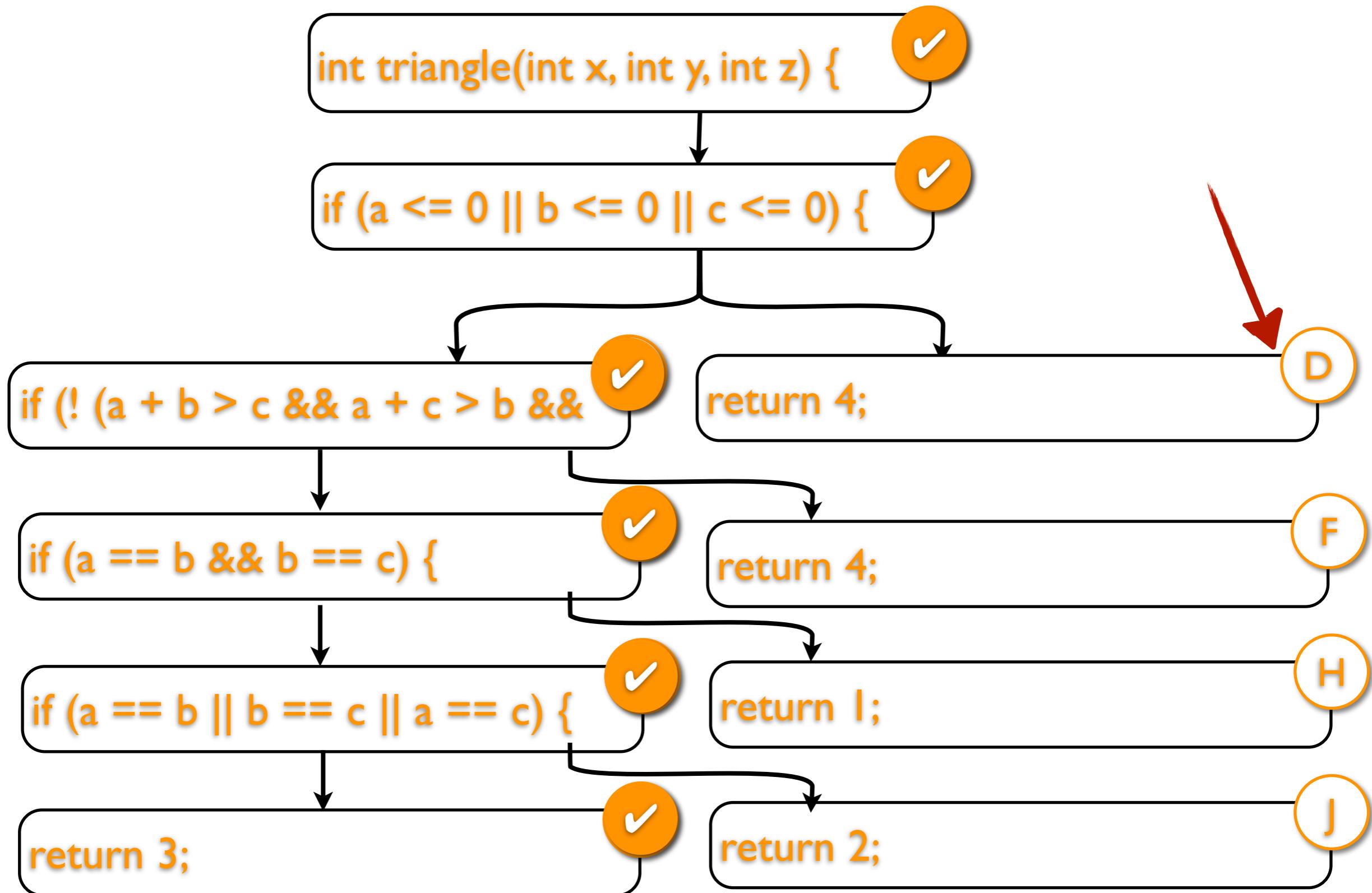




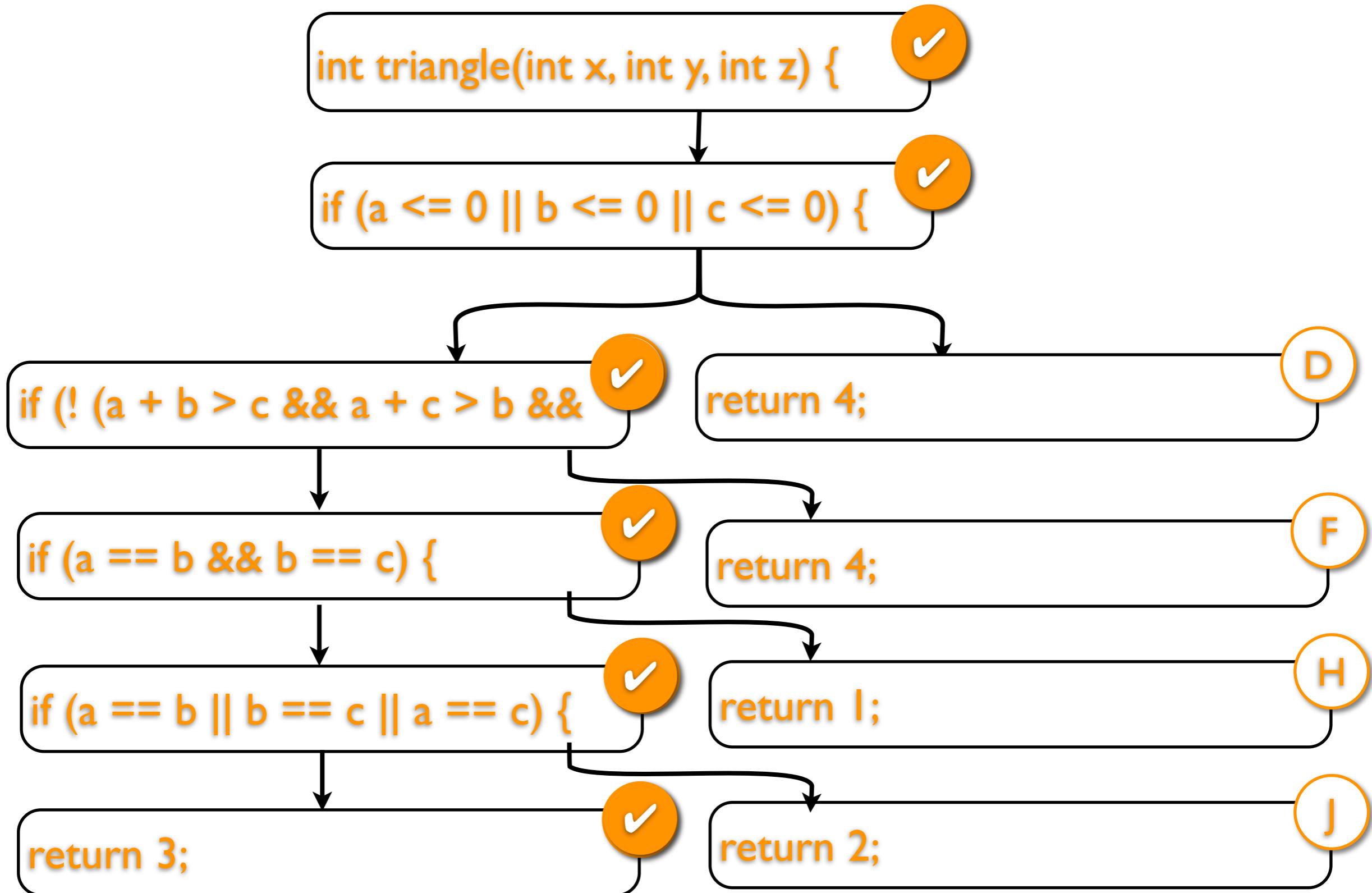
(21, 53, 38)



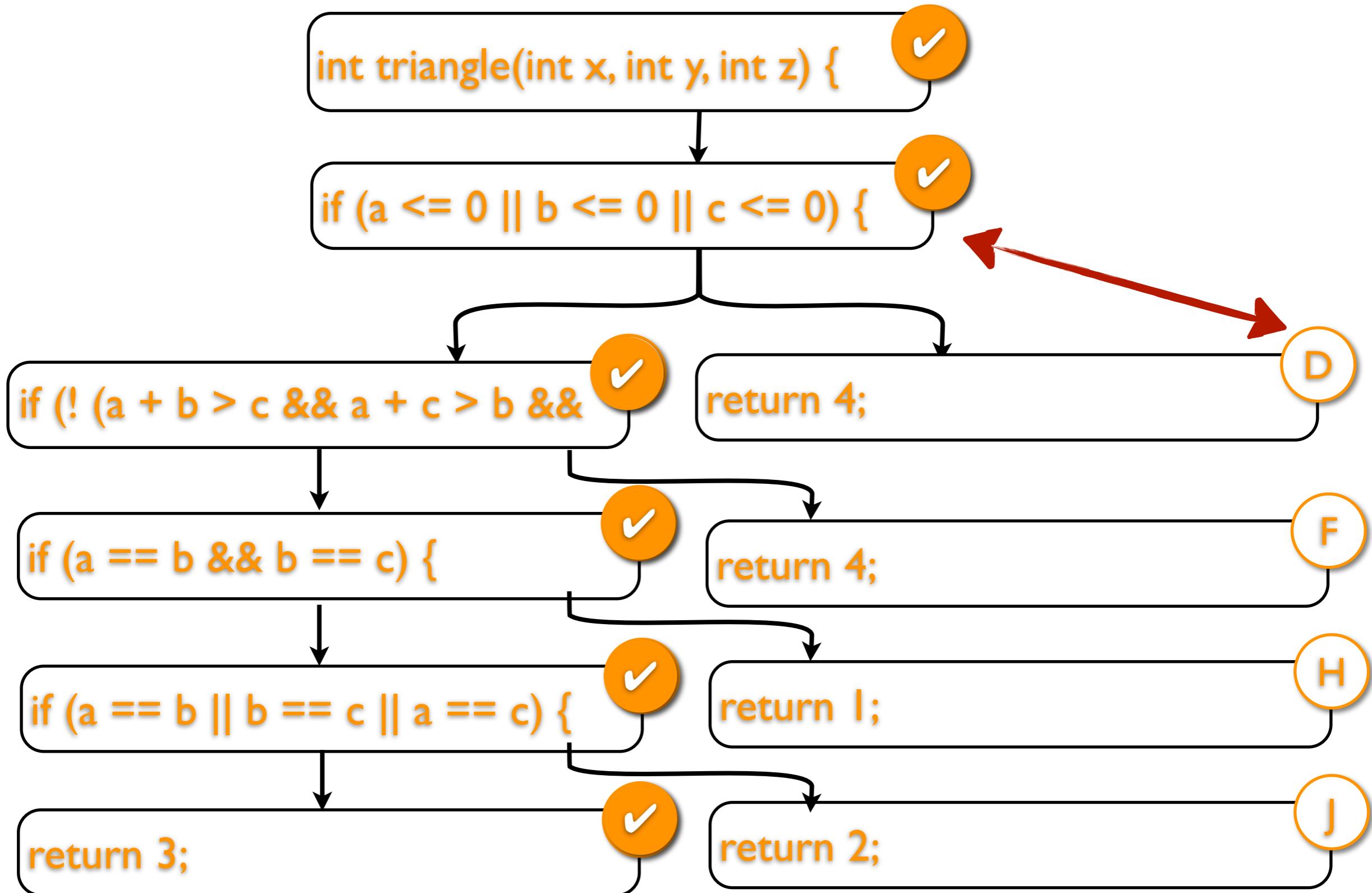
(21, 53, 38)



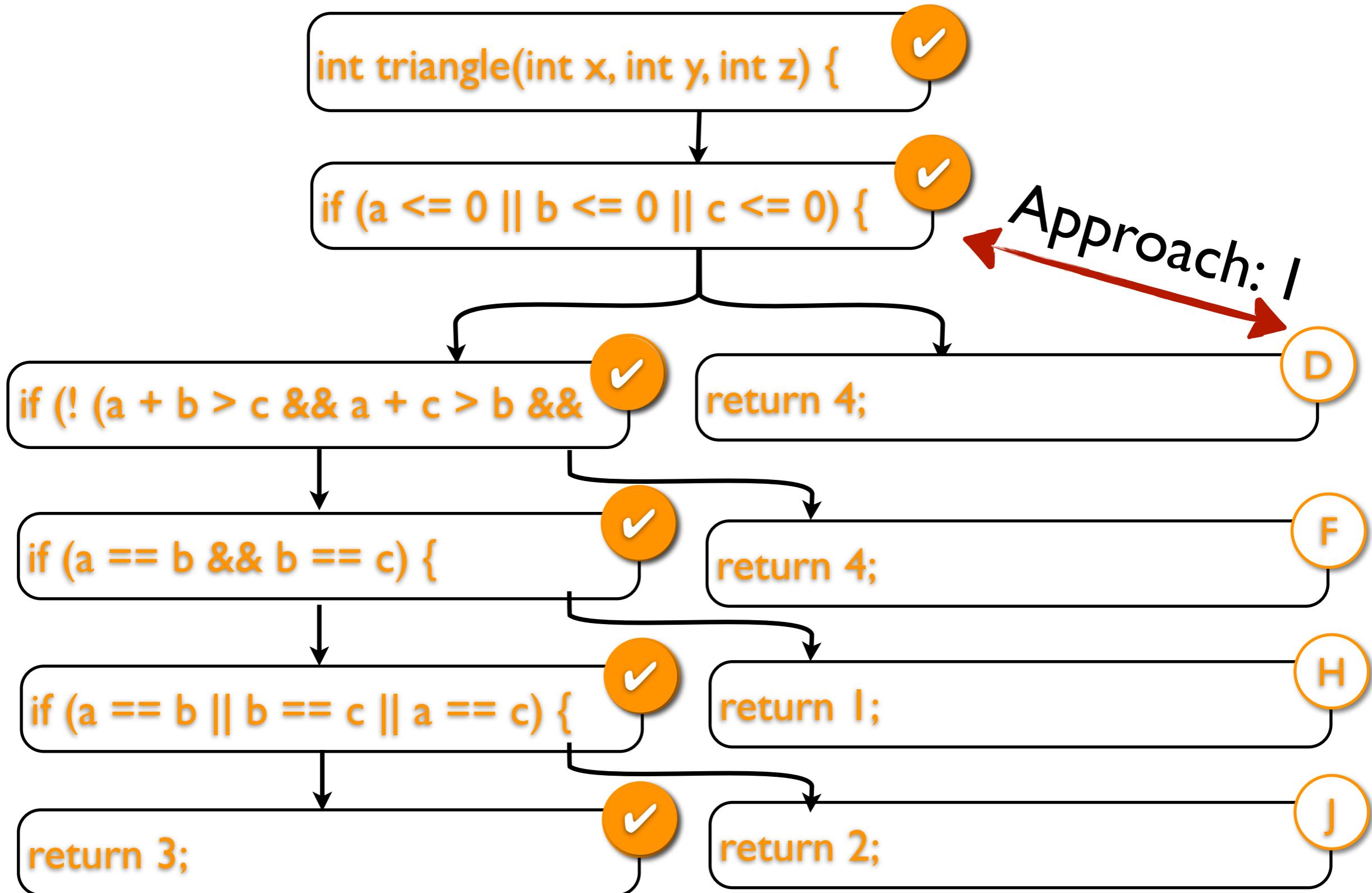
(21, 53, 38)



(21, 53, 38)



(21, 53, 38)



# Branch Distance

# Branch Distance

- Input (21, 53, 38)

# Branch Distance

- Input (21, 53, 38)
- Distance to ( $a \leq 0 \parallel b \leq 0 \parallel c \leq 0$ )?

# Branch Distance

- Input (21, 53, 38)
- Distance to  $(a \leq 0 \parallel b \leq 0 \parallel c \leq 0)?$
- Minimum of distance to  $a \leq 0, b \leq 0, c \leq 0$

# Branch Distance

- Input (21, 53, 38)
- Distance to ( $a \leq 0 \parallel b \leq 0 \parallel c \leq 0$ )?
- Minimum of distance to  $a \leq 0, b \leq 0, c \leq 0$
- $a \leq 0$

# Branch Distance

- Input (21, 53, 38)
- Distance to ( $a \leq 0 \parallel b \leq 0 \parallel c \leq 0$ )?
- Minimum of distance to  $a \leq 0, b \leq 0, c \leq 0$
- $a \leq 0$
- $a - b \leq 0 ? 0 : (a - b) + K$  ( $K = 1$ )

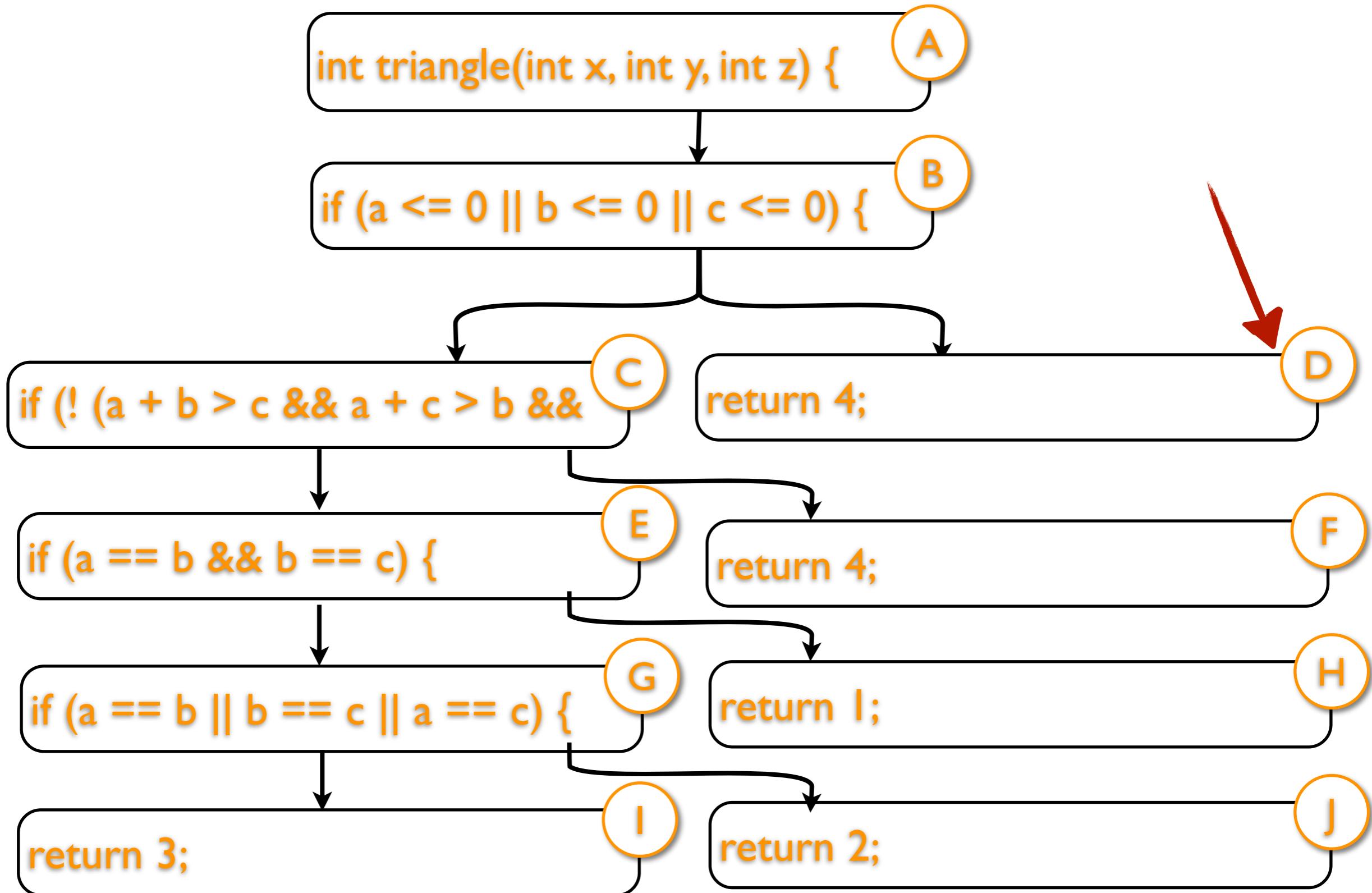
# Branch Distance

- Input (21, 53, 38)
- Distance to ( $a \leq 0 \parallel b \leq 0 \parallel c \leq 0$ )?
- Minimum of distance to  $a \leq 0, b \leq 0, c \leq 0$
- $a \leq 0$
- $a - b \leq 0 ? 0 : (a - b) + K$  ( $K = 1$ )
- Minimum of (21, 53, 38) = 22

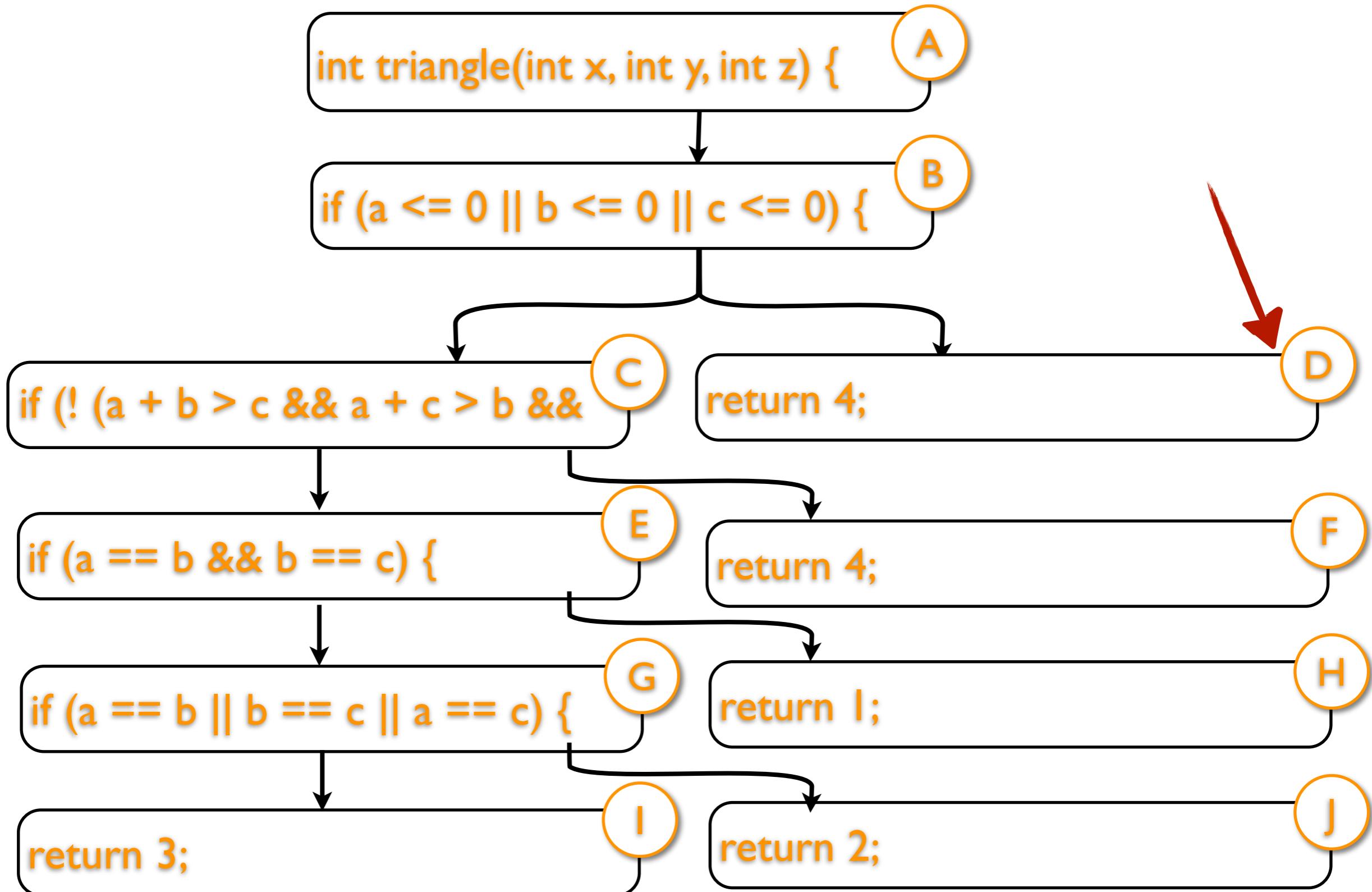
# Fitness

- Approach level:  $I$
- Branch distance: 22
- Fitness =  $I + 22/(22+I) = 1.96$

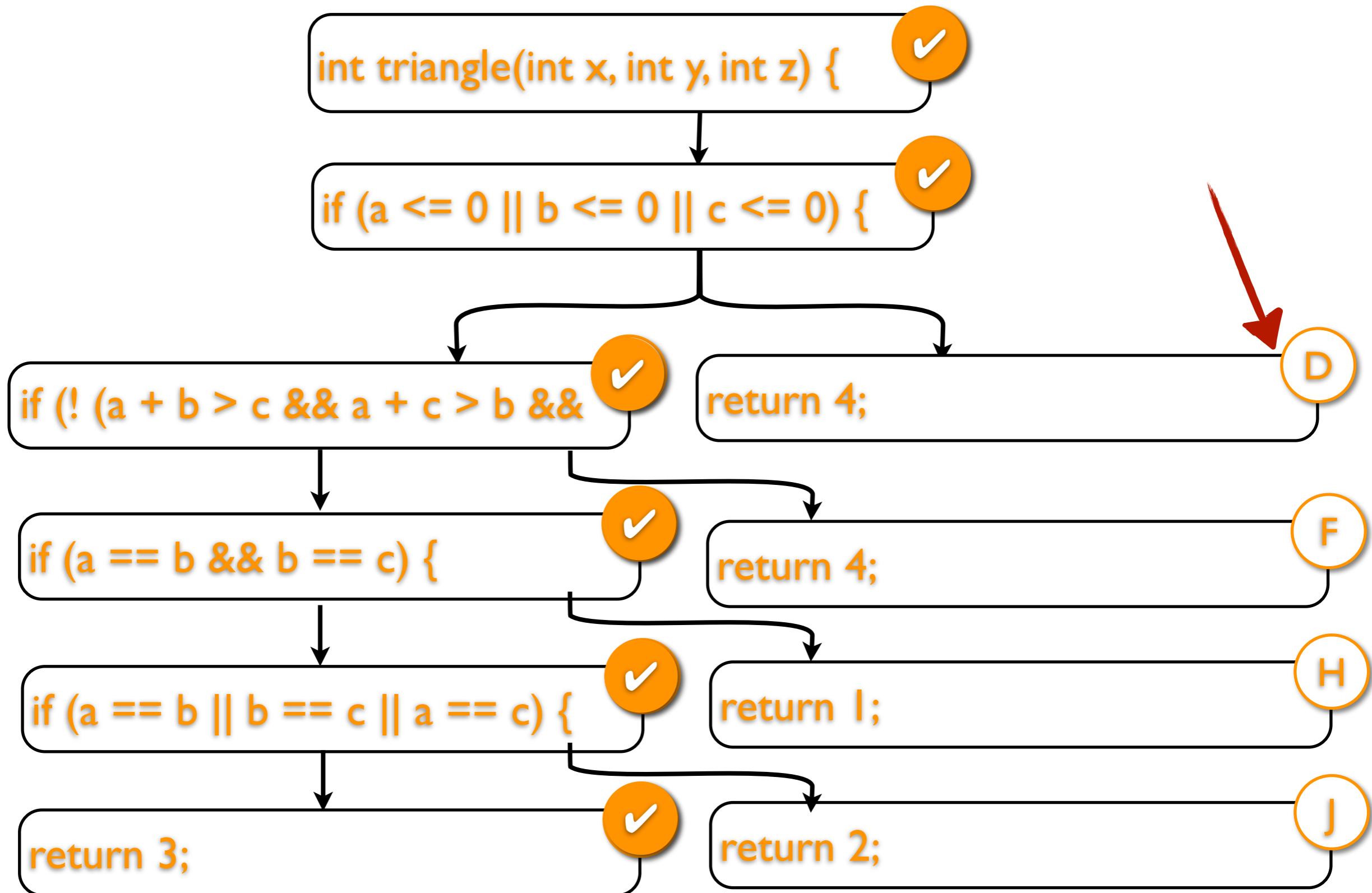
(21, 53, 38)



(20, 53, 38)



(20, 53, 38)



# Search

# Search

- Next best solution:

# Search

- Next best solution:
- (20, 53, 38) - Approach I, Branch 2I

# Search

- Next best solution:
- (20, 53, 38) - Approach I, Branch 21
- (19, 53, 38) - Approach I, Branch 20

# Search

- Next best solution:
- (20, 53, 38) - Approach I, Branch 21
- (19, 53, 38) - Approach I, Branch 20
- (18, 53, 38) - Approach I, Branch 19

# Search

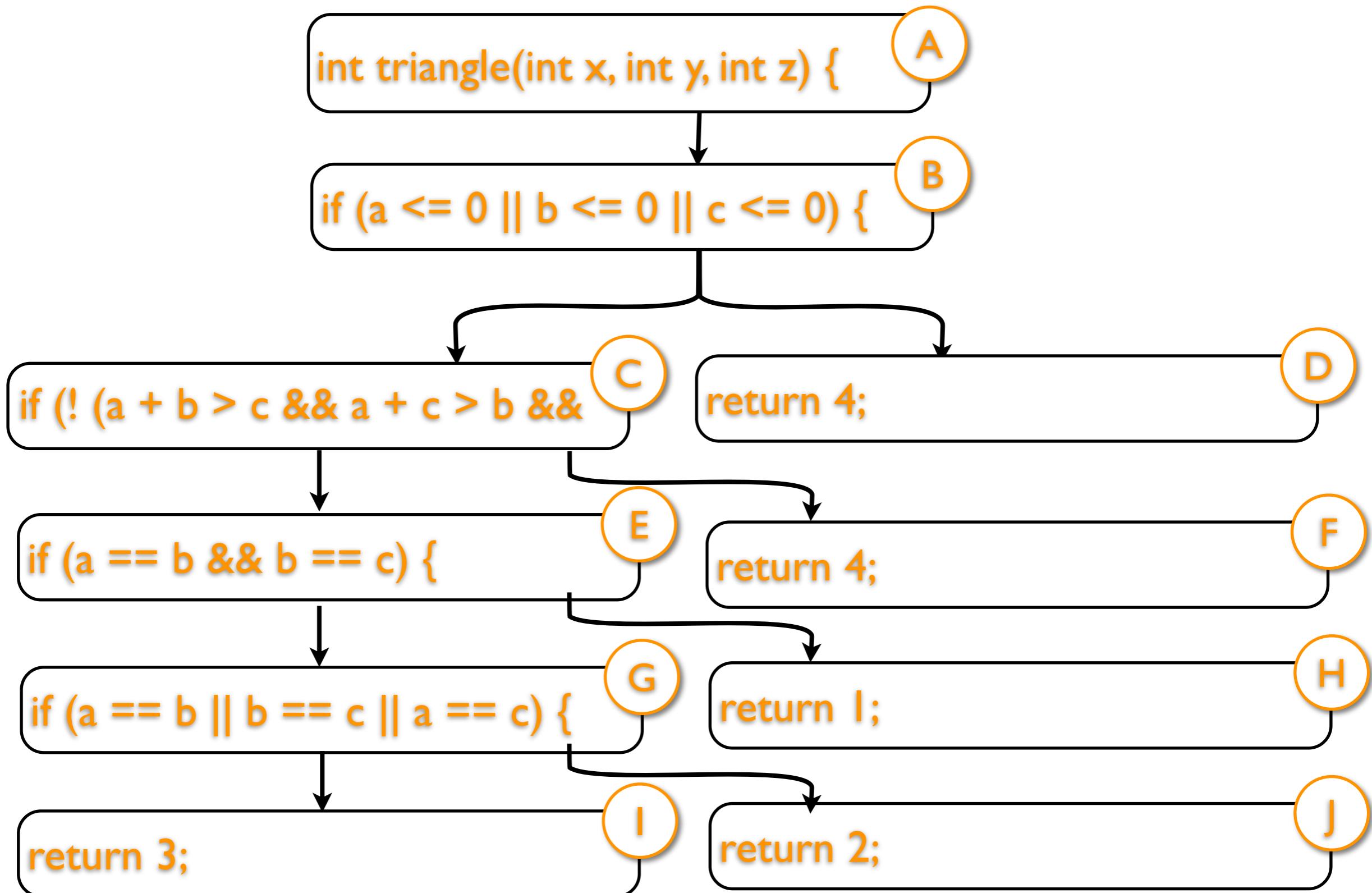
- Next best solution:
- (20, 53, 38) - Approach I, Branch 21
- (19, 53, 38) - Approach I, Branch 20
- (18, 53, 38) - Approach I, Branch 19
- (17, 53, 38) - Approach I, Branch 18

# Search

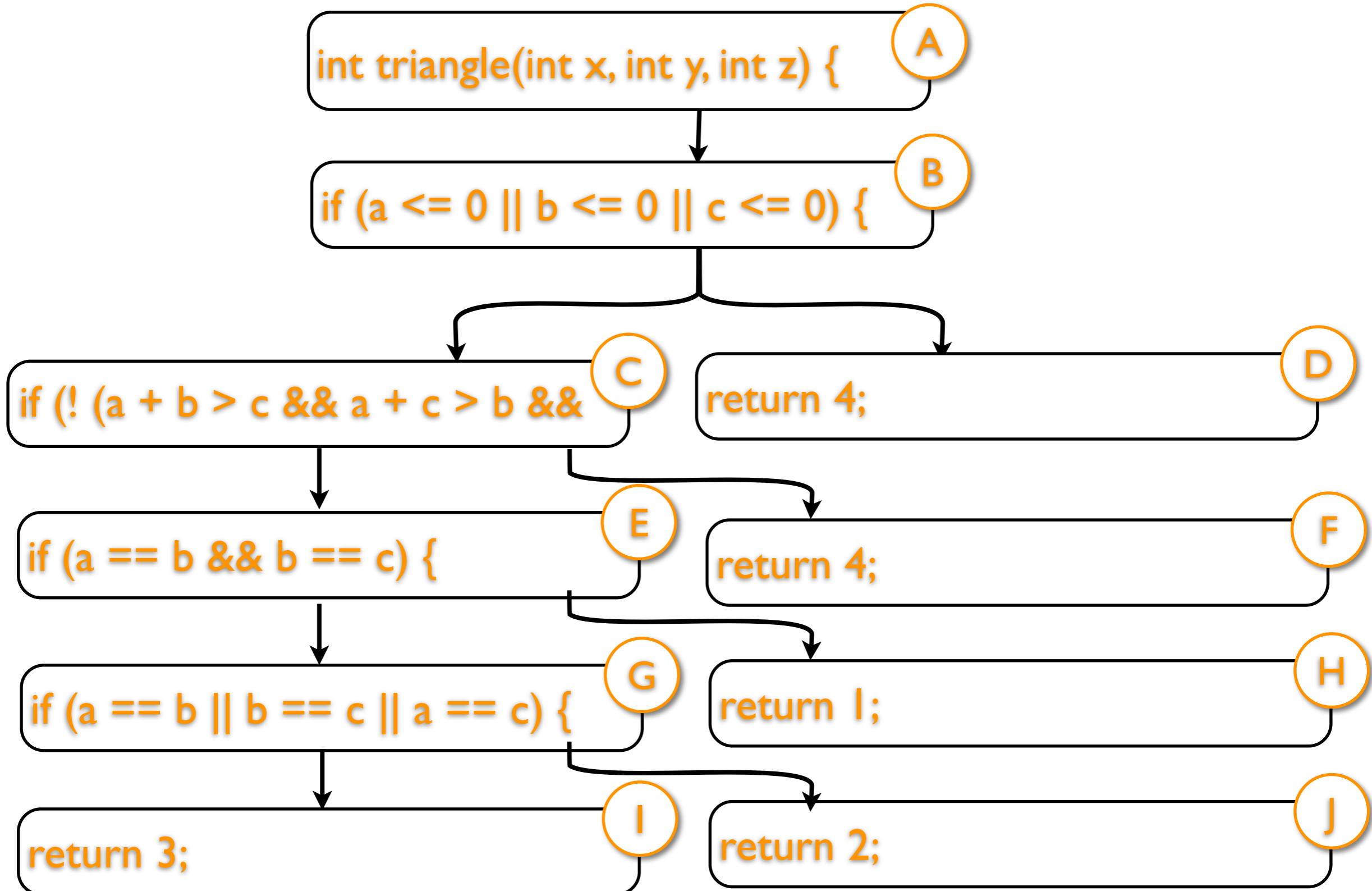
- Next best solution:
- (20, 53, 38) - Approach I, Branch 21
- (19, 53, 38) - Approach I, Branch 20
- (18, 53, 38) - Approach I, Branch 19
- (17, 53, 38) - Approach I, Branch 18
- (16, 53, 38) - Approach I, Branch 17

# Search

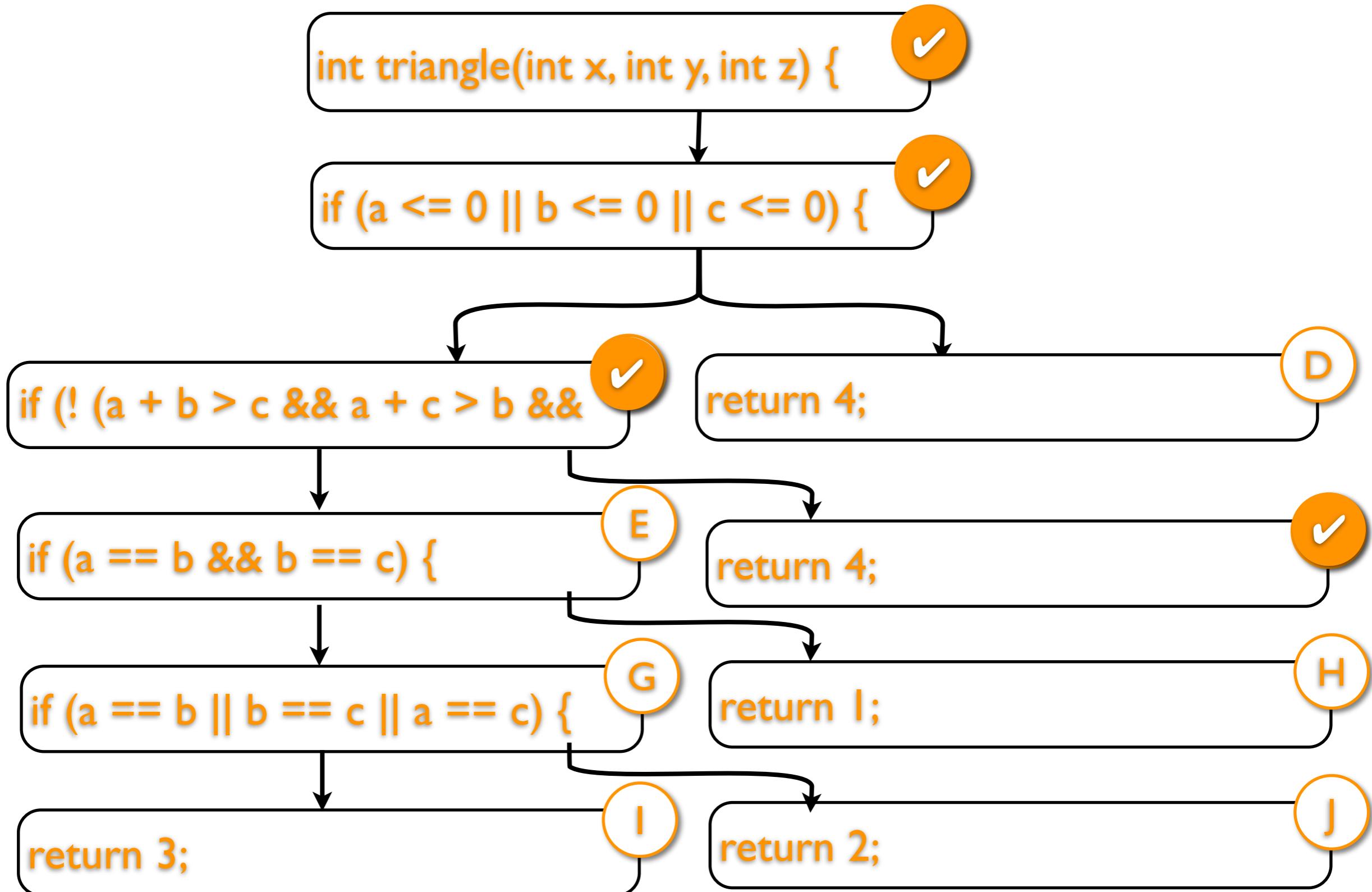
- Next best solution:
- (20, 53, 38) - Approach I, Branch 21
- (19, 53, 38) - Approach I, Branch 20
- (18, 53, 38) - Approach I, Branch 19
- (17, 53, 38) - Approach I, Branch 18
- (16, 53, 38) - Approach I, Branch 17
- (15, 53, 38)



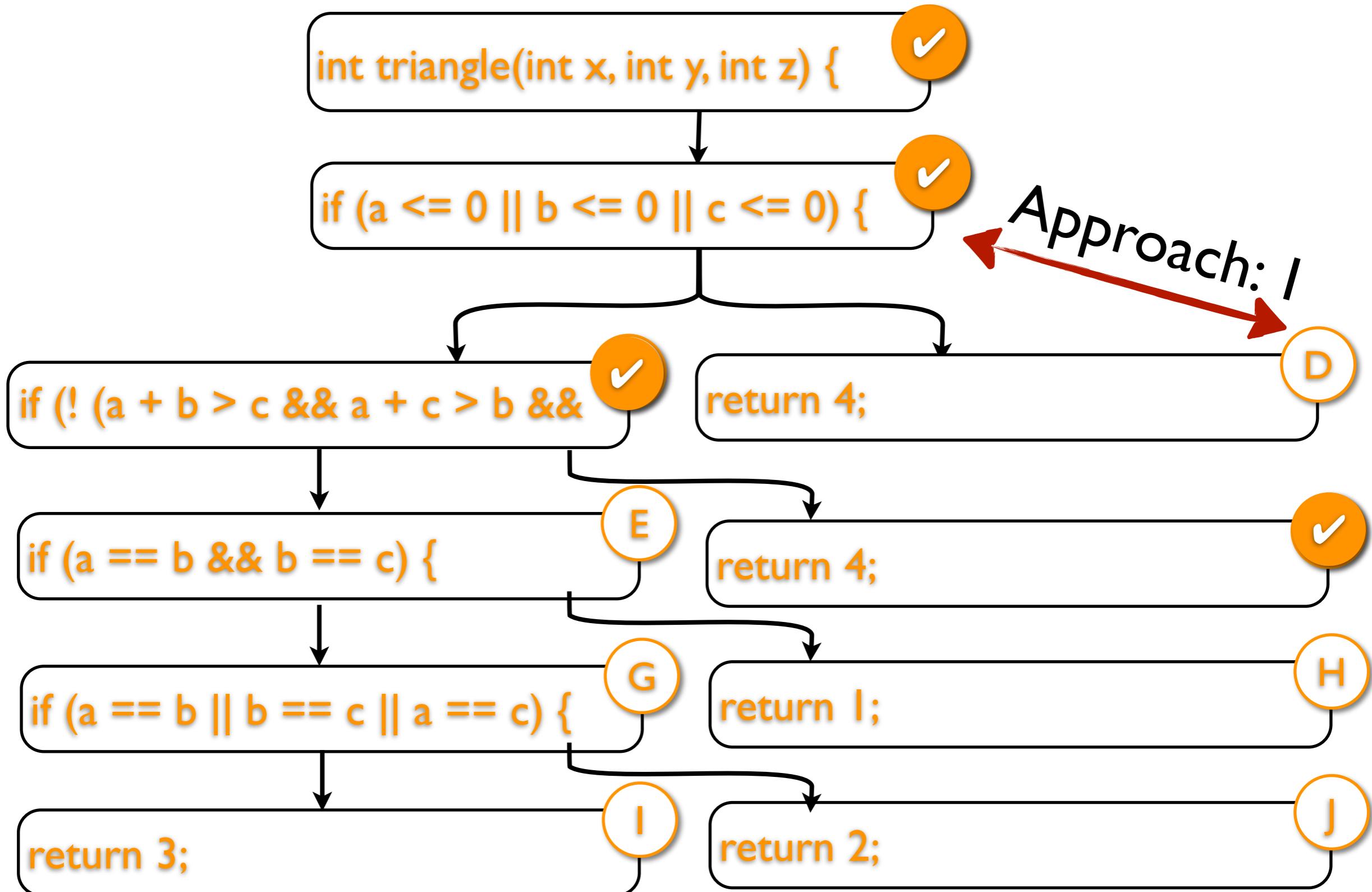
(15, 53, 38)



(15, 53, 38)



(15, 53, 38)



# Search

# Search

- (15, 53, 38) - Approach I, Branch 16

# Search

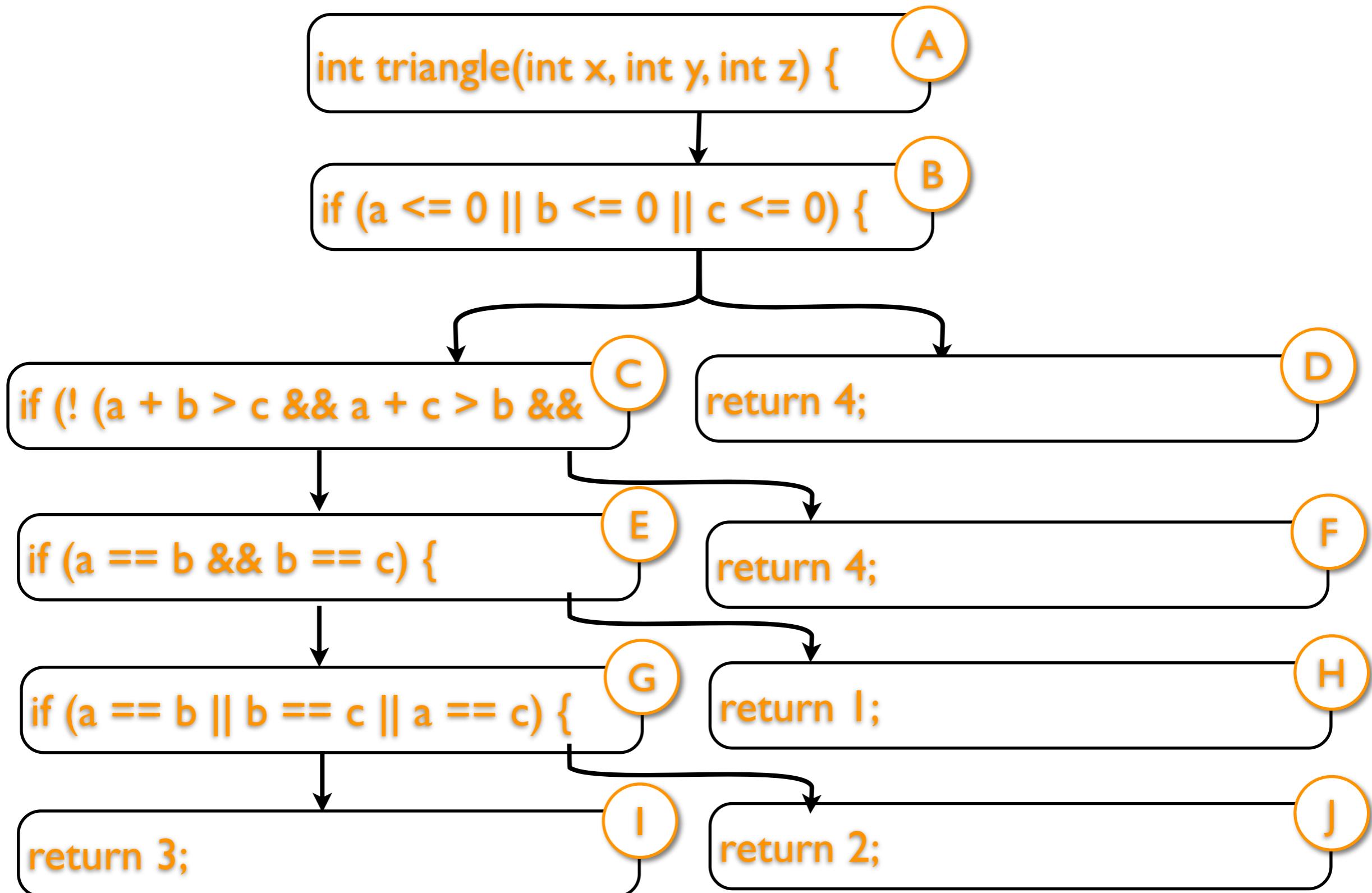
- (15, 53, 38) - Approach I, Branch 16
- (14, 53, 38) - Approach I, Branch 15

# Search

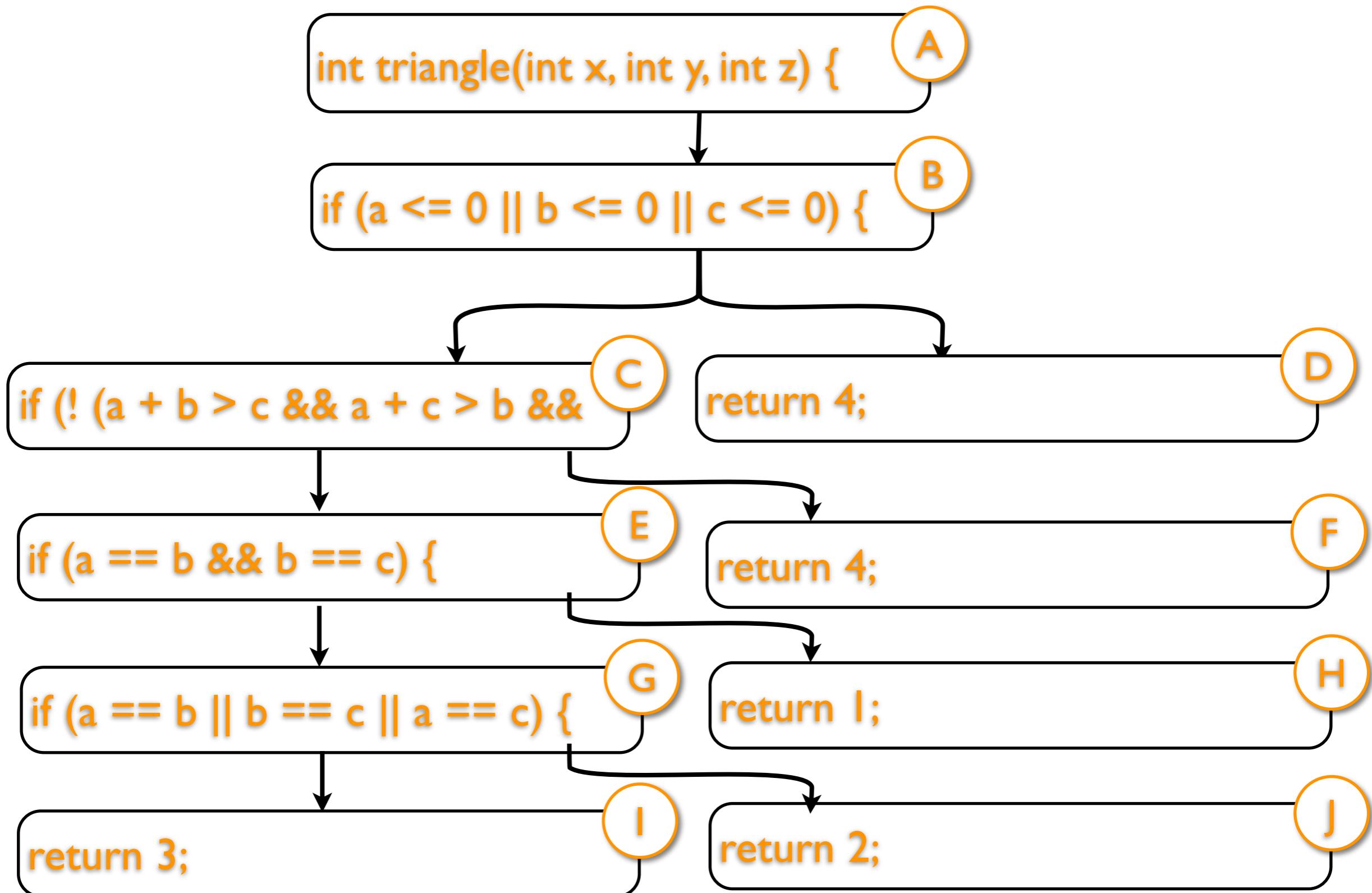
- (15, 53, 38) - Approach I, Branch 16
- (14, 53, 38) - Approach I, Branch 15
- ...

# Search

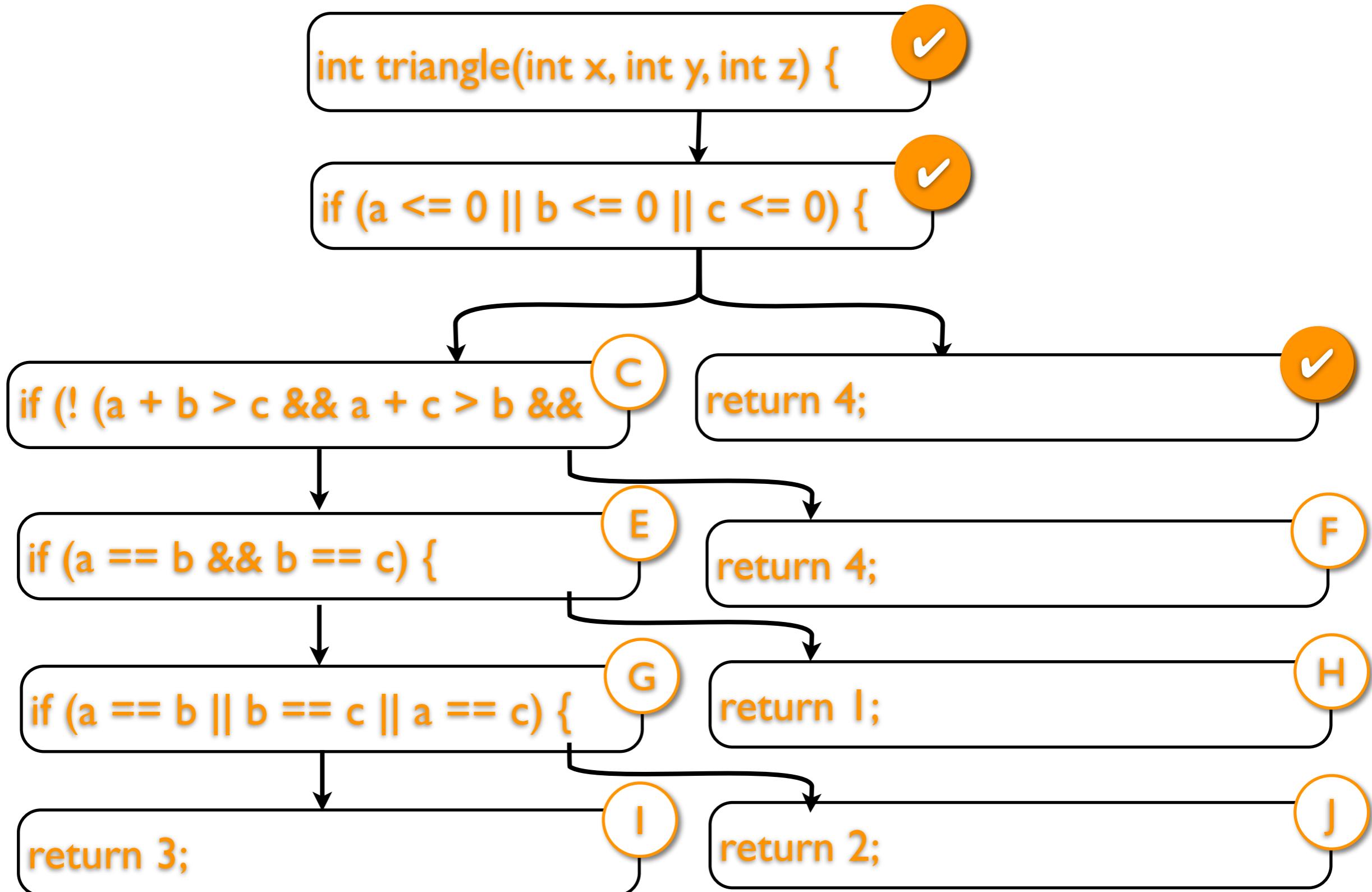
- (15, 53, 38) - Approach 1, Branch 16
- (14, 53, 38) - Approach 1, Branch 15
- ...
- (0, 53, 38) - Approach 0, Branch 0



(0, 53, 38)



(0, 53, 38)



```
int triangle(int a, int b, int c) {  
  
    if (a <= 0 || b <= 0 || c <= 0) {  
        return 4; // invalid  
    }  
    if (!(a + b > c && a + c > b && b + c > a)) {  
        return 4; // invalid  
    }  
    if (a == b && b == c) {  
        return 1; // equilateral  
    }  
    if (a == b || b == c || a == c) {  
        return 2; // isosceles  
    }  
  
    return 3; // scalene  
}
```

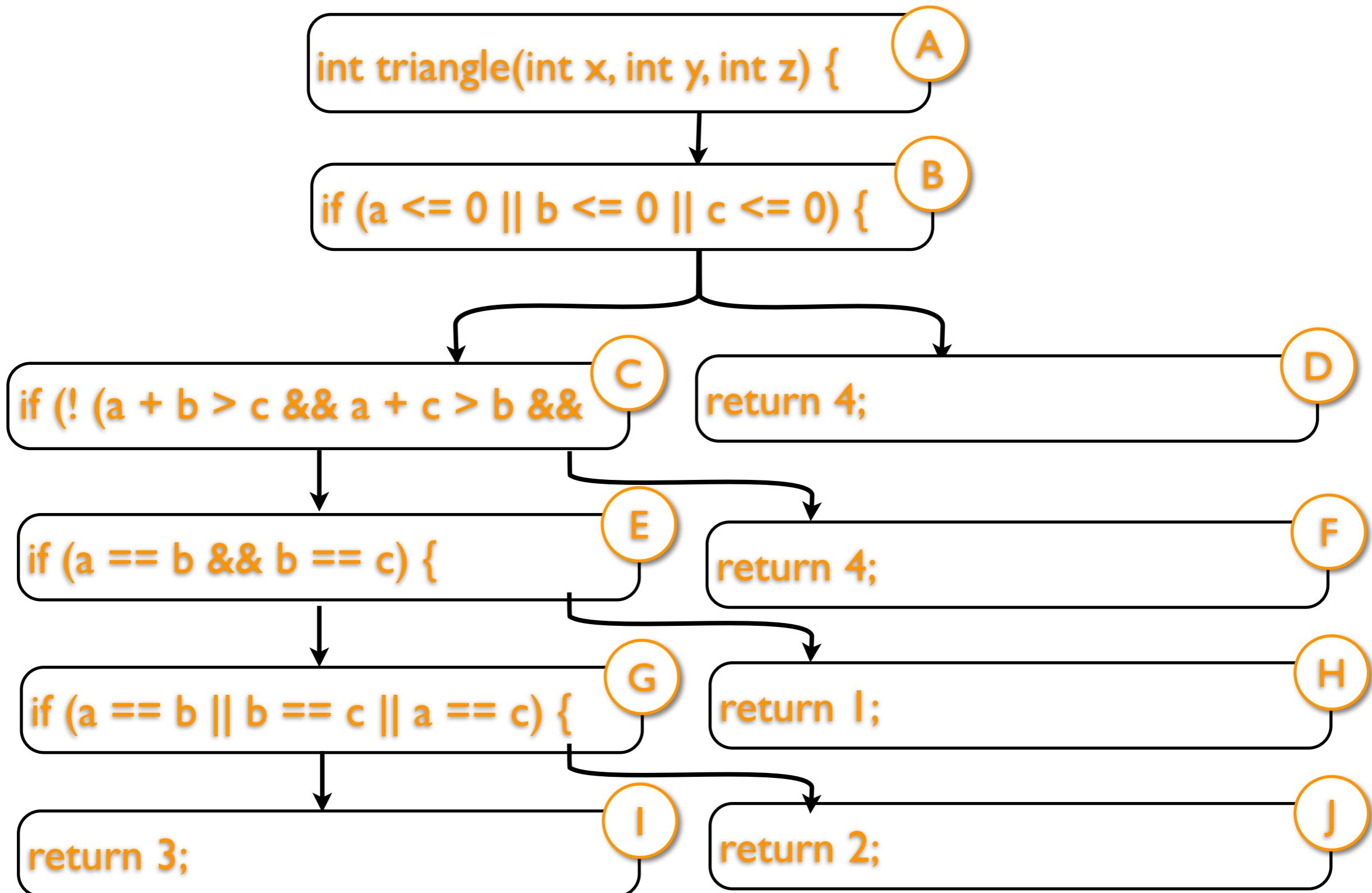
```
int triangle(int a, int b, int c) {  
  
    if (a <= 0 || b <= 0 || c <= 0) {  
        return 4; // invalid ←  
    }  
    if (!(a + b > c && a + c > b && b + c > a)) {  
        return 4; // invalid  
    }  
    if (a == b && b == c) {  
        return 1; // equilateral  
    }  
    if (a == b || b == c || a == c) {  
        return 2; // isosceles  
    }  
  
    return 3; // scalene  
}
```

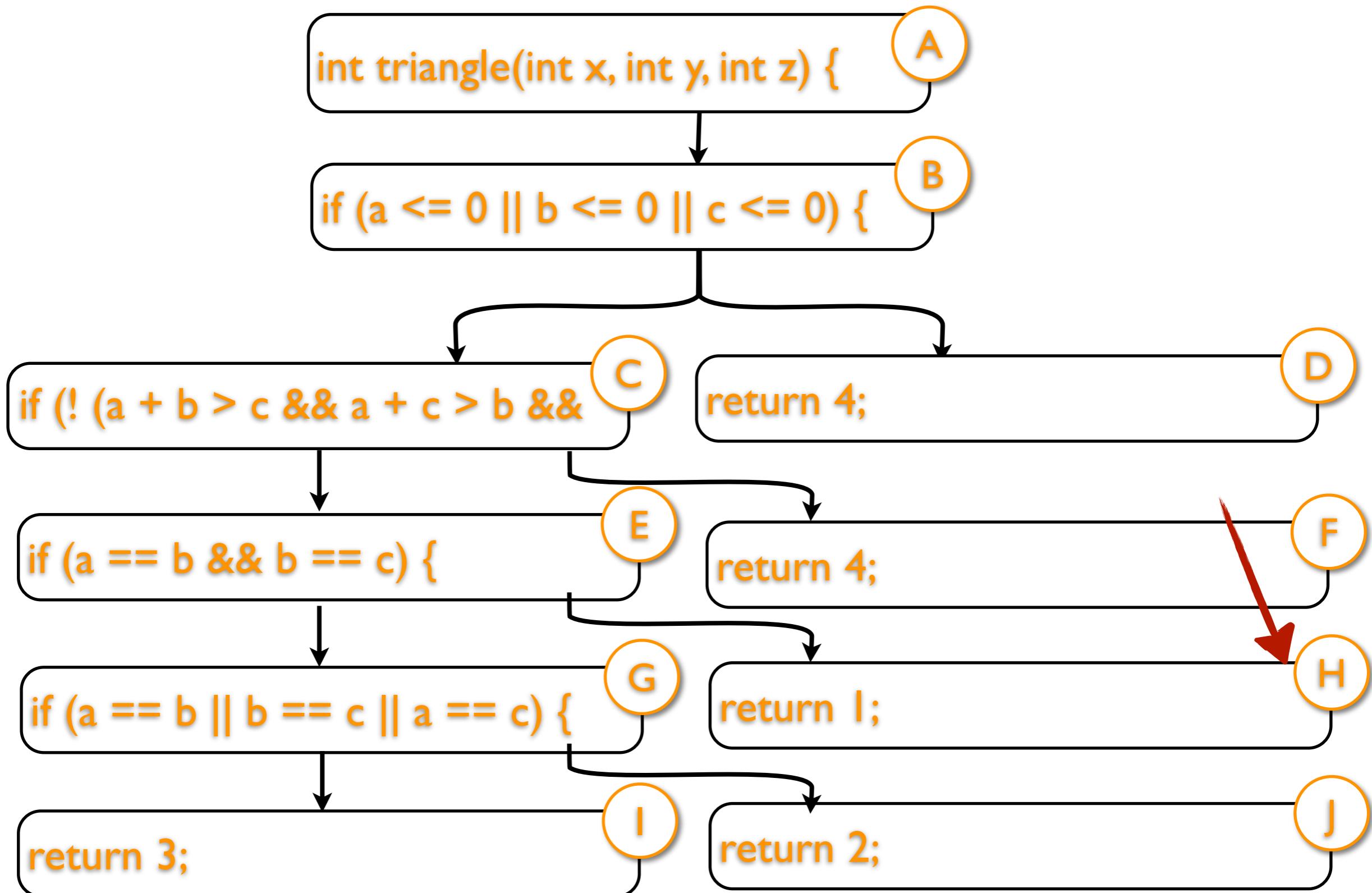
```
int triangle(int a, int b, int c) {  
  
    if (a <= 0 || b <= 0 || c <= 0) {  
        return 4; // invalid ←  
    }  
    if (!(a + b > c && a + c > b && b + c > a)) {  
        return 4; // invalid  
    }  
    if (a == b && b == c) {  
        return 1; // equilateral  
    }  
    if (a == b || b == c || a == c) {  
        return 2; // isosceles  
    }  
  
    return 3; // scalene  
}
```

(0, 53, 38)

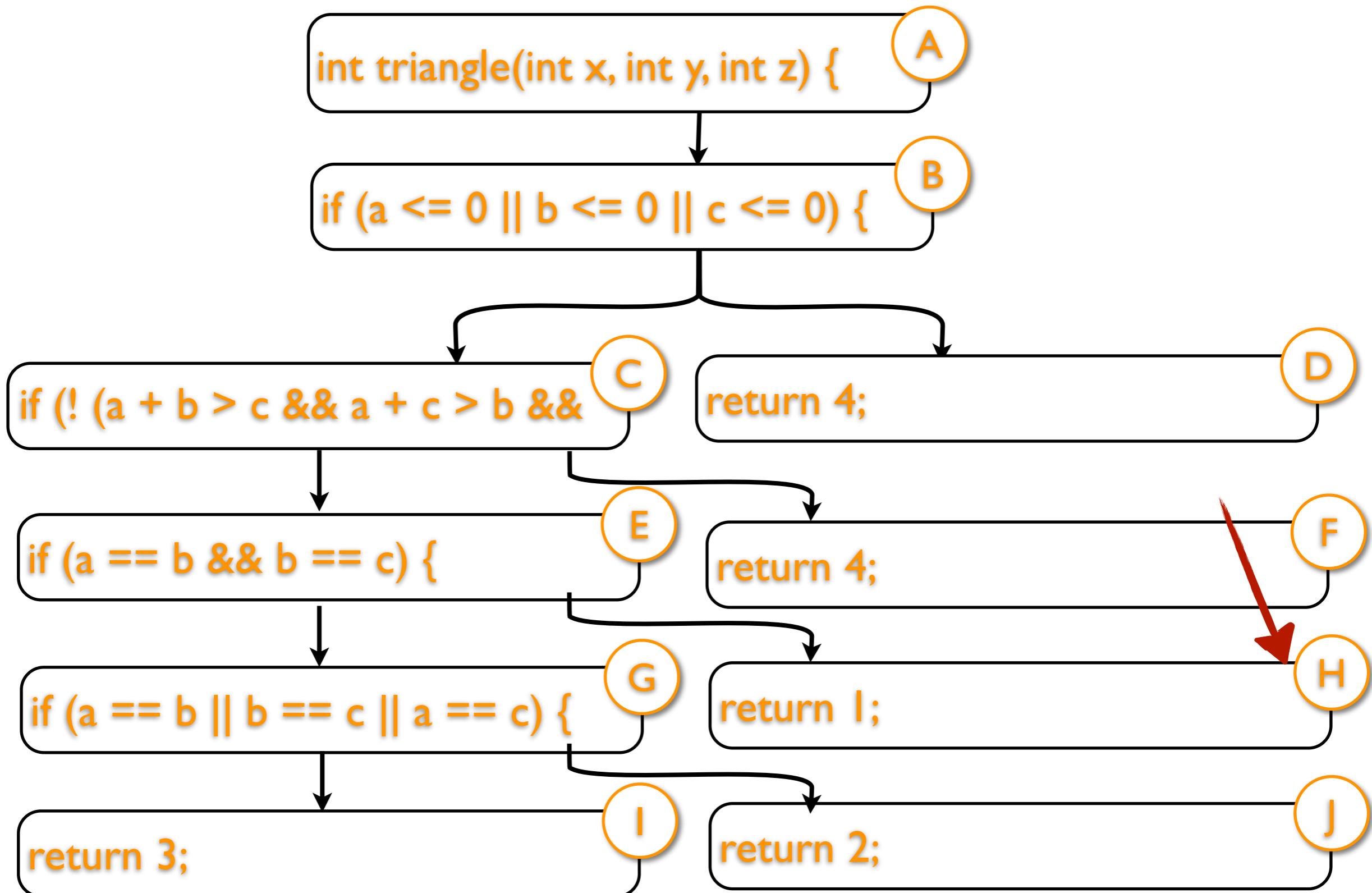
```
int triangle(int a, int b, int c) {  
  
    if (a <= 0 || b <= 0 || c <= 0) {  
        return 4; // invalid  
    }  
    if (!(a + b > c && a + c > b && b + c > a)) {  
        return 4; // invalid  
    }  
    if (a == b && b == c) {  
        return 1; // equilateral  
    }  
    if (a == b || b == c || a == c) {  
        return 2; // isosceles  
    }  
  
    return 3; // scalene  
}
```

(0, 53, 38)

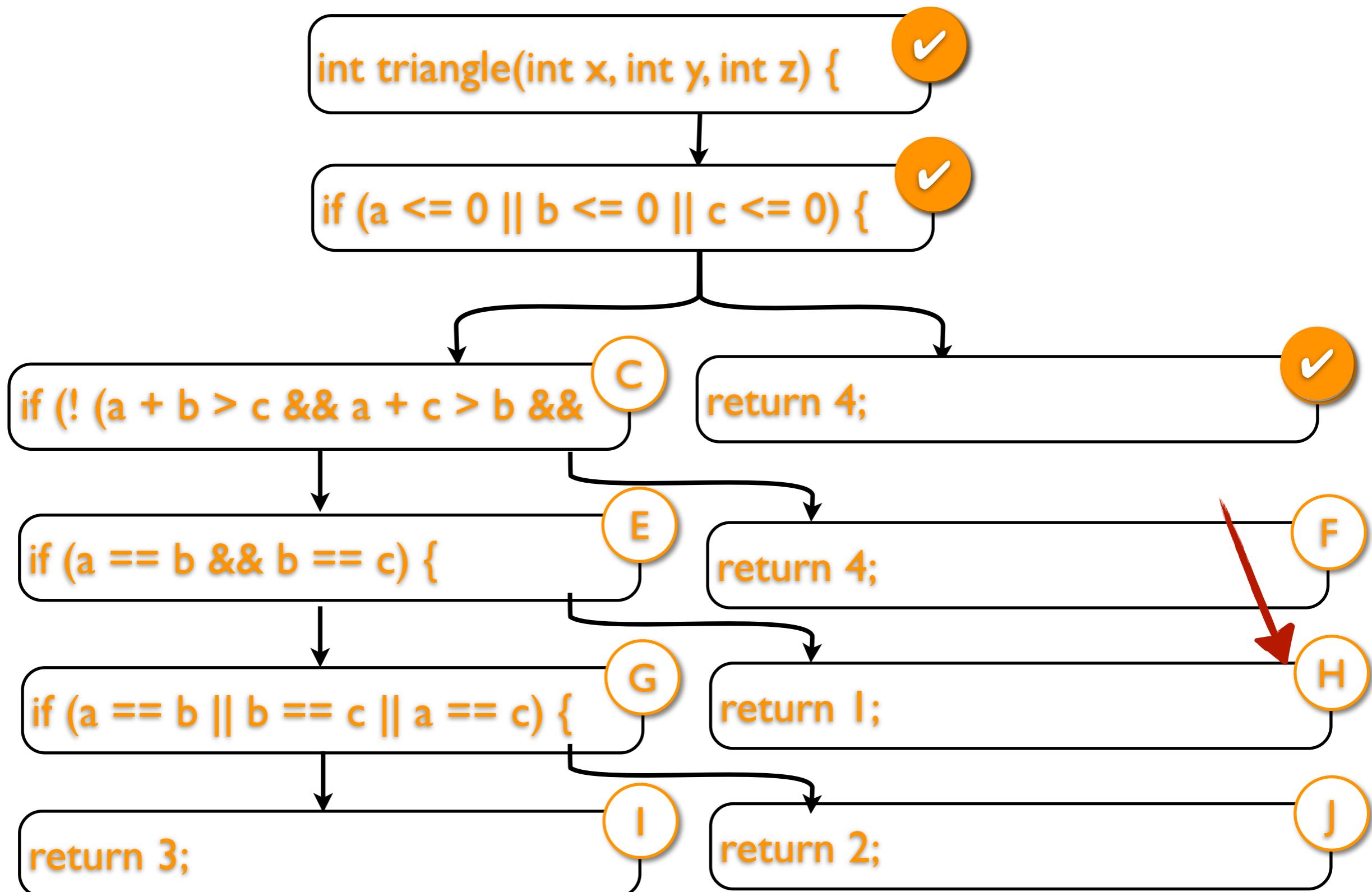




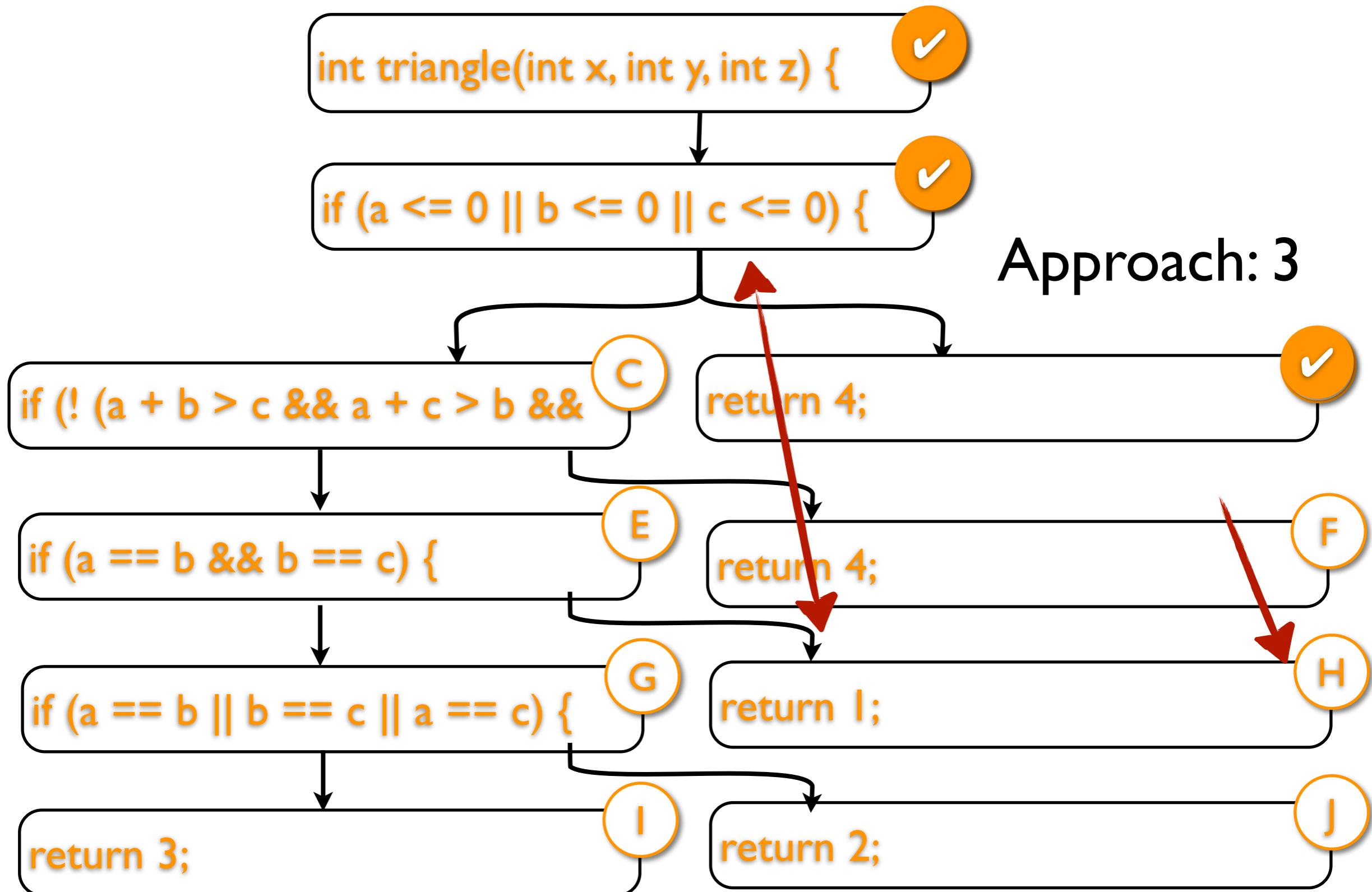
(5, 0, 9)



(5, 0, 9)



(5, 0, 9)



# Approach: 3

# Branch Distance

# Branch Distance

- Input (5, 0, 9)

# Branch Distance

- Input (5, 0, 9)
- Distance to !( $a \leq 0 \parallel b \leq 0 \parallel c \leq 0$ )?

# Branch Distance

- Input (5, 0, 9)
- Distance to !( $a \leq 0 \parallel b \leq 0 \parallel c \leq 0$ )?
- Sum of distance to  $a>0, b>0, c>0$

# Branch Distance

- Input (5, 0, 9)
- Distance to !( $a \leq 0 \parallel b \leq 0 \parallel c \leq 0$ )?
- Sum of distance to  $a>0, b>0, c>0$
- $b - a < 0 ? 0 : (b - a) + K$  ( $K = 1$ )

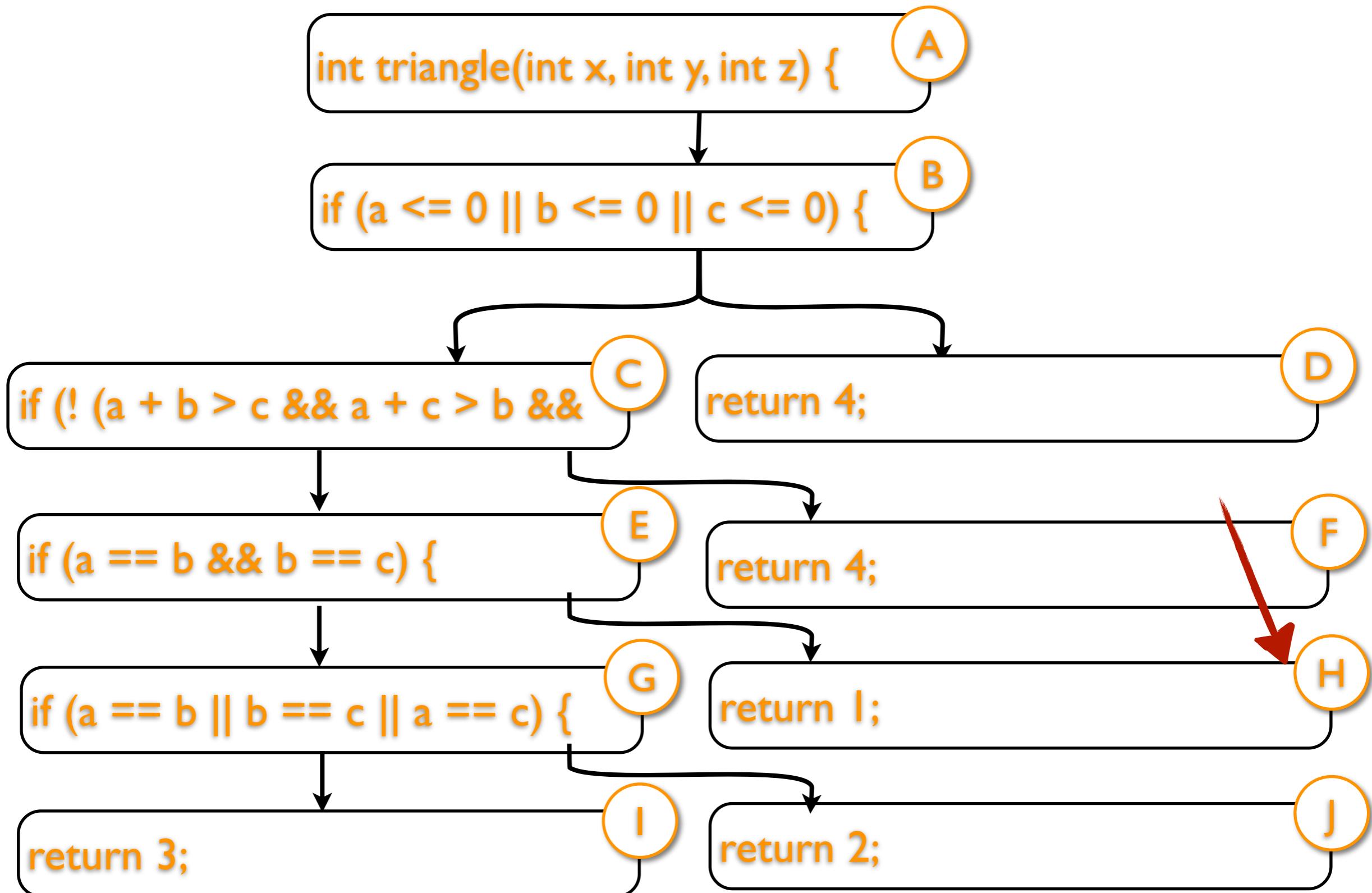
# Branch Distance

- Input (5, 0, 9)
- Distance to !( $a \leq 0 \parallel b \leq 0 \parallel c \leq 0$ )?
- Sum of distance to  $a>0, b>0, c>0$
- $b - a < 0 ? 0 : (b - a) + K$  ( $K = I$ )
- $0 + I + 0$

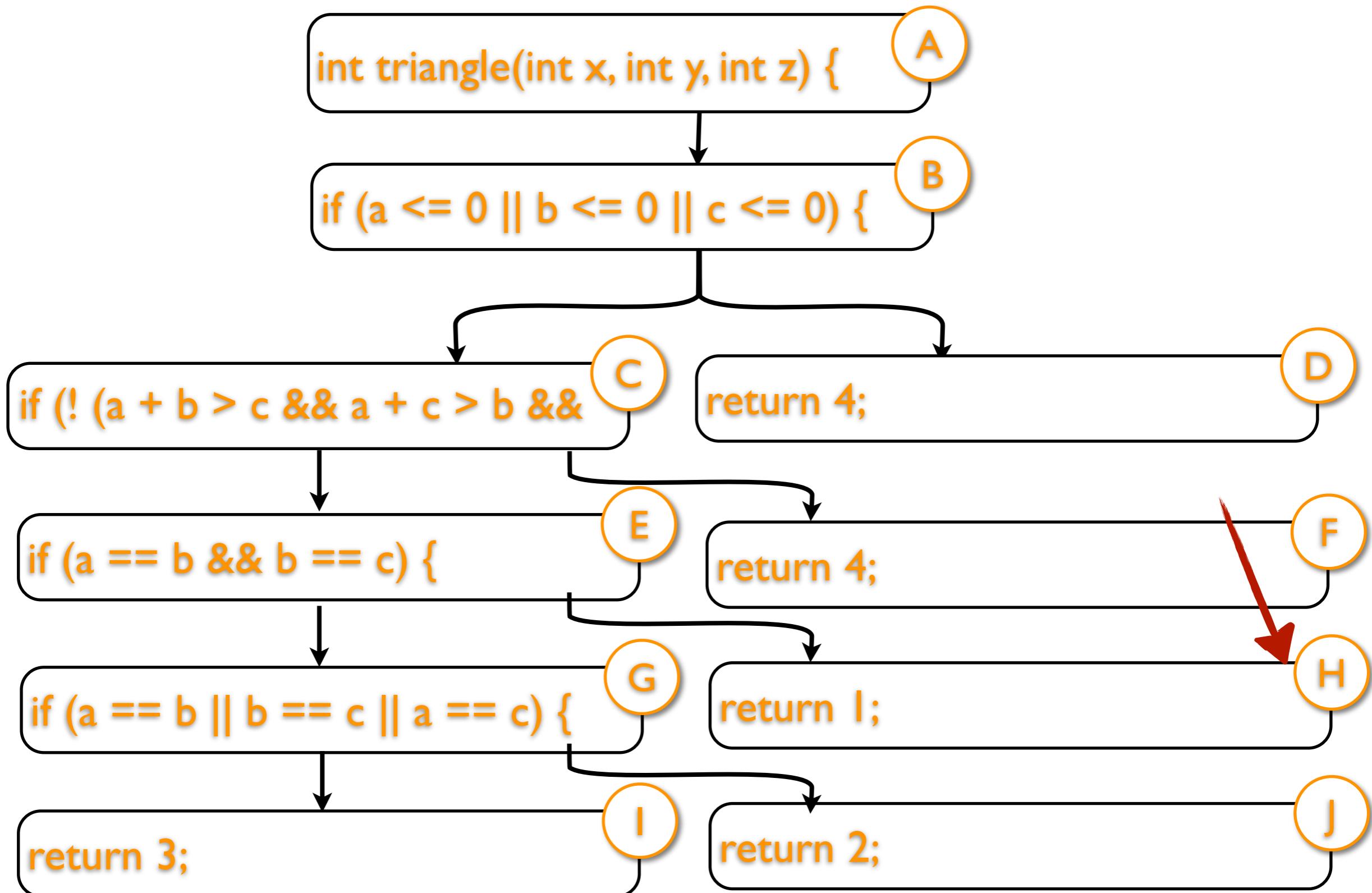
# Fitness

- Input (5, 0, 9)
- Approach level: 3
- Branch distance: 1
- Fitness =  $3 + 1/(1+1) = 3.5$

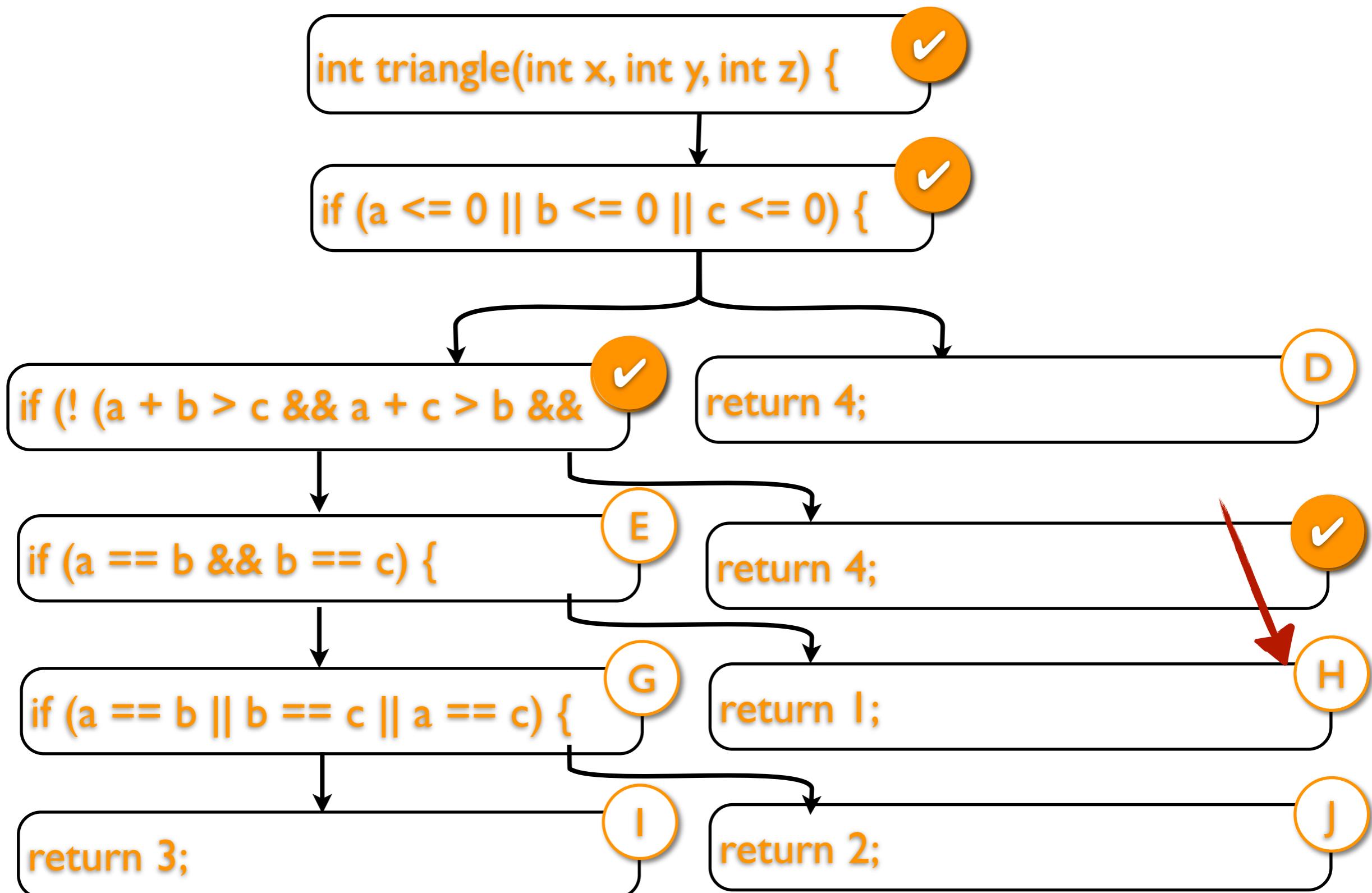
(5, 0, 9)



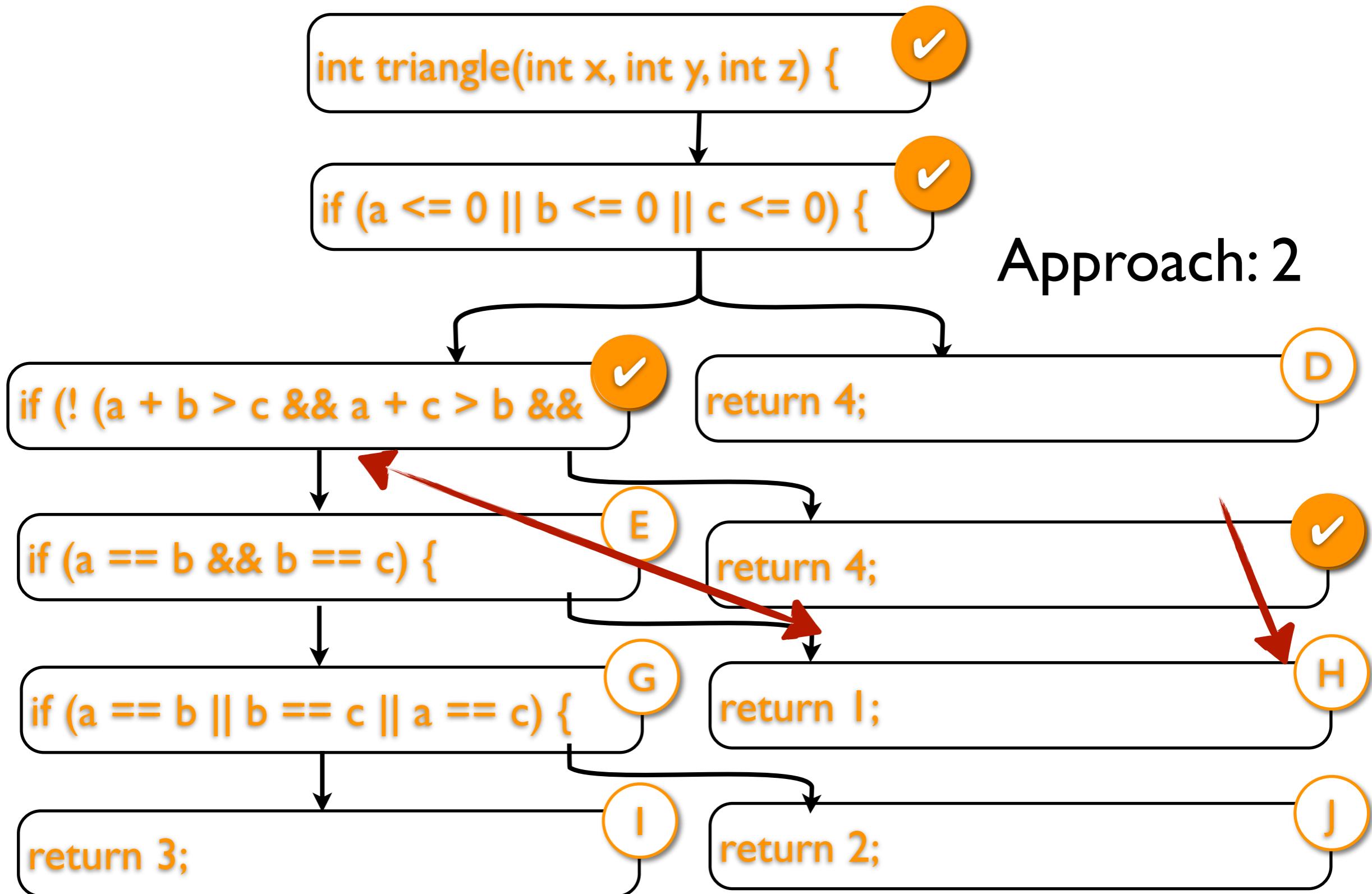
(5, 1, 9)



(5, 1, 9)



(5, 1, 9)



# Branch Distance

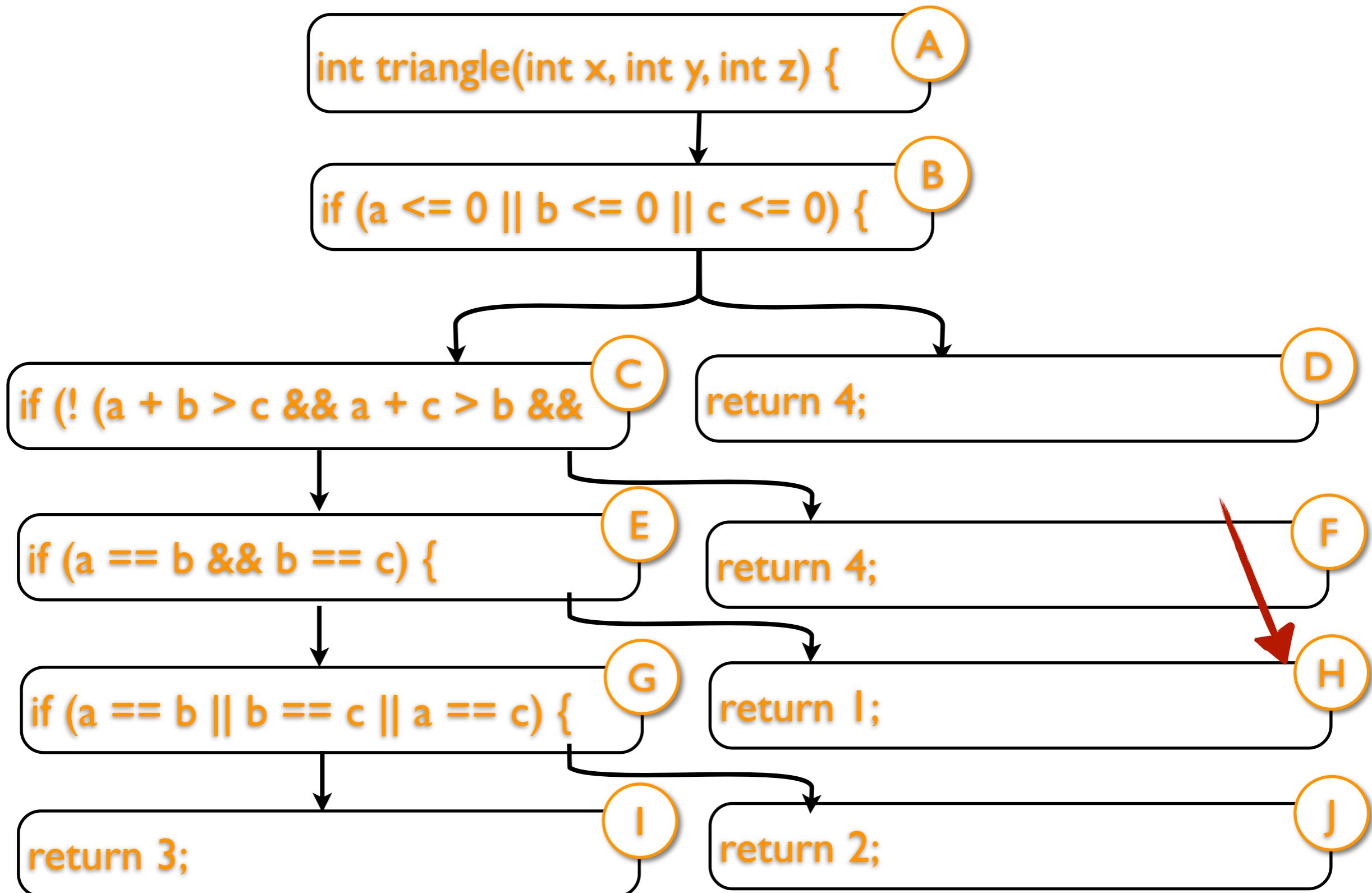
- Input (5, 1, 9)
- Distance to !(a == b && b == c)?
- Sum of distance to a==b + b==c
- $\text{abs}(a-b) = 0 ? 0 : \text{abs}(a-b) + K$  ( $K = 1$ )
- $5 + 9$

# Fitness

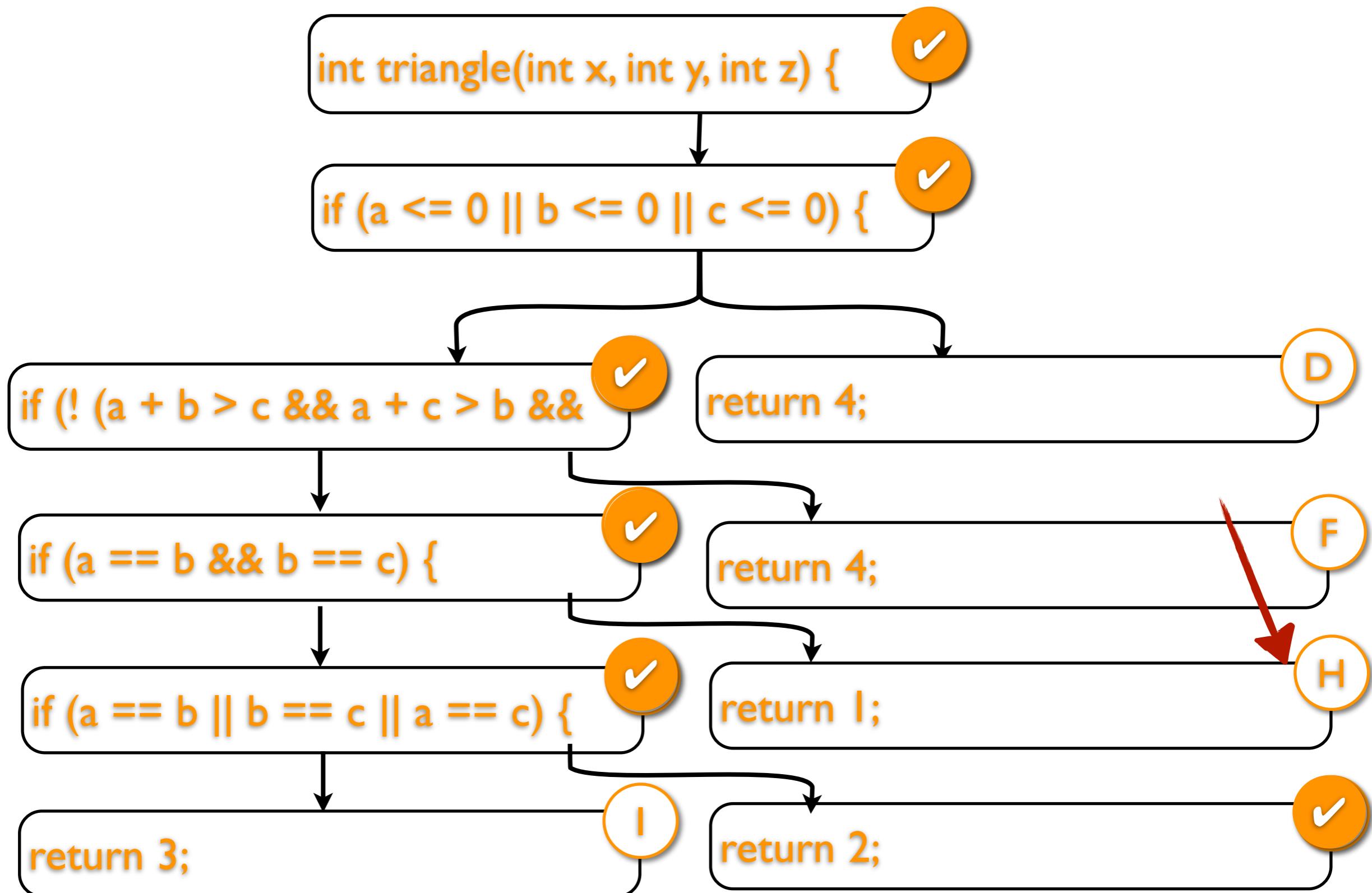
- Input (5, 1, 9)
- Approach level: 2
- Branch distance: 14
- Fitness =  $2 + 14/(14+1) = 2.9$

# Search

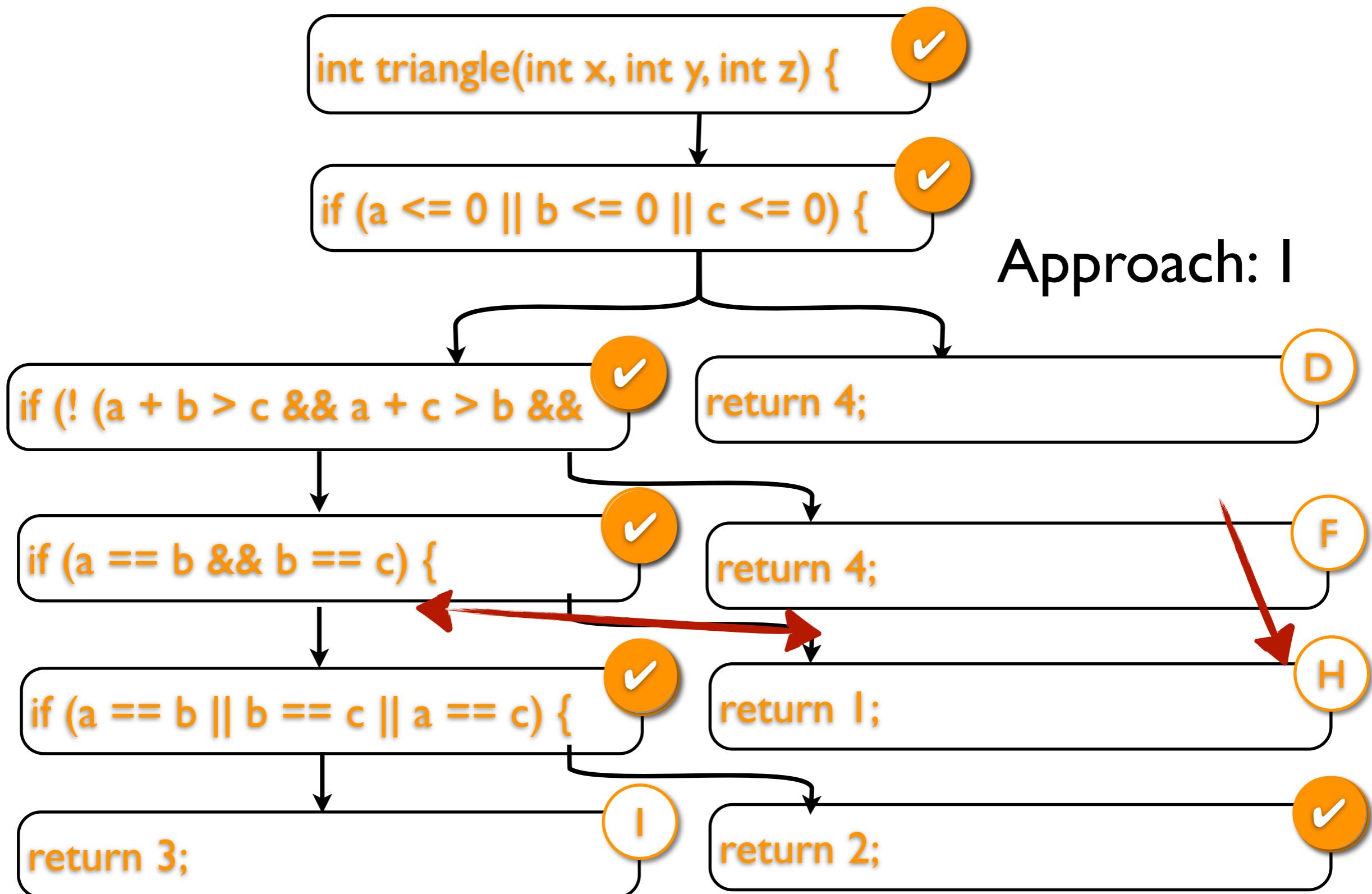
- (5, 2, 9) - Approach 2, Branch 12
- (5, 3, 9) - Approach 2, Branch 10
- (5, 4, 9) - Approach 2, Branch 8
- (5, 5, 9)



(5, 5, 9)



(5, 5, 9)



# Branch Distance

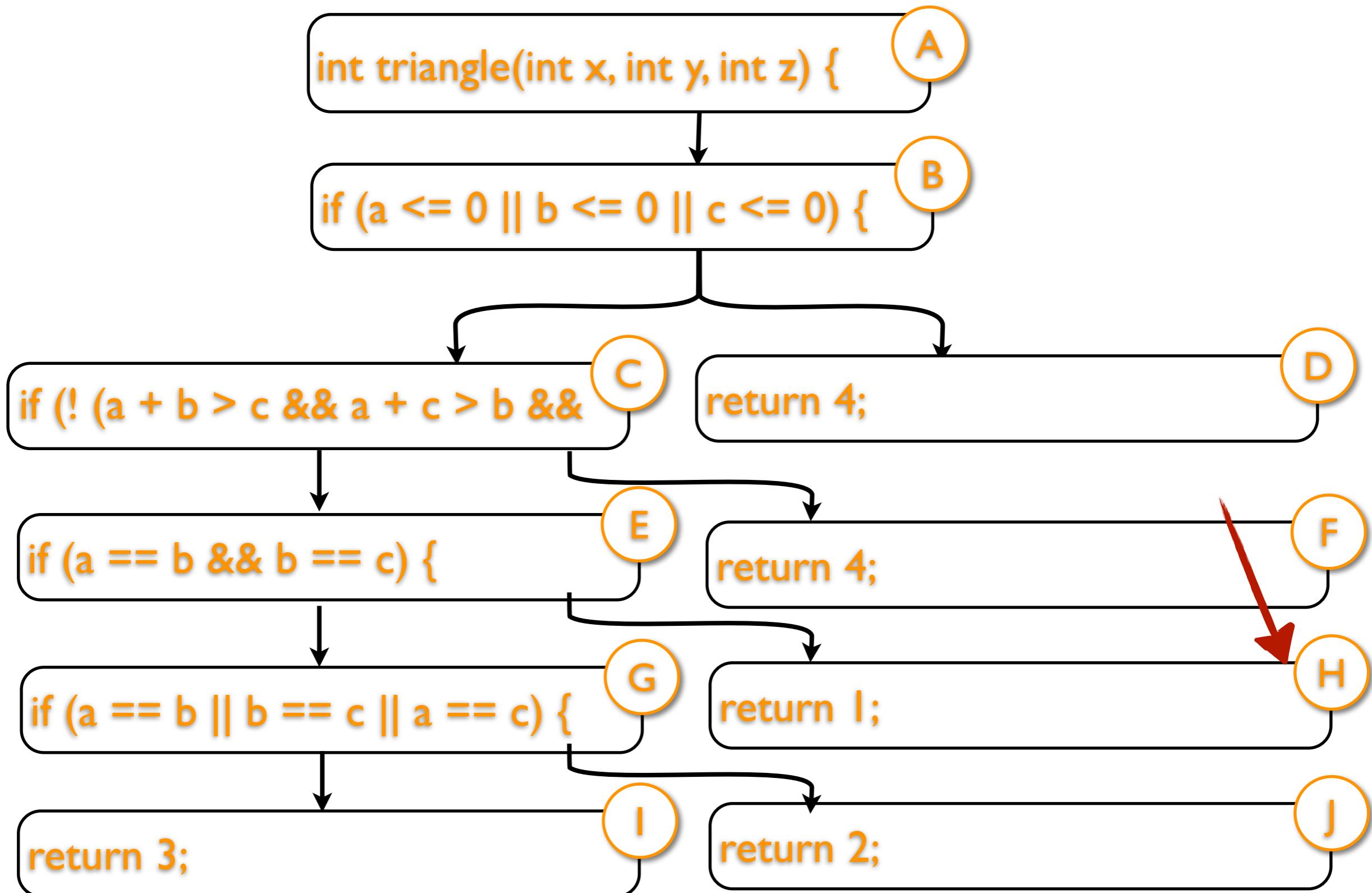
- Input (5, 5, 9)
- Distance to !(a == b && b == c)?
- Sum of distance to a==b + b==c
- $\text{abs}(a-b) = 0 ? 0 : \text{abs}(a-b) + K$  ( $K = 1$ )
- $0 + 5$

# Fitness

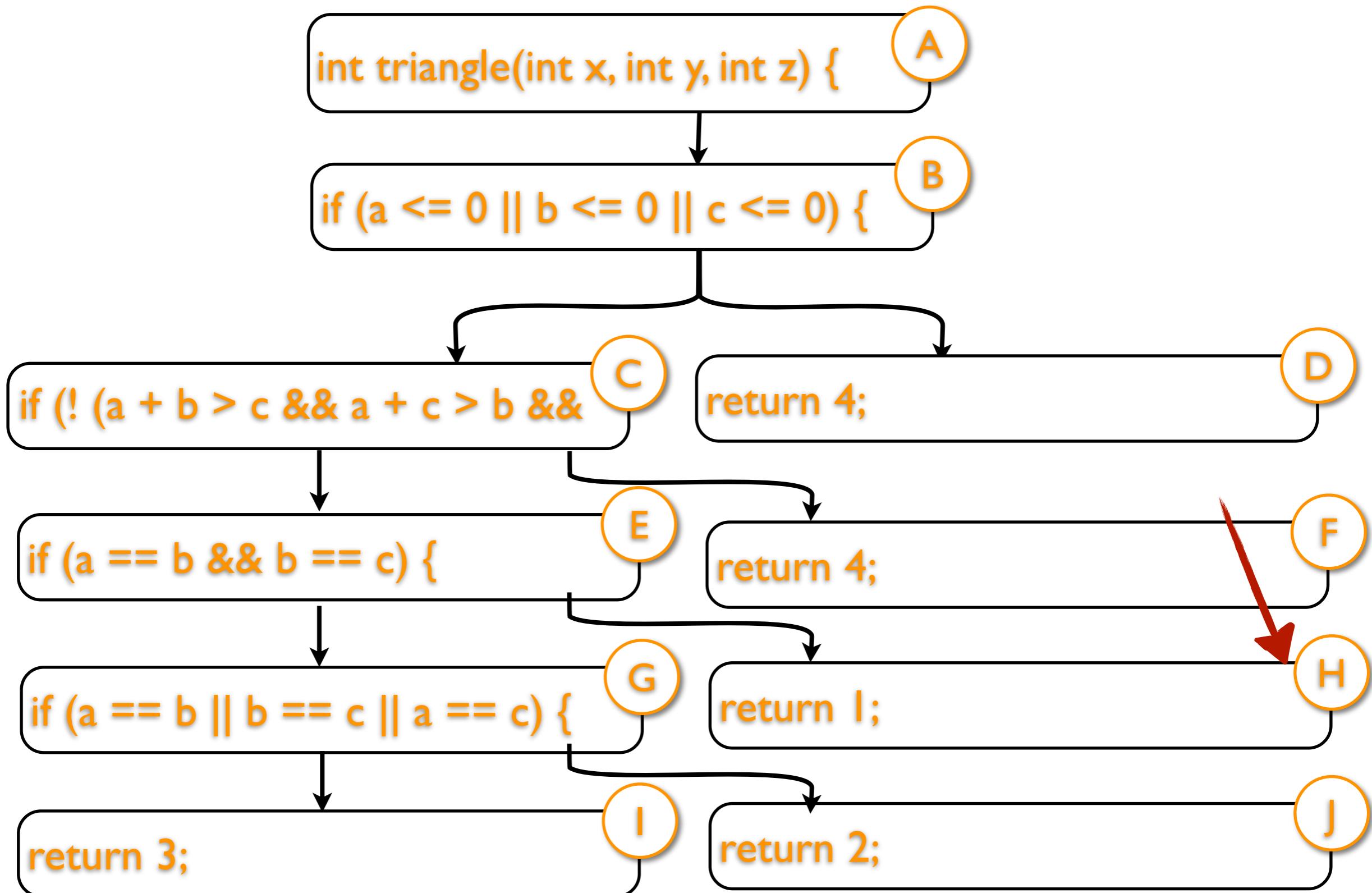
- Input (5, 5, 9)
- Approach level: 1
- Branch distance: 5
- Fitness =  $2 + 5/(5+1) = 1.8$

# Search

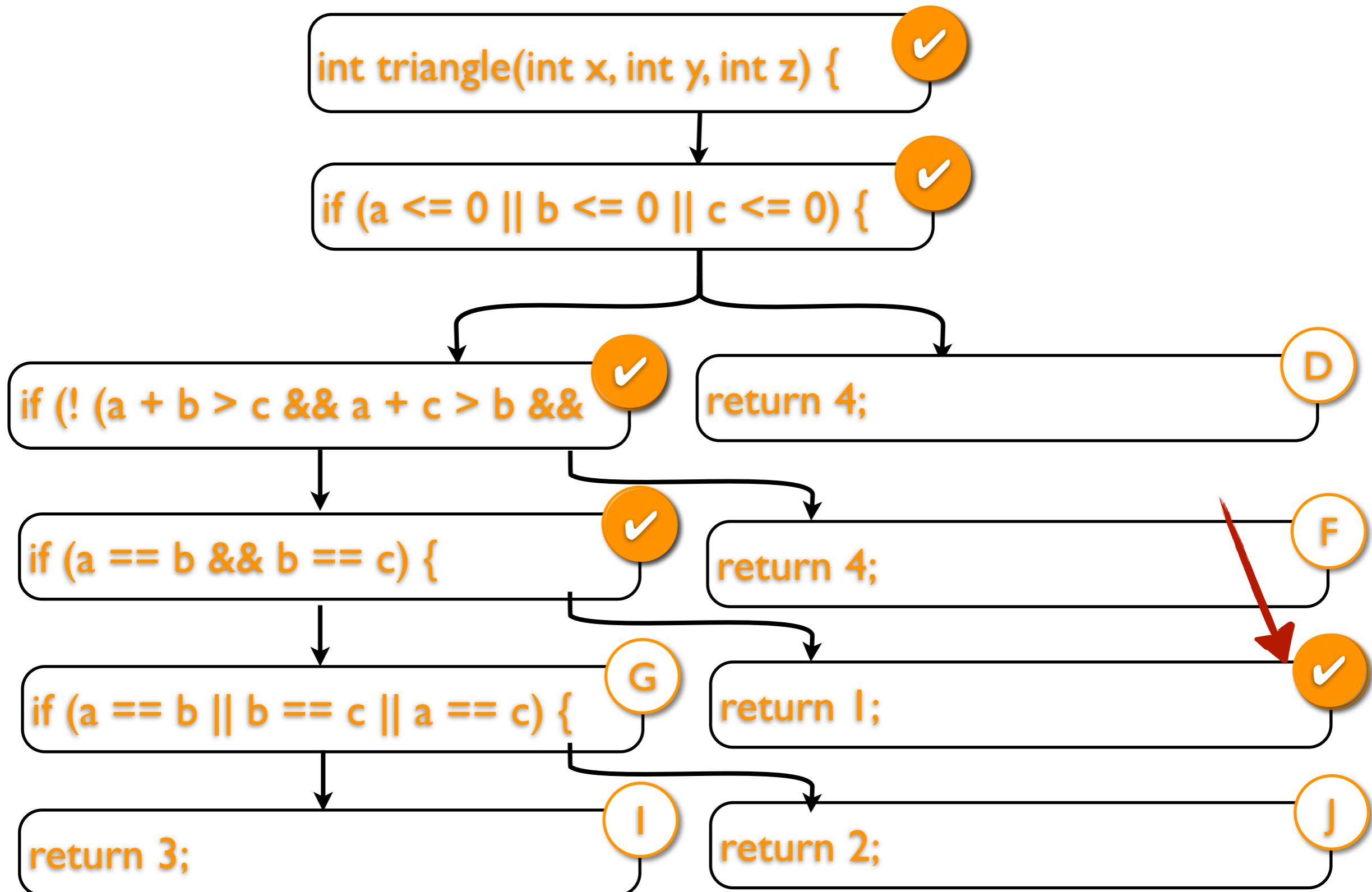
- (5, 5, 8) - Approach I, Branch 4
- (5, 5, 7) - Approach I, Branch 3
- (5, 5, 6) - Approach I, Branch 2
- (5, 5, 5) - Approach 0, Branch 0



(5, 5, 5)



(5, 5, 5)



```
int triangle(int a, int b, int c) {  
  
    if (a <= 0 || b <= 0 || c <= 0) {  
        return 4; // invalid  
    }  
    if (!(a + b > c && a + c > b && b + c > a)) {  
        return 4; // invalid  
    }  
    if (a == b && b == c) {  
        return 1; // equilateral  
    }  
    if (a == b || b == c || a == c) {  
        return 2; // isosceles  
    }  
  
    return 3; // scalene  
}
```

```
int triangle(int a, int b, int c) {  
  
    if (a <= 0 || b <= 0 || c <= 0) {  
        return 4; // invalid ←  
    }  
    if (!(a + b > c && a + c > b && b + c > a)) {  
        return 4; // invalid  
    }  
    if (a == b && b == c) {  
        return 1; // equilateral  
    }  
    if (a == b || b == c || a == c) {  
        return 2; // isosceles  
    }  
  
    return 3; // scalene  
}
```

```
int triangle(int a, int b, int c) {  
  
    if (a <= 0 || b <= 0 || c <= 0) {  
        return 4; // invalid ←  
    }  
    if (!(a + b > c && a + c > b && b + c > a)) {  
        return 4; // invalid  
    }  
    if (a == b && b == c) {  
        return 1; // equilateral  
    }  
    if (a == b || b == c || a == c) {  
        return 2; // isosceles  
    }  
  
    return 3; // scalene  
}
```

(0, 53, 38)

```
int triangle(int a, int b, int c) {  
  
    if (a <= 0 || b <= 0 || c <= 0) {  
        return 4; // invalid  
    }  
    if (!(a + b > c && a + c > b && b + c > a)) {  
        return 4; // invalid  
    }  
    if (a == b && b == c) {  
        return 1; // equilateral  
    }  
    if (a == b || b == c || a == c) {  
        return 2; // isosceles  
    }  
  
    return 3; // scalene  
}
```

(0, 53, 38)

```
int triangle(int a, int b, int c) {  
  
    if (a <= 0 || b <= 0 || c <= 0) {  
        return 4; // invalid  
    }  
    if (!(a + b > c && a + c > b && b + c > a)) {  
        return 4; // invalid  
    }  
    if (a == b && b == c) {  
        return 1; // equilateral  
    }  
    if (a == b || b == c || a == c) {  
        return 2; // isosceles  
    }  
  
    return 3; // scalene  
}
```

(0, 53, 38)

(5, 5, 5)

```
int triangle(int a, int b, int c) {  
  
    if (a <= 0 || b <= 0 || c <= 0) {  
        return 4; // invalid  
    }  
    if (!(a + b > c && a + c > b && b + c > a)) {  
        return 4; // invalid  
    }  
    if (a == b && b == c) {  
        return 1; // equilateral  
    }  
    if (a == b || b == c || a == c) {  
        return 2; // isosceles  
    }  
  
    return 3; // scalene  
}
```

```
int triangle(int a, int b, int c) {  
  
    if (a <= 0 || b <= 0 || c <= 0) {  
        return 4; // invalid ←  
    }  
    if (!(a + b > c && a + c > b && b + c > a)) {  
        return 4; // invalid  
    }  
    if (a == b && b == c) {  
        return 1; // equilateral  
    }  
    if (a == b || b == c || a == c) {  
        return 2; // isosceles  
    }  
  
    return 3; // scalene  
}
```

```
int triangle(int a, int b, int c) {  
  
    if (a <= 0 || b <= 0 || c <= 0) {  
        return 4; // invalid ←  
    }  
    if (!(a + b > c && a + c > b && b + c > a)) {  
        return 4; // invalid  
    }  
    if (a == b && b == c) {  
        return 1; // equilateral  
    }  
    if (a == b || b == c || a == c) {  
        return 2; // isosceles  
    }  
  
    return 3; // scalene  
}
```

(0, 53, 38)

```
int triangle(int a, int b, int c) {  
  
    if (a <= 0 || b <= 0 || c <= 0) {  
        return 4; // invalid  
    }  
    if (!(a + b > c && a + c > b && b + c > a)) {  
        return 4; // invalid  
    }  
    if (a == b && b == c) {  
        return 1; // equilateral  
    }  
    if (a == b || b == c || a == c) {  
        return 2; // isosceles  
    }  
  
    return 3; // scalene  
}
```

(0, 53, 38)

```
int triangle(int a, int b, int c) {  
  
    if (a <= 0 || b <= 0 || c <= 0) {  
        return 4; // invalid  
    }  
    if (!(a + b > c && a + c > b && b + c > a)) {  
        return 4; // invalid  
    }  
    if (a == b && b == c) {  
        return 1; // equilateral  
    }  
    if (a == b || b == c || a == c) {  
        return 2; // isosceles  
    }  
  
    return 3; // scalene  
}
```

(0, 53, 38)

(5, 5, 5)

```
int triangle(int a, int b, int c) {  
  
    if (a <= 0 || b <= 0 || c <= 0) {  
        return 4; // invalid ← (0, 53, 38)  
    }  
    if (!(a + b > c && a + c > b && b + c > a)) {  
        return 4; // invalid  
    }  
    if (a == b && b == c) {  
        return 1; // equilateral ← (5, 5, 5)  
    }  
    if (a == b || b == c || a == c) {  
        return 2; // isosceles ←  
    }  
  
    return 3; // scalene  
}
```

(0, 53, 38)

(5, 5, 5)

# GA Example

# GA Example

I	0	0	0	0	0	I	I	O	I
0	I	0	I	I	I	I	0	0	I
I	I	I	I	0	I	I	0	0	I
0	0	0	0	I	I	I	I	O	I

# GA Example

I	0	0	0	0	0	I	I	O	I
0	I	0	I	I	I	I	0	0	I
I	I	I	I	0	I	I	0	0	I
0	0	0	0	I	I	I	I	O	I

(4, I, 5)

# GA Example

I	0	0	0	0	0	I	I	O	I
0	I	I	0	I	I	I	0	0	I
I	I	I	I	0	I	I	0	0	I
0	0	0	0	0	I	I	I	O	O

(4, I, 5)

(2, 7, I)

# GA Example

I	0	0	0	0	0	I	I	O	I
0	I	I	0	I	I	I	0	0	I
I	I	I	I	0	I	I	0	0	I
0	0	0	0	0	I	I	I	O	O

(4, I, 5)

(2, 7, I)

(7, 5, I)

# GA Example

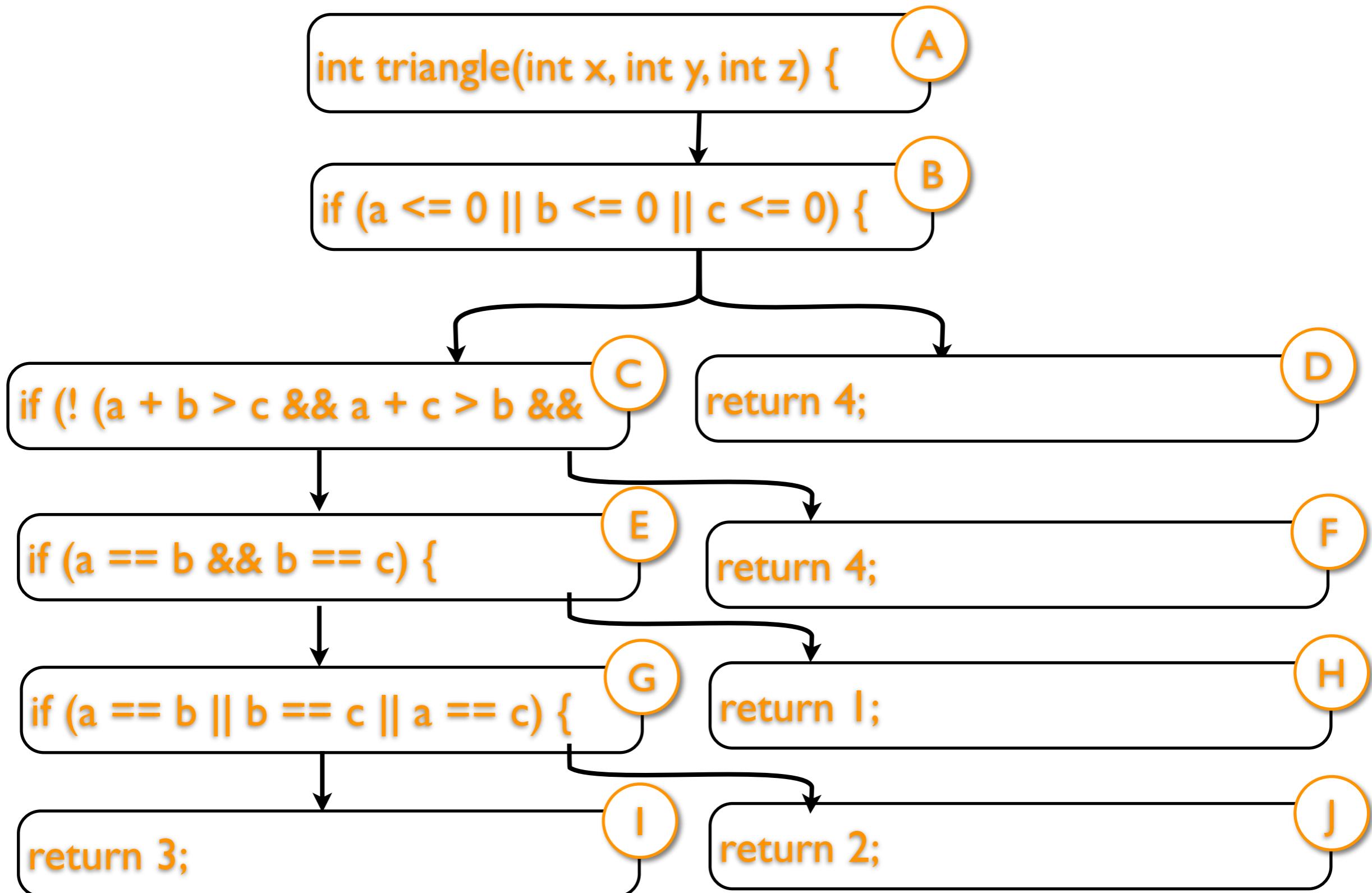
I	0	0	0	0	0	I	I	O	I
0	I	I	0	I	I	I	0	0	I
I	I	I	I	0	I	I	0	0	I
0	0	0	0	0	I	I	I	O	O

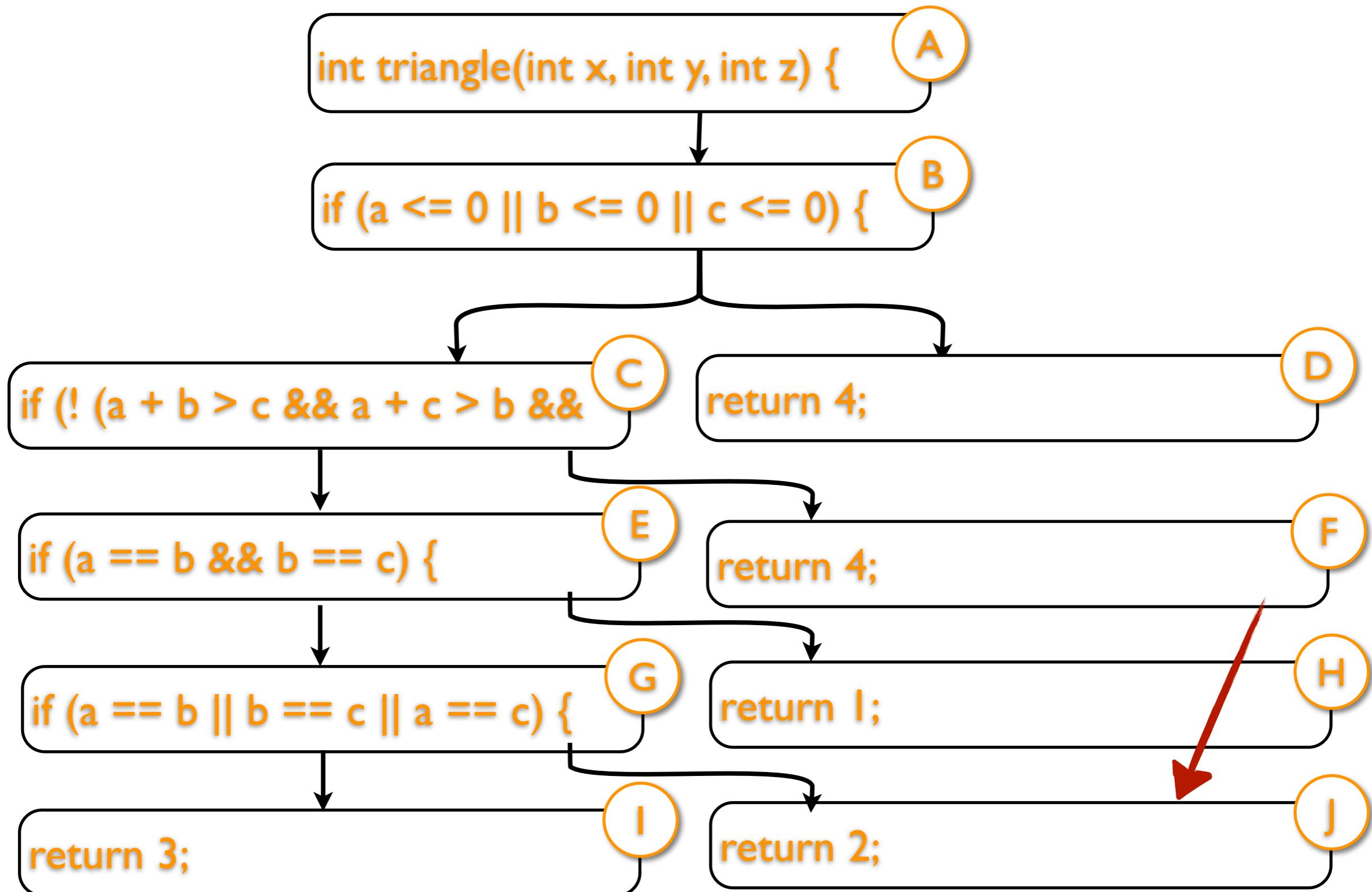
(4, I, 5)

(2, 7, I)

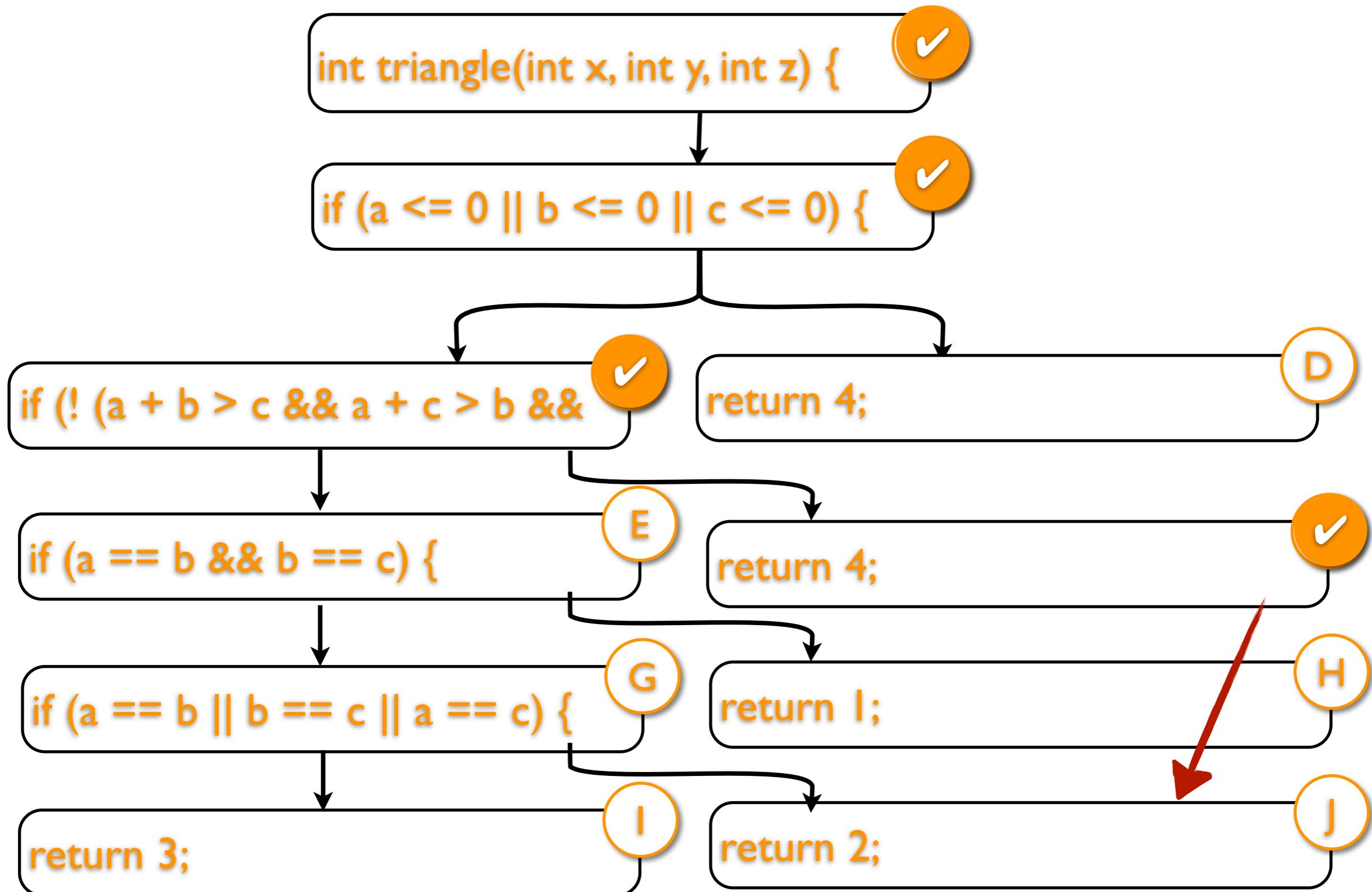
(7, 5, I)

(0, 3, 6)

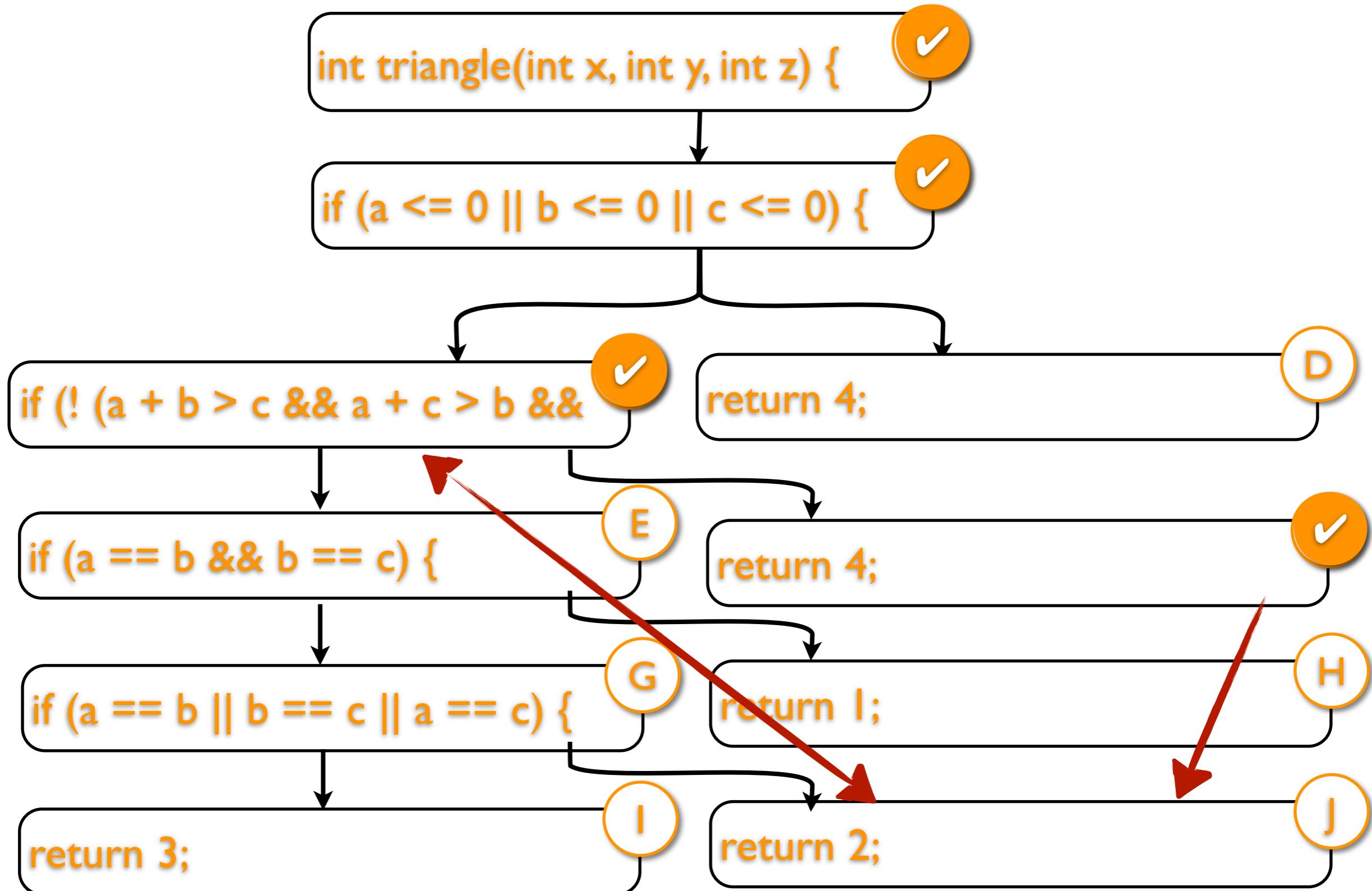




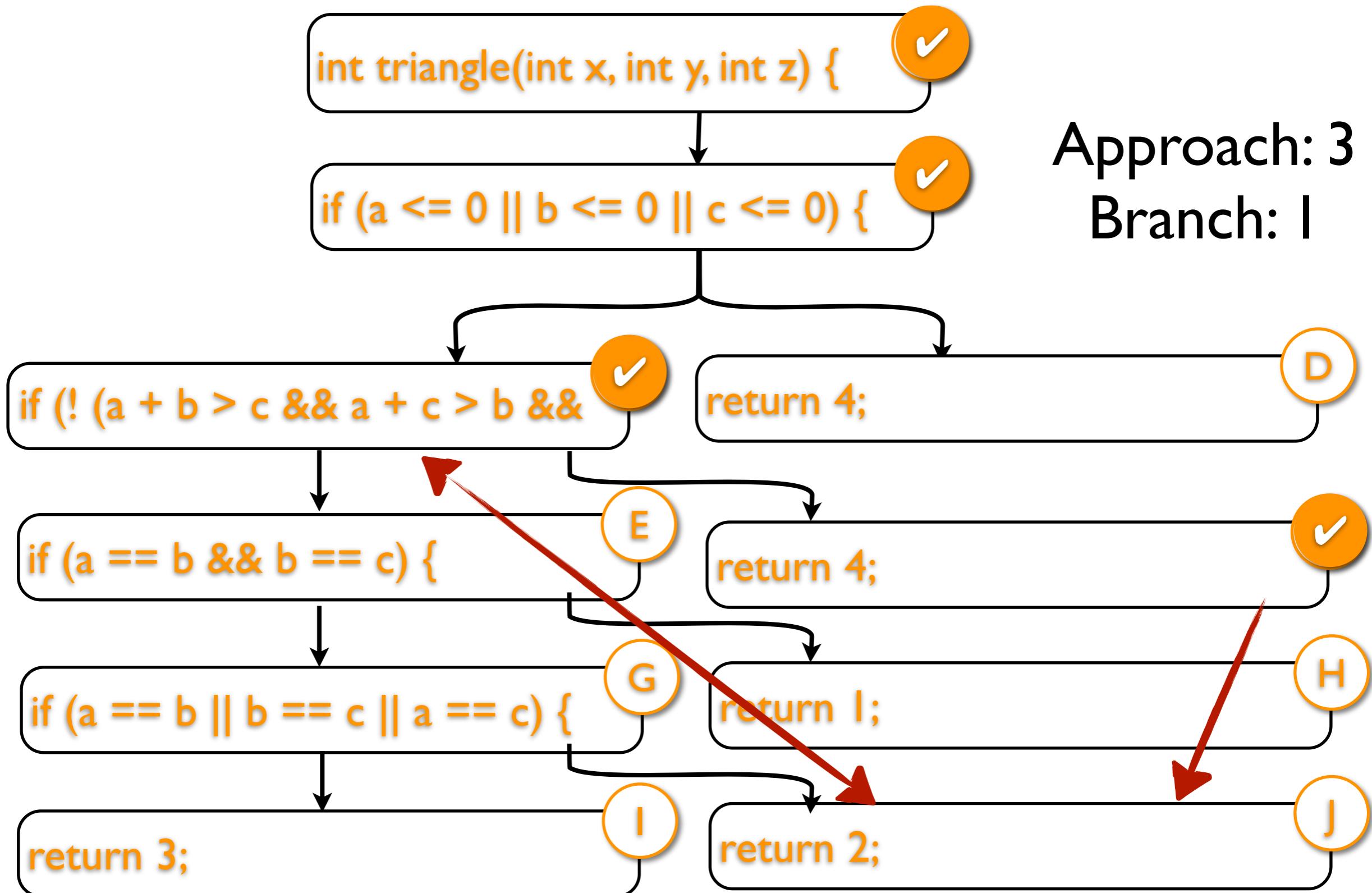
(4, 1, 5)

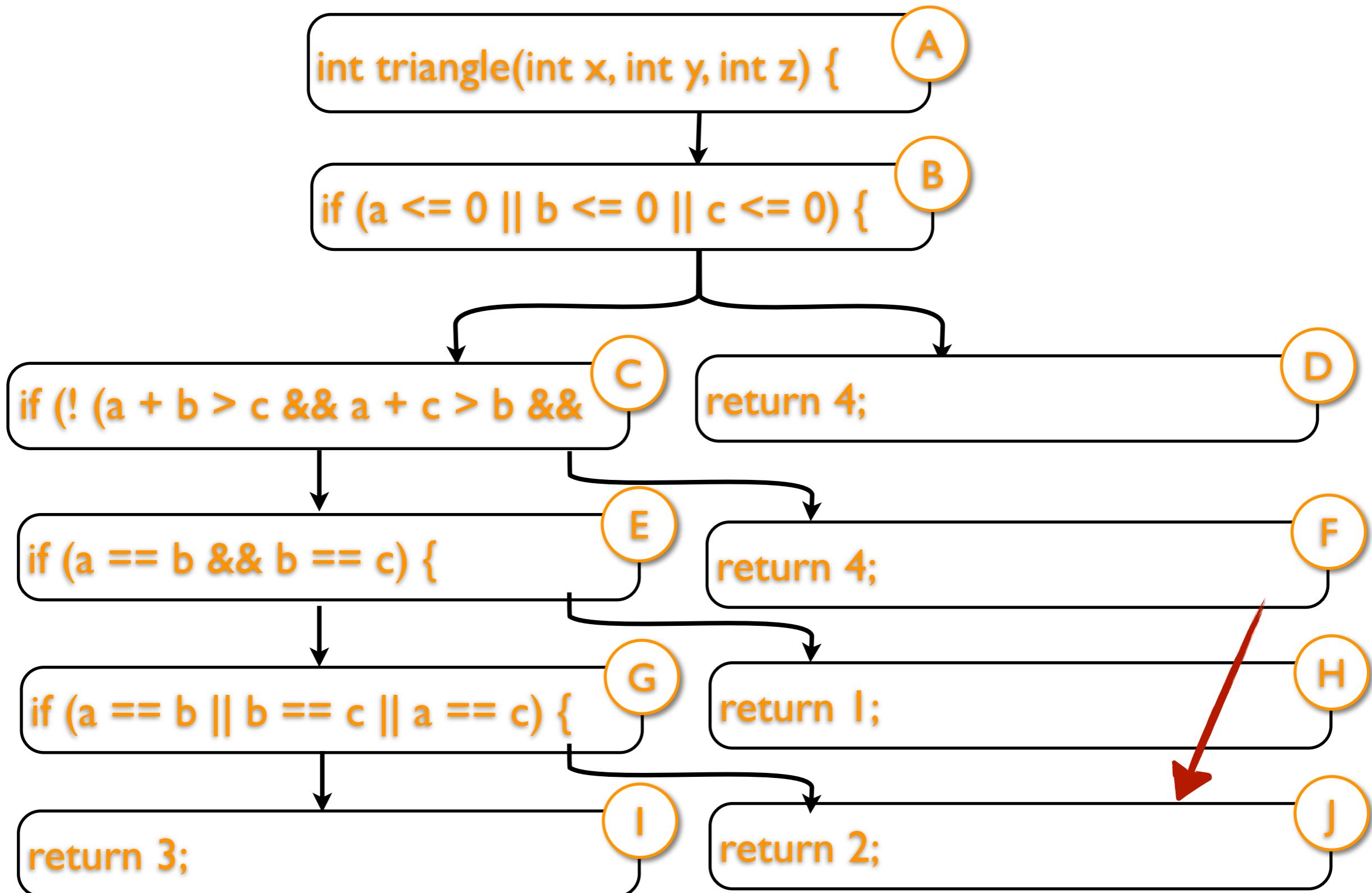


(4, 1, 5)

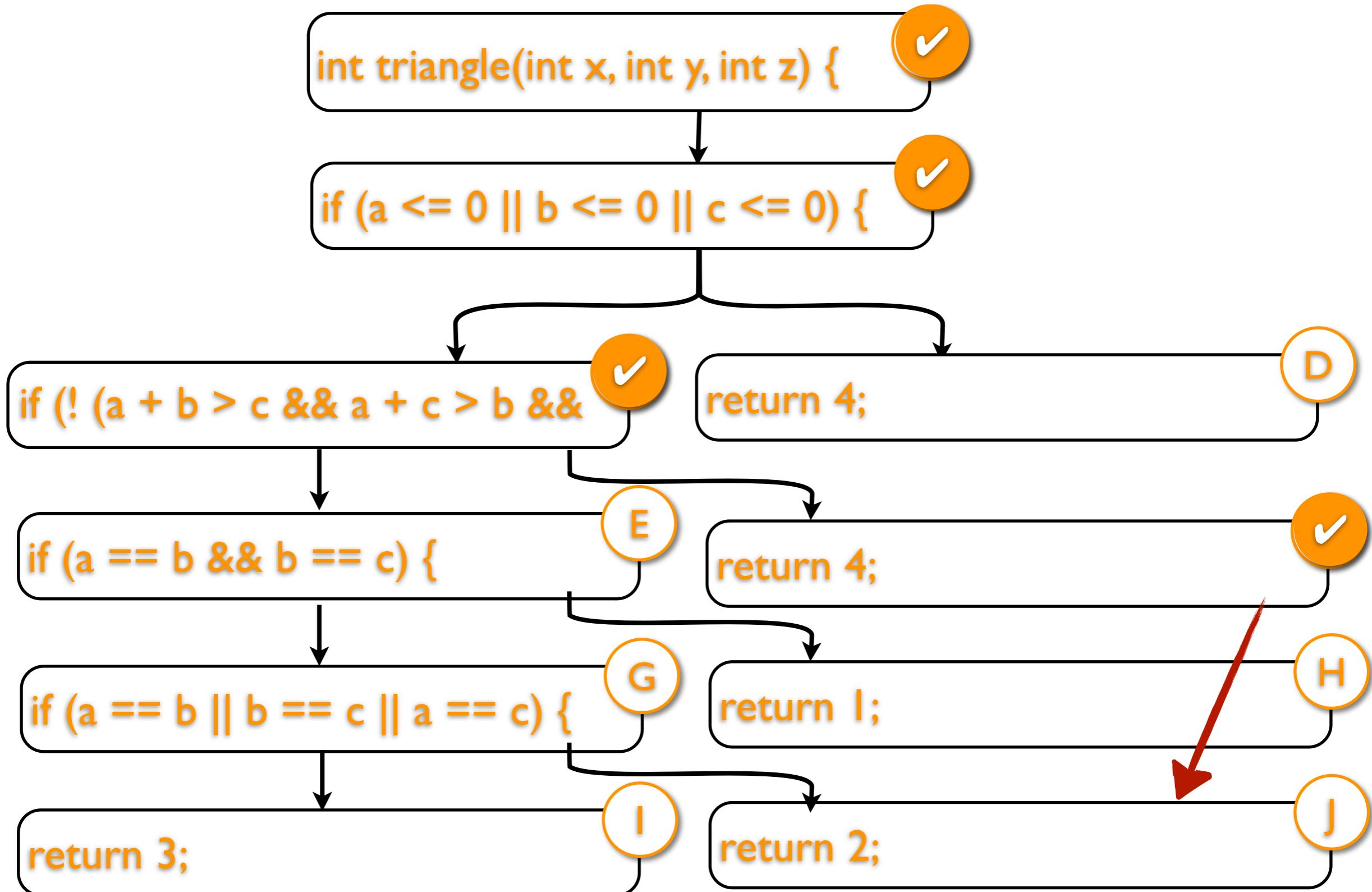


(4, 1, 5)

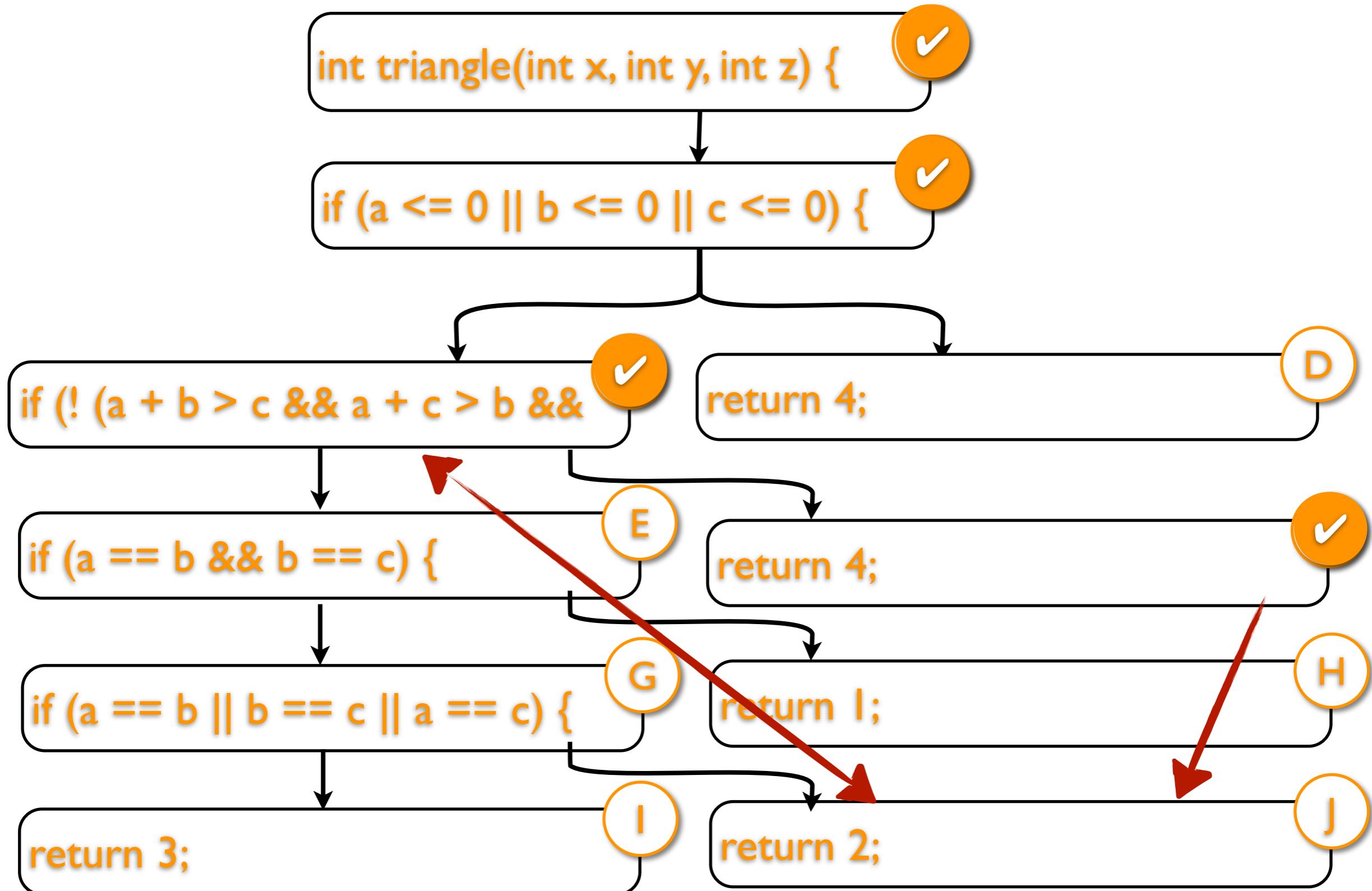




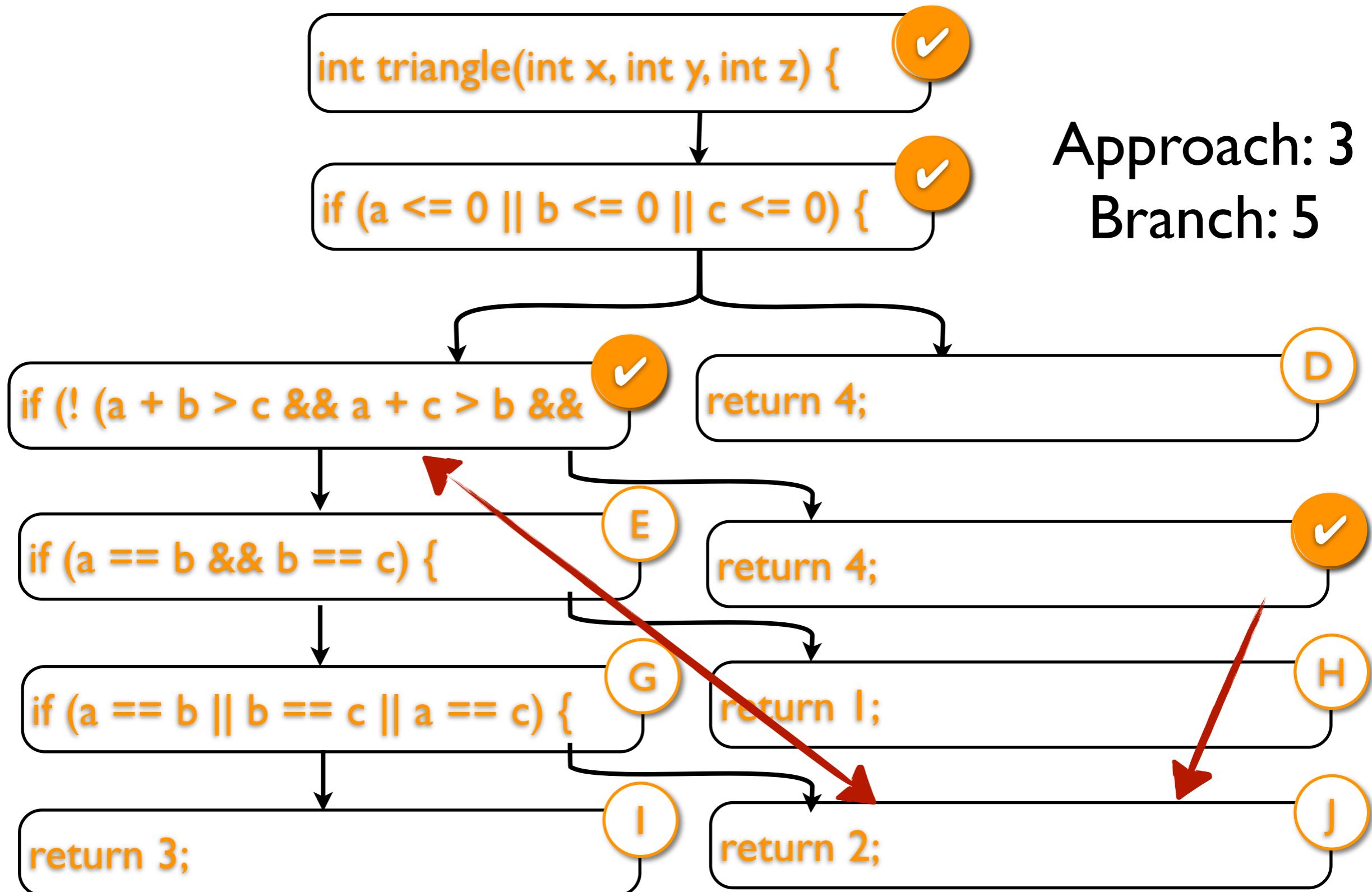
(2, 7, 1)



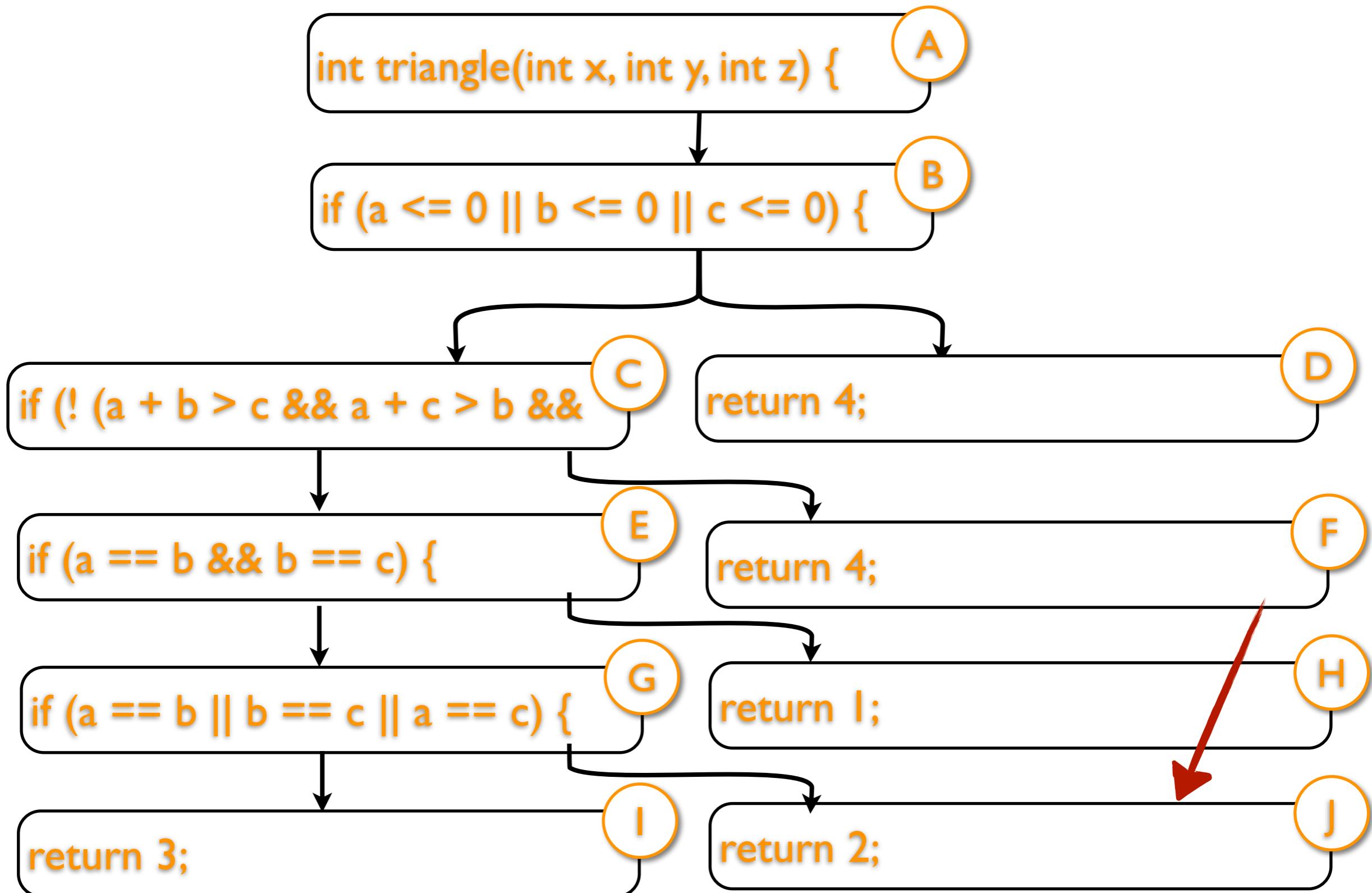
(2, 7, 1)



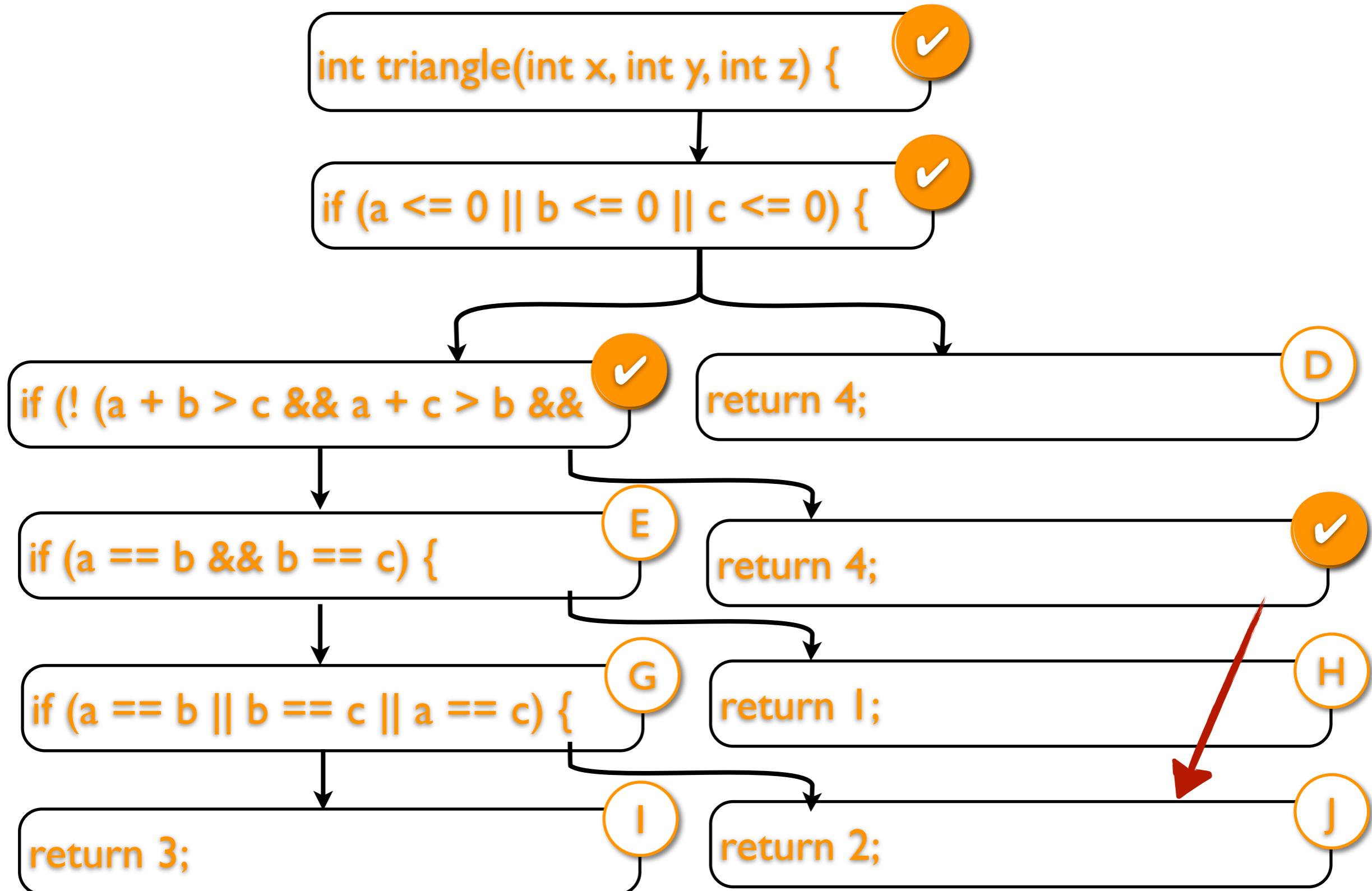
(2, 7, 1)



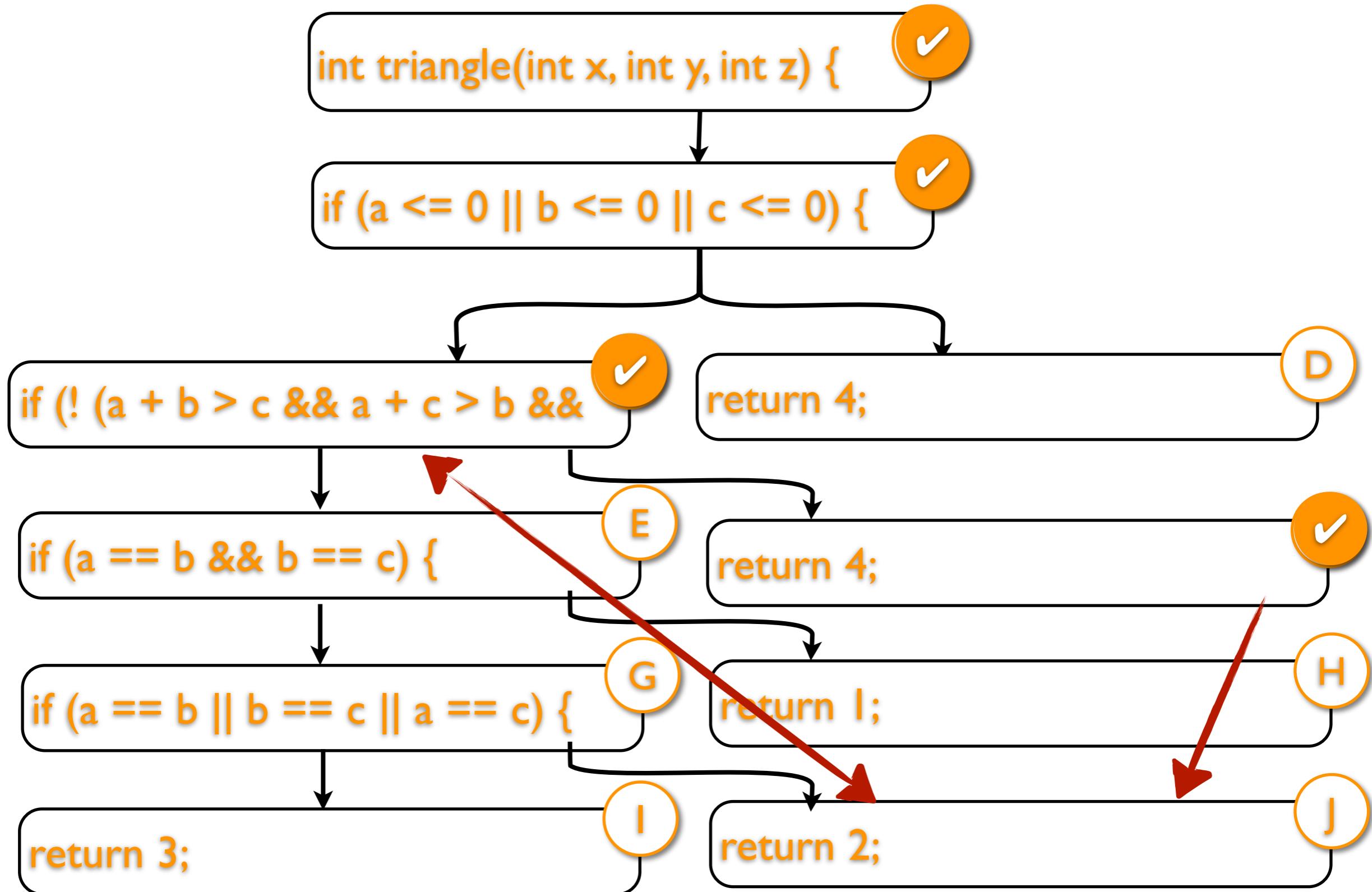
Approach: 3  
Branch: 5



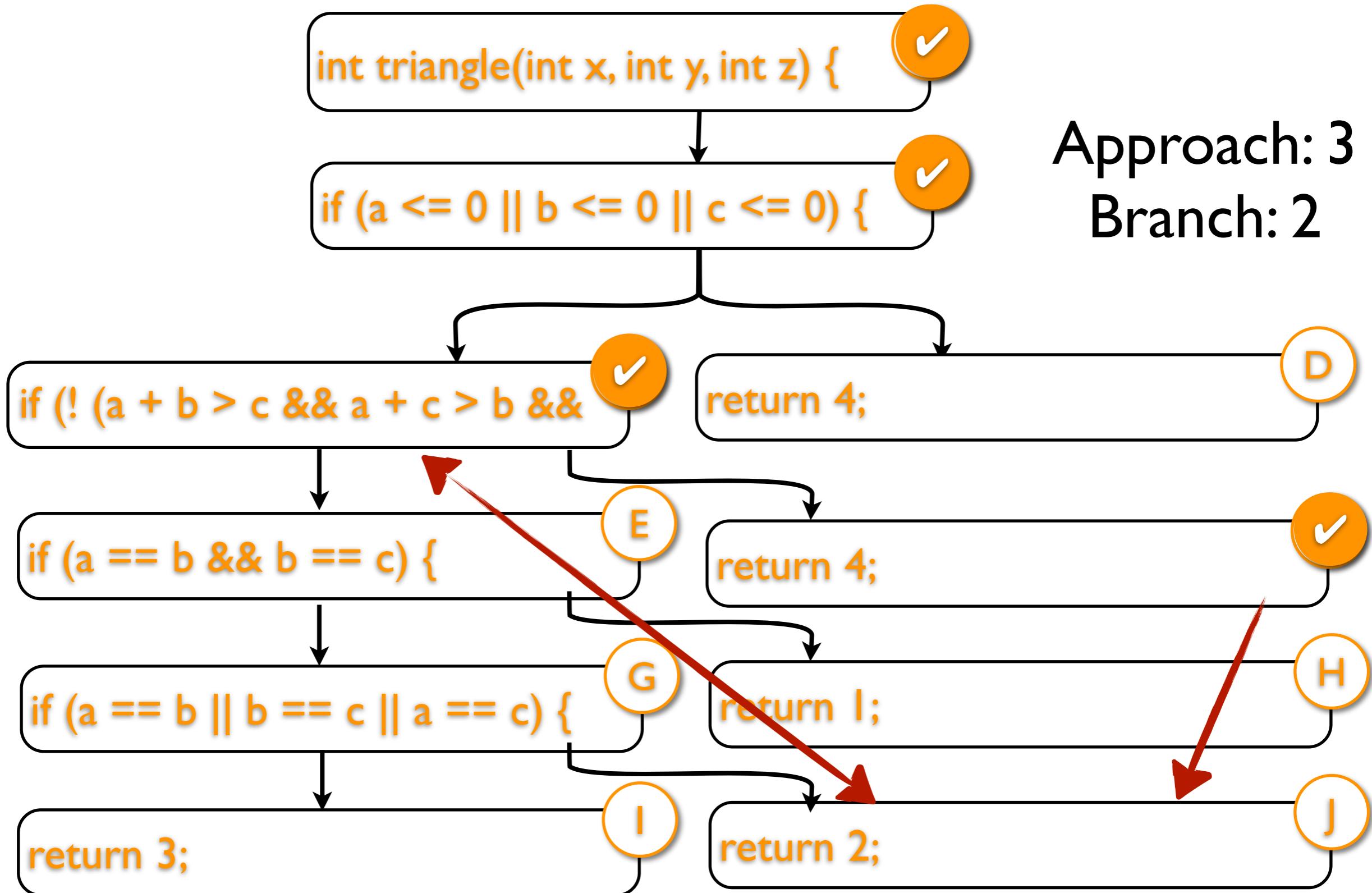
(7, 5, 1)



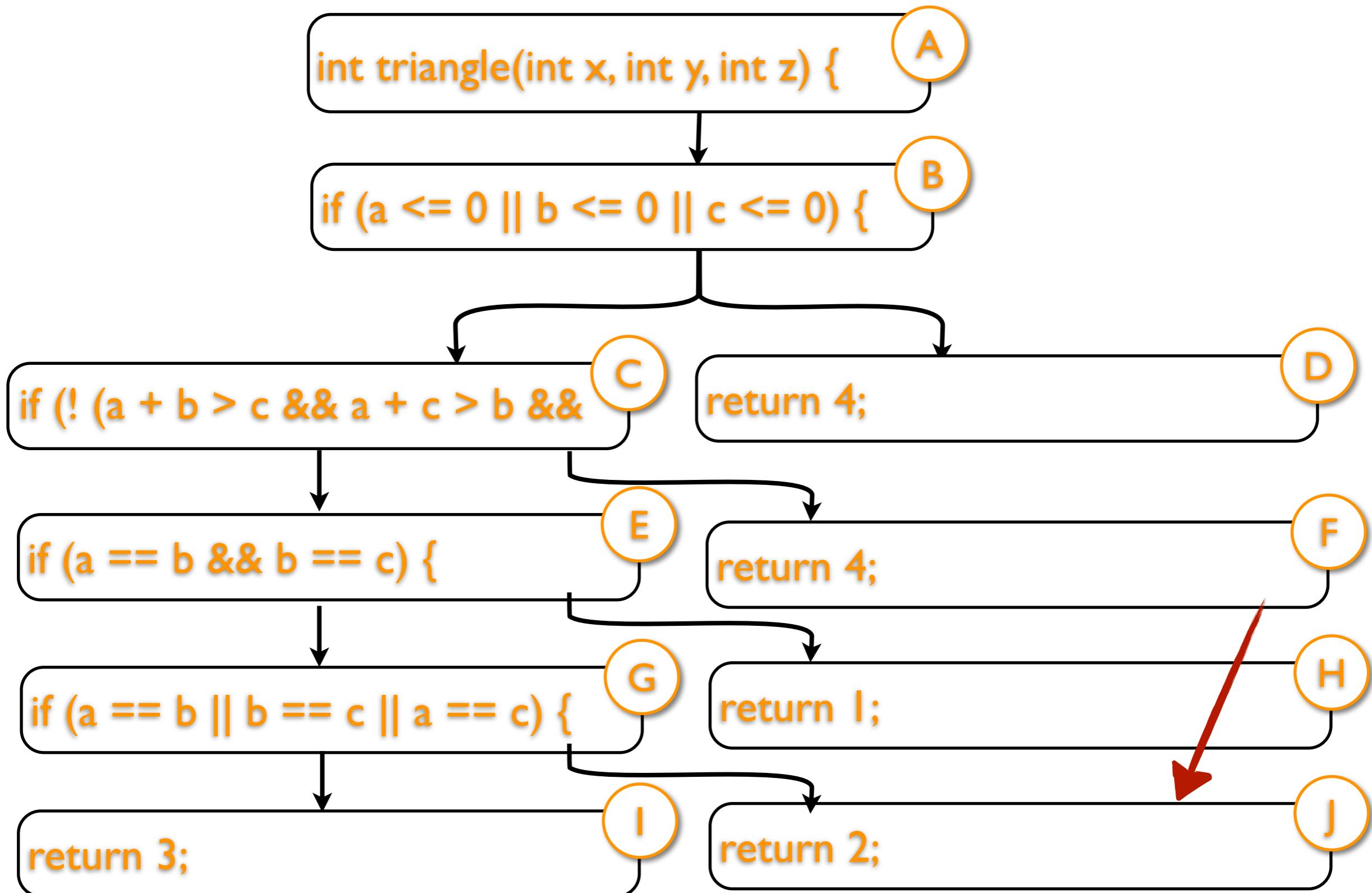
(7, 5, 1)



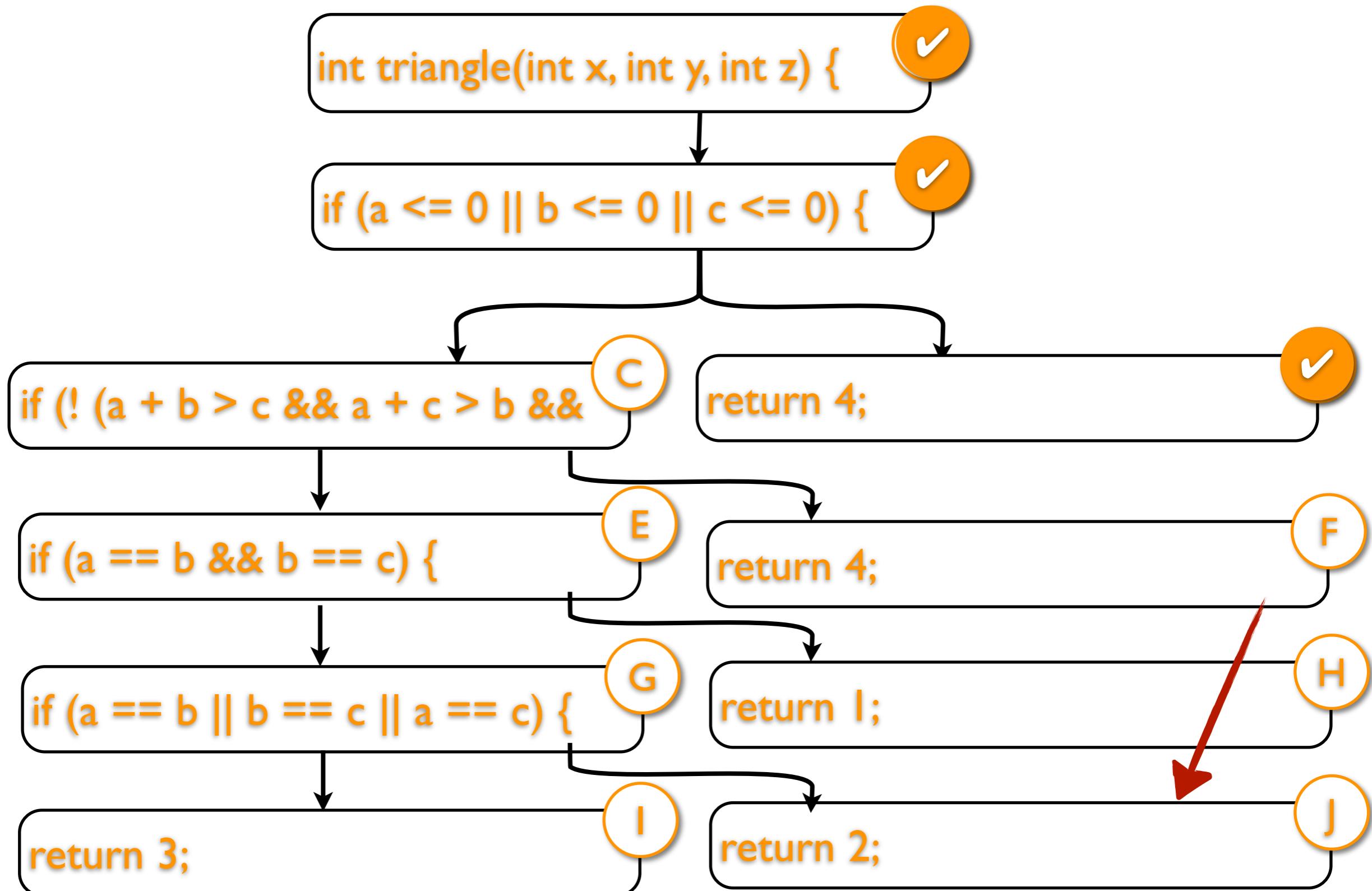
(7, 5, 1)



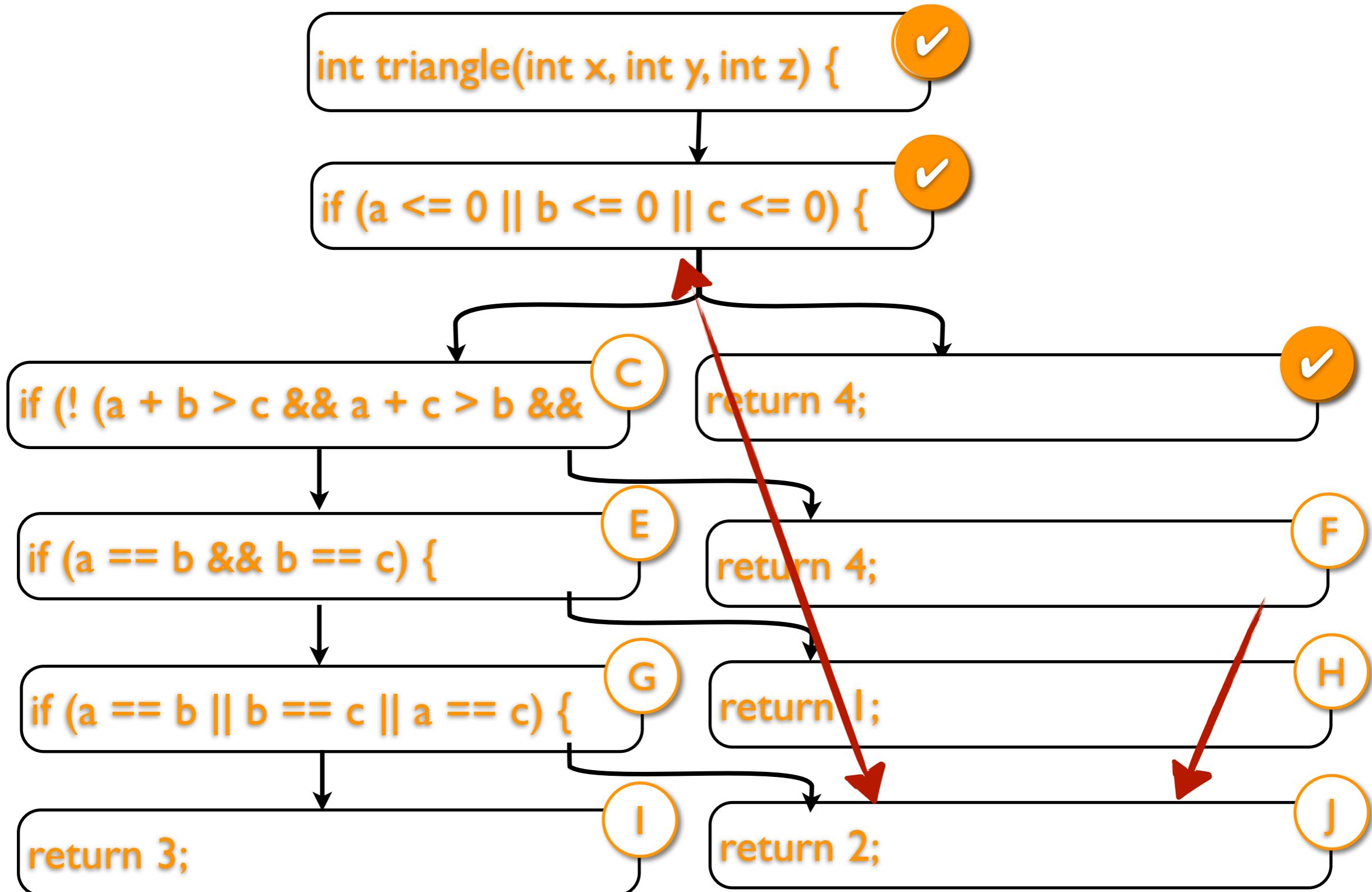
Approach: 3  
Branch: 2



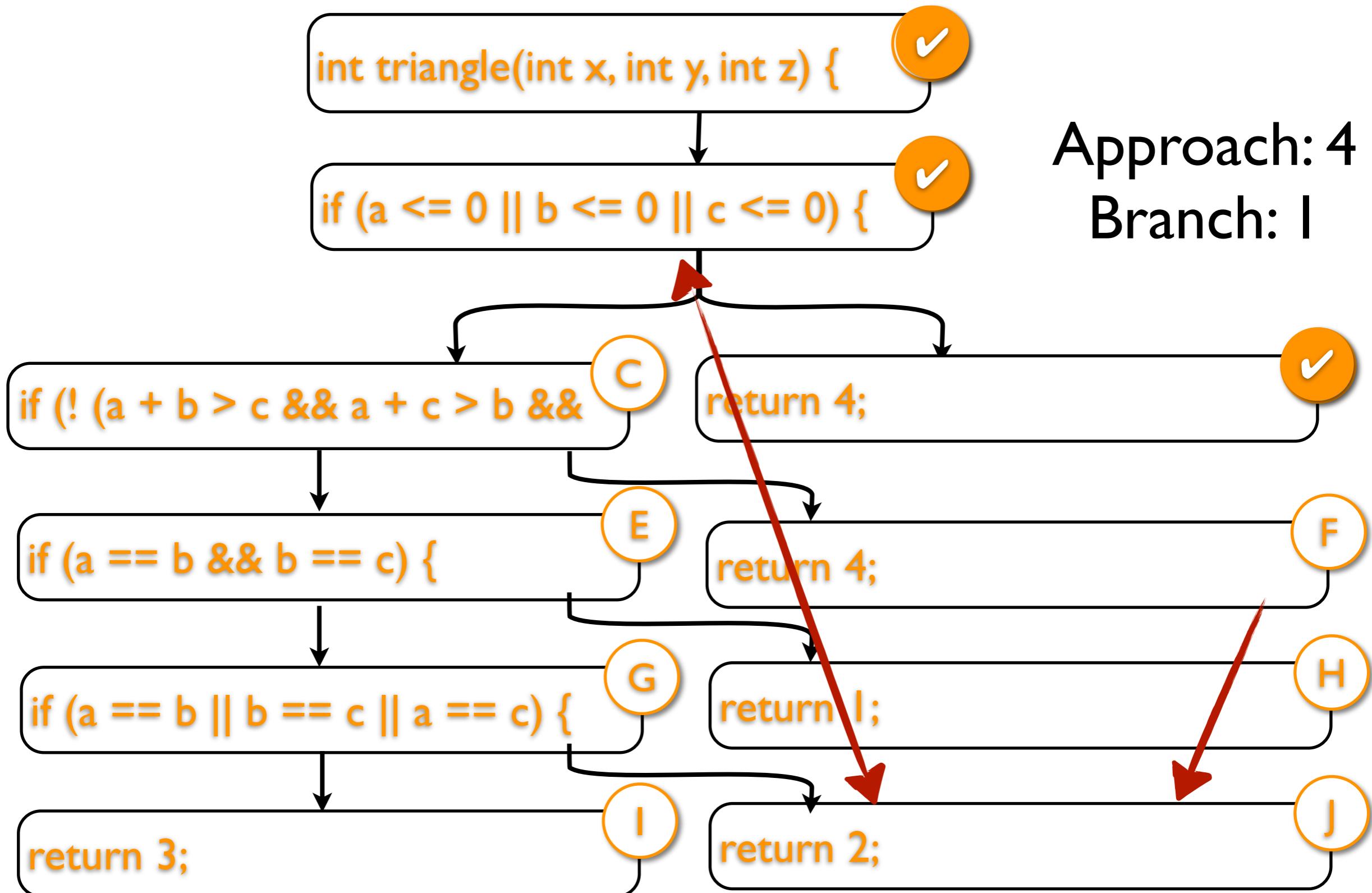
(0, 3, 6)



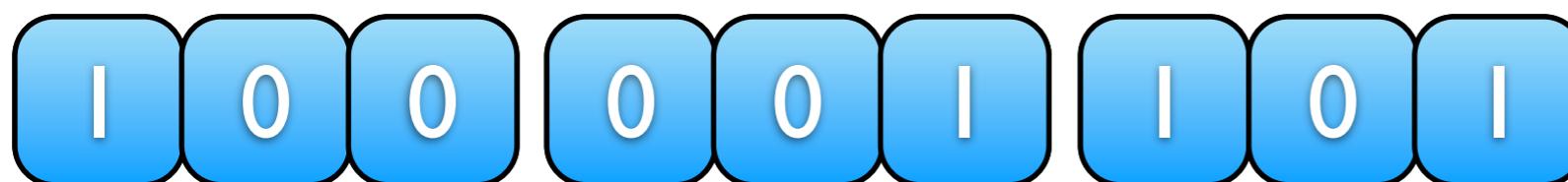
(0, 3, 6)



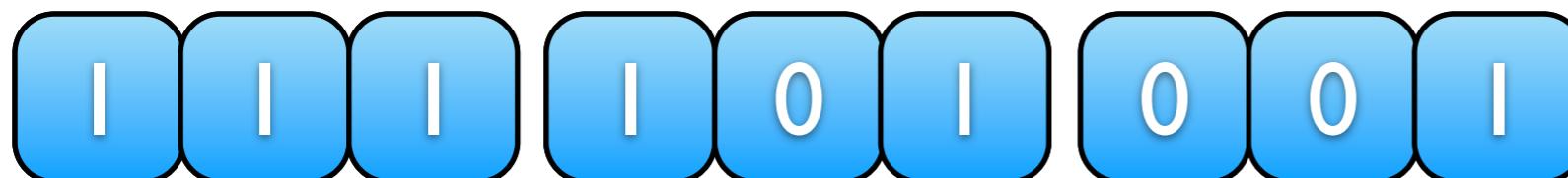
(0, 3, 6)



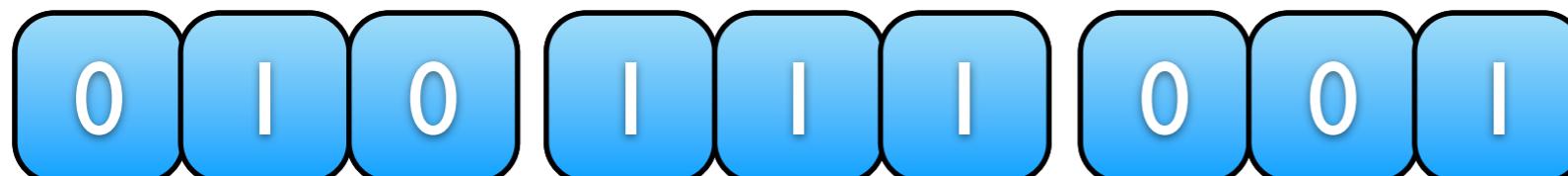
Approach: 4  
Branch: I



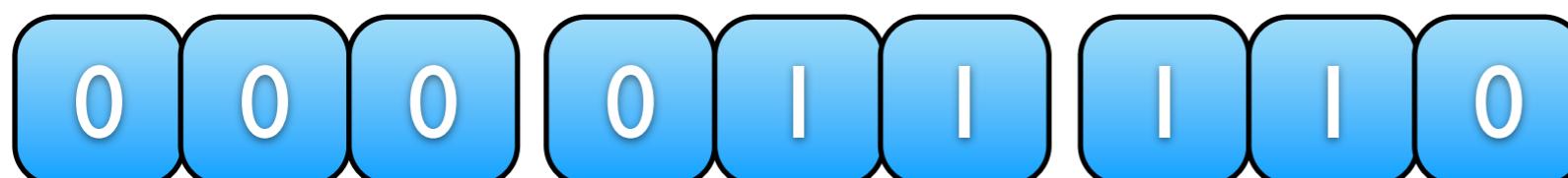
(4,1,5) A: 3, B: 1



(7,5,1) A: 3, B: 2



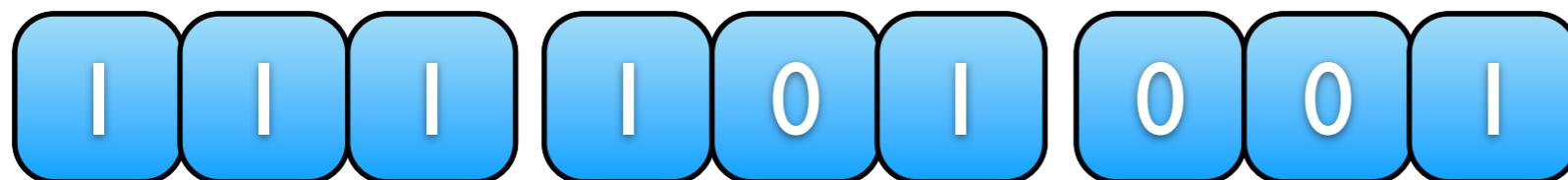
(2,7,1) A: 3, B: 5



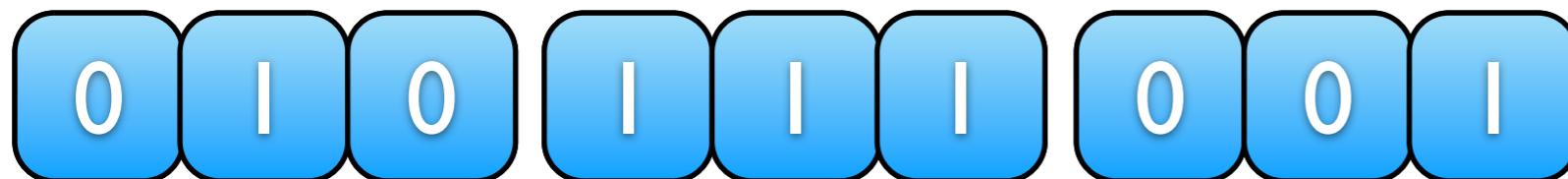
(0,3,6) A: 4, B: 1



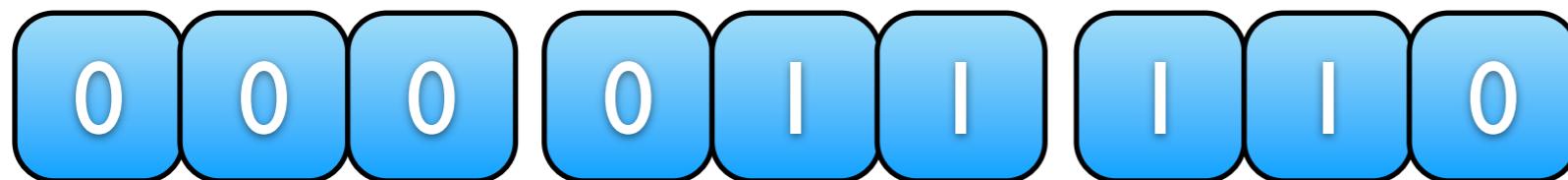
(4,1,5) A: 3, B: 1



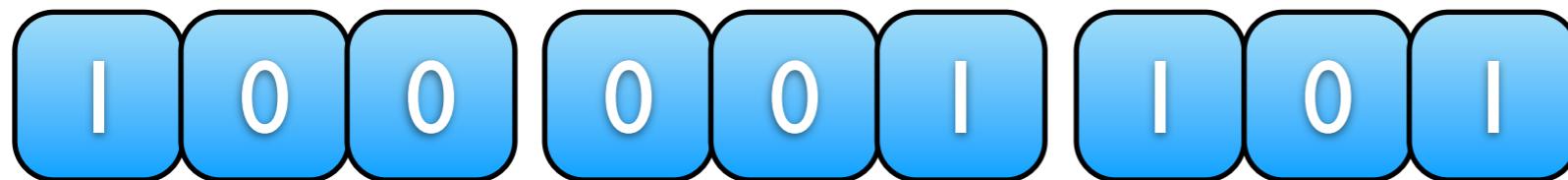
(7,5,1) A: 3, B: 2

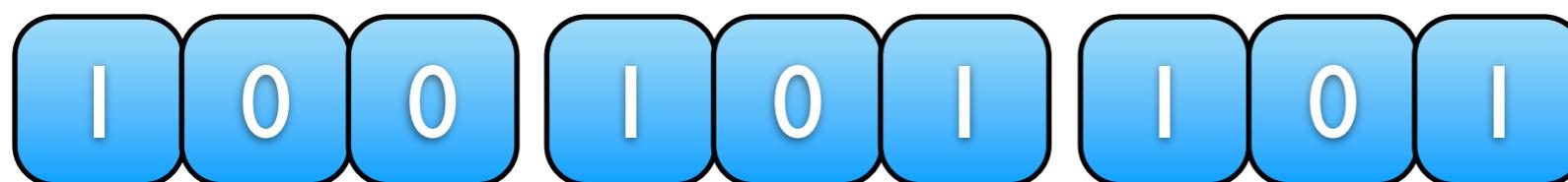


(2,7,1) A: 3, B: 5

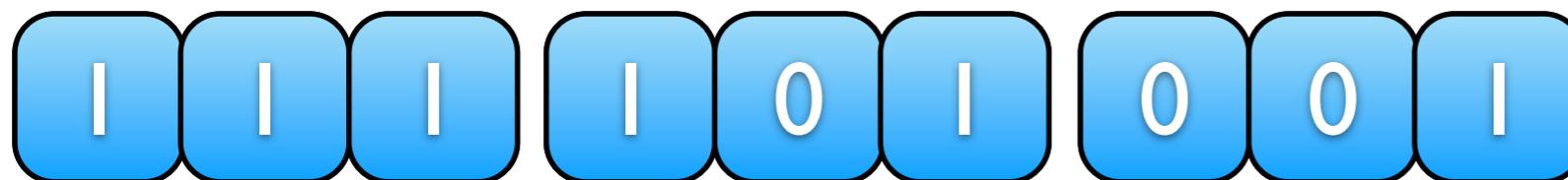


(0,3,6) A: 4, B: 1

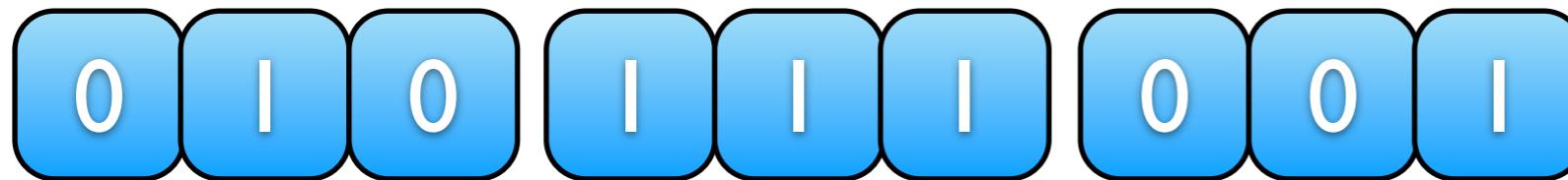




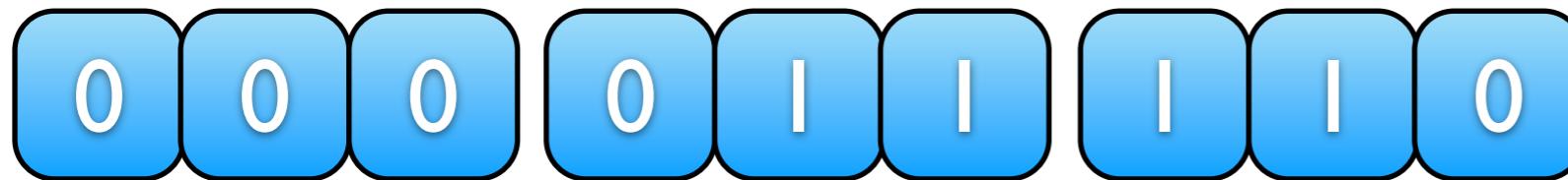
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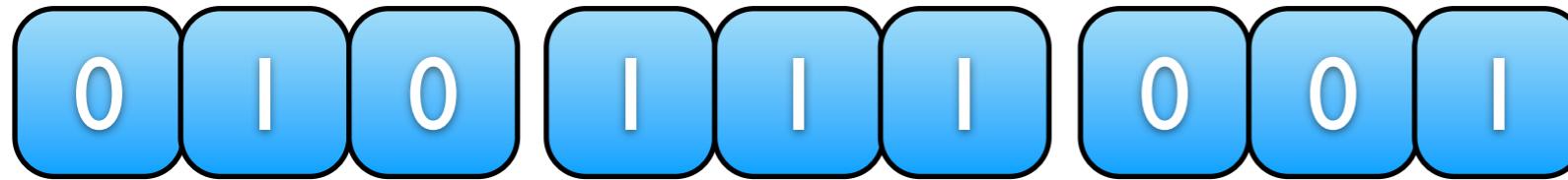
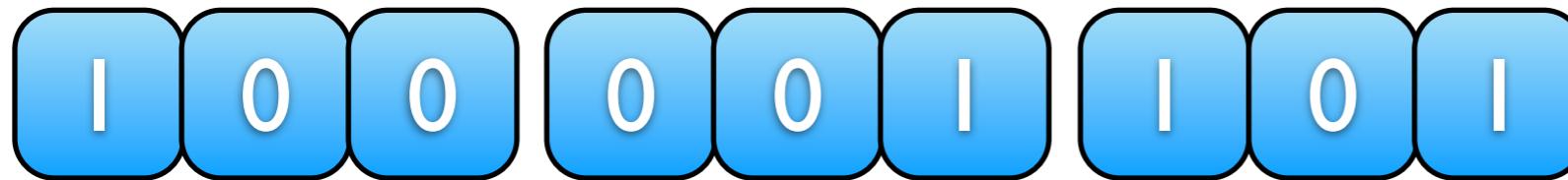
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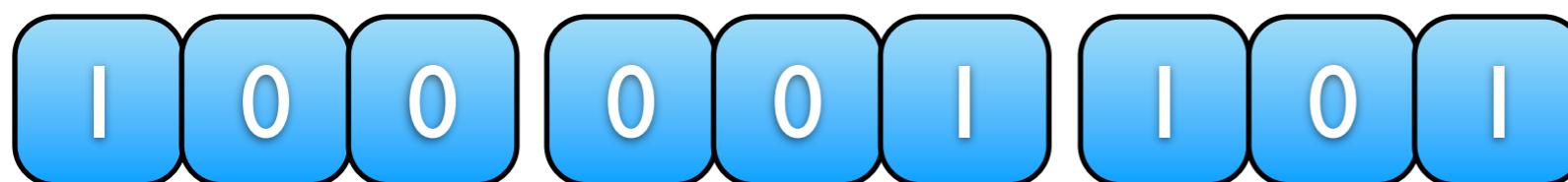


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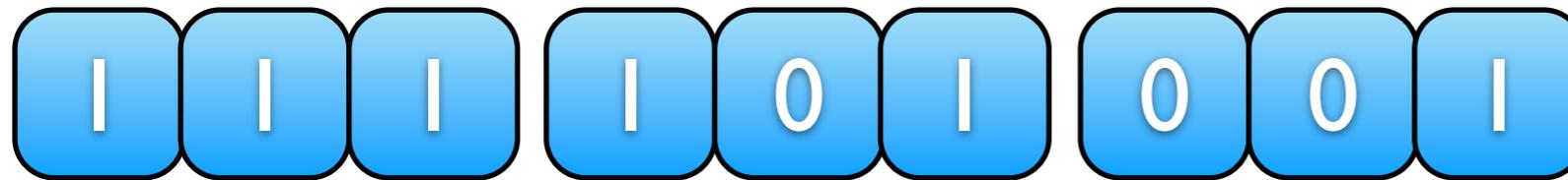


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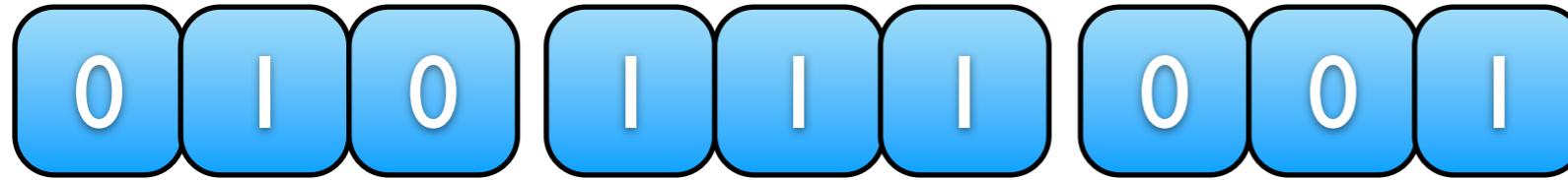




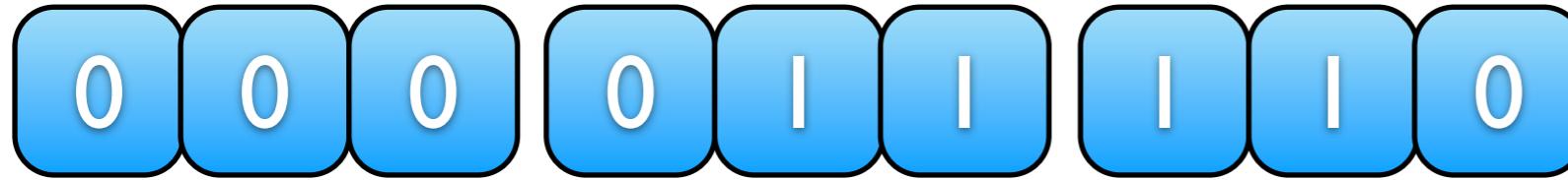
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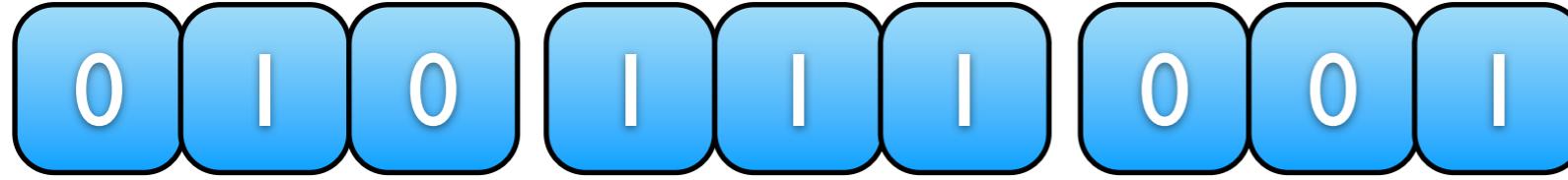
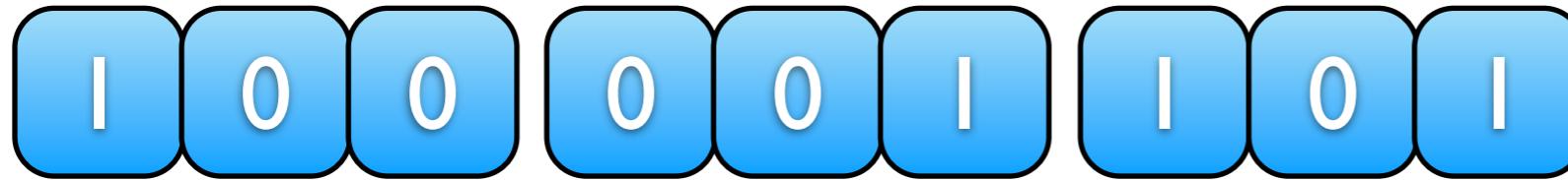
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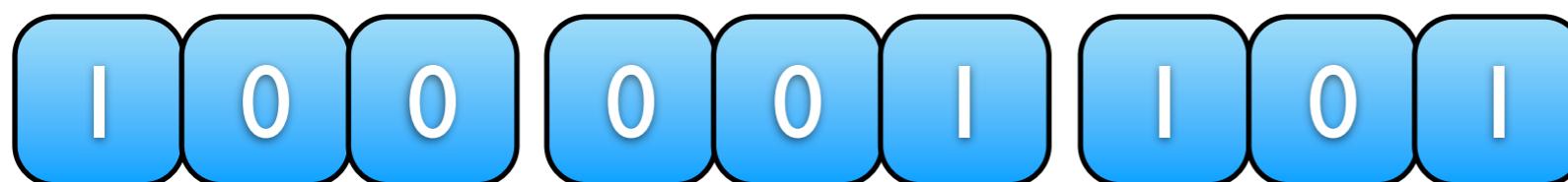


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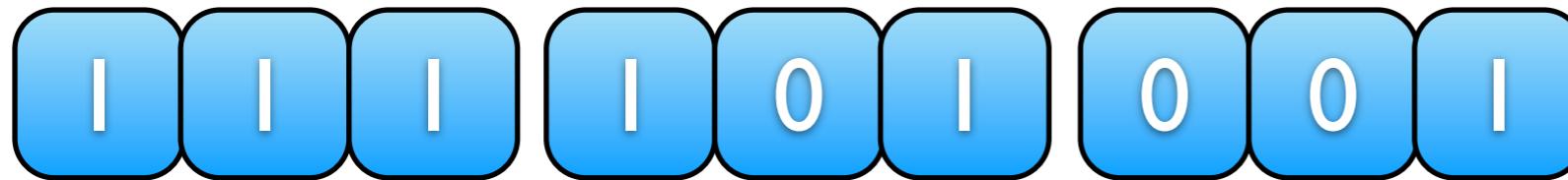


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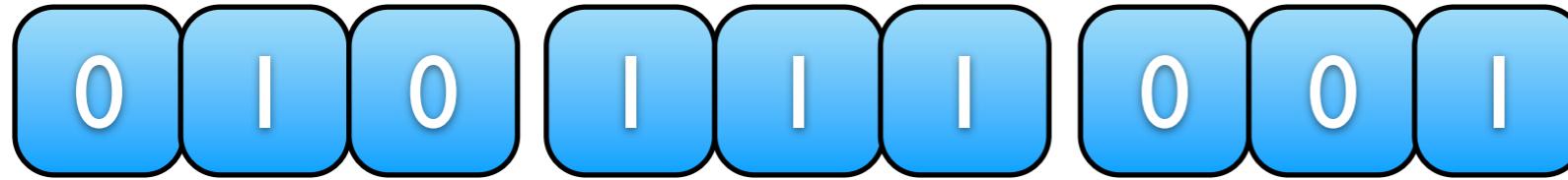




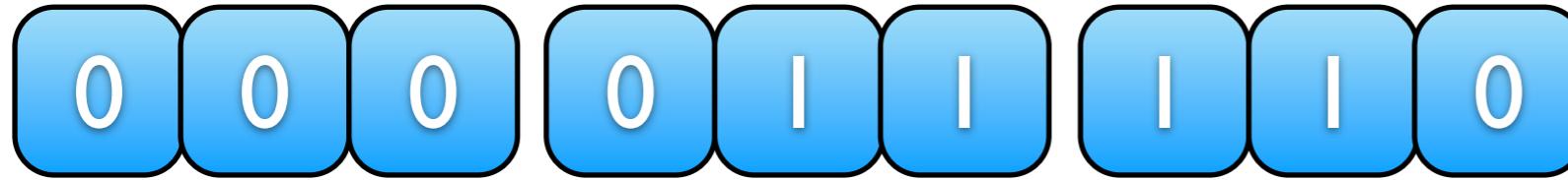
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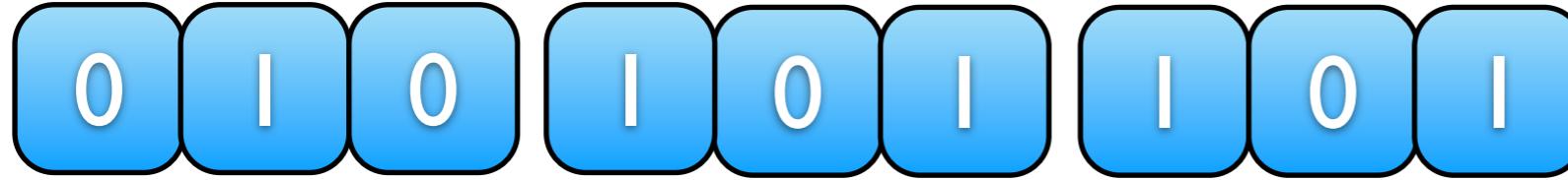
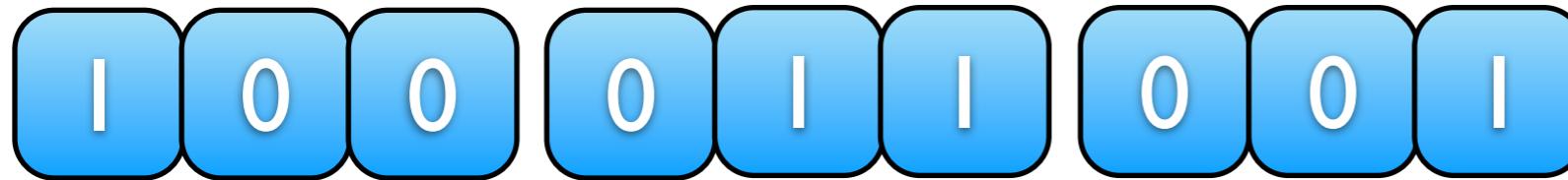
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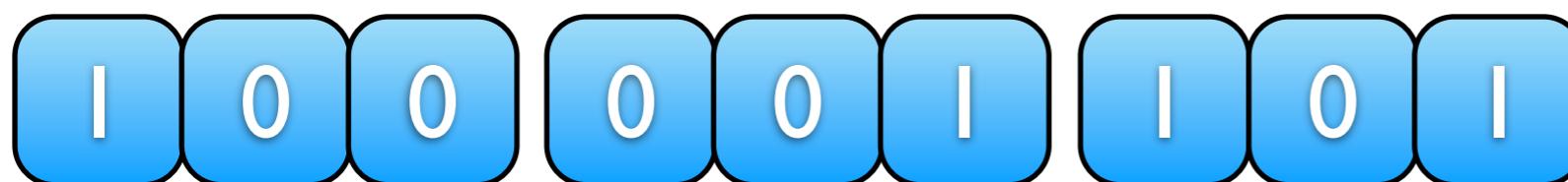


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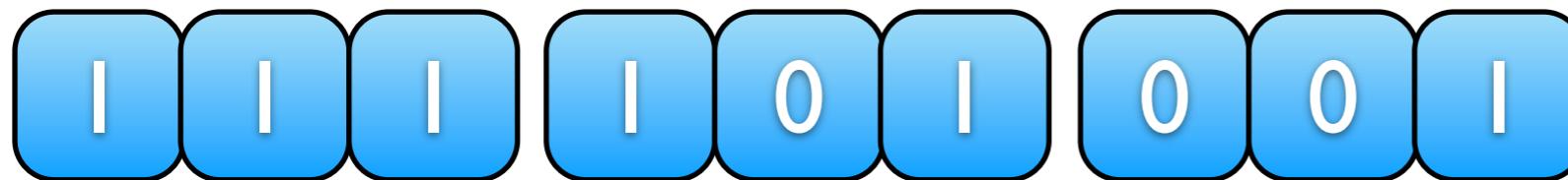


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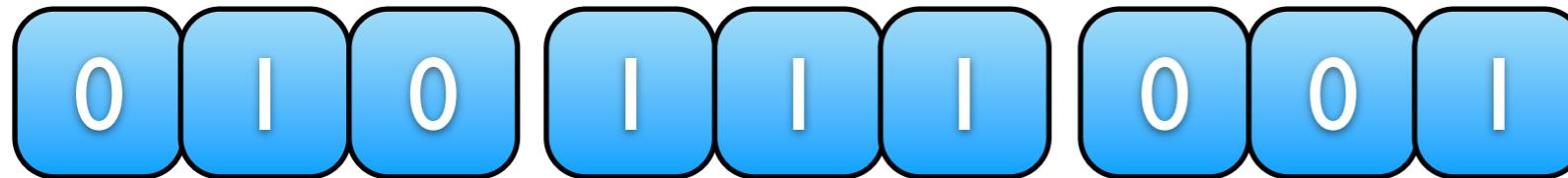




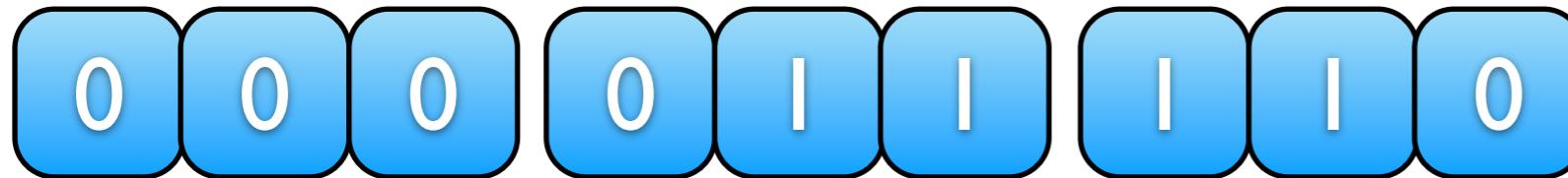
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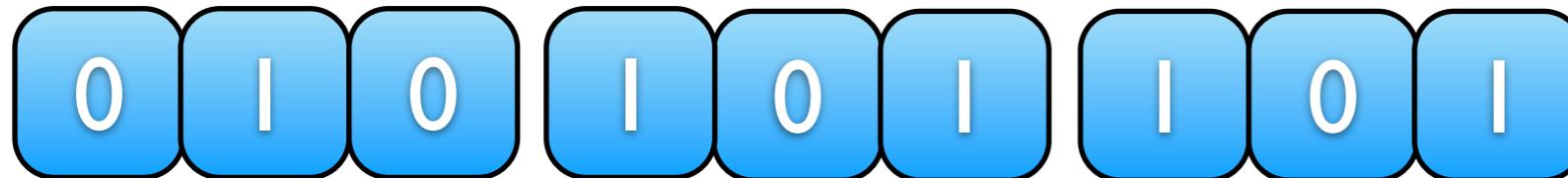
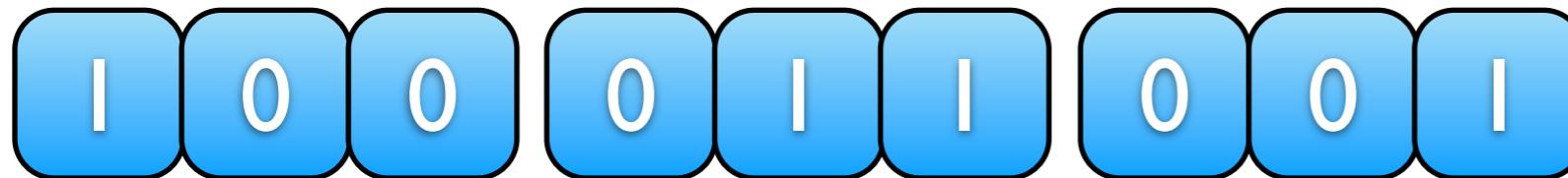
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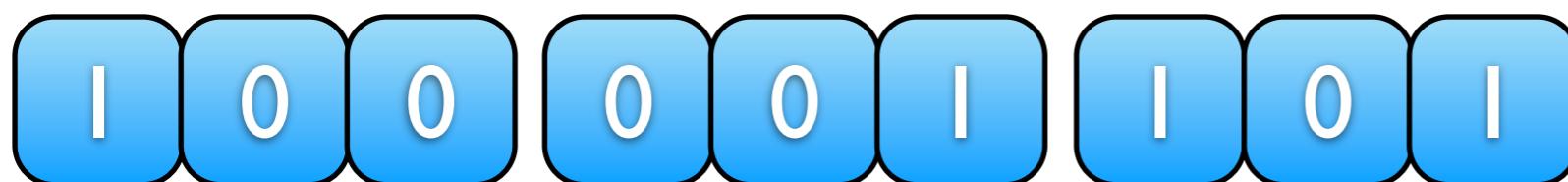


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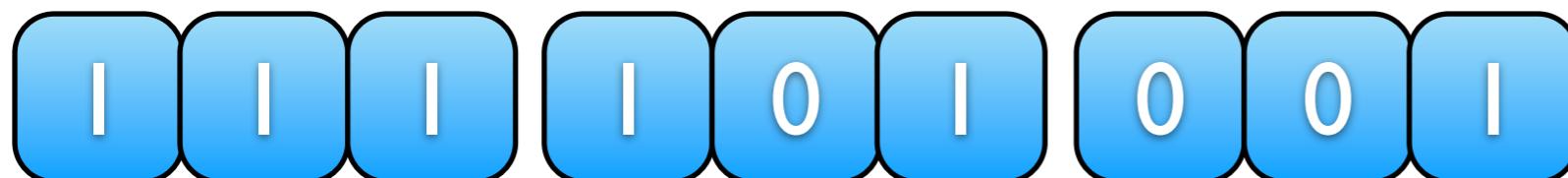


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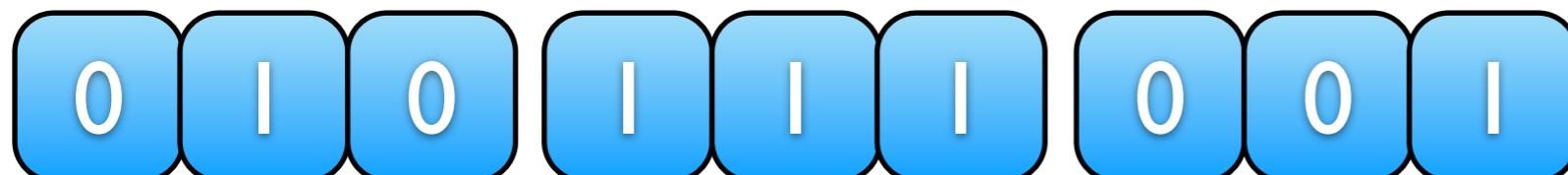




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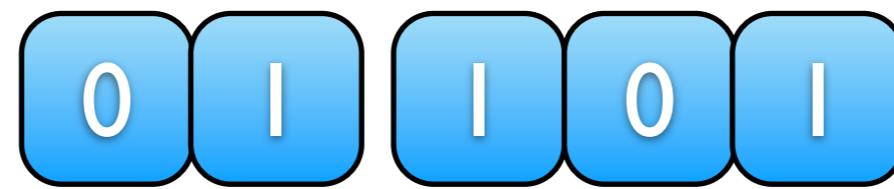
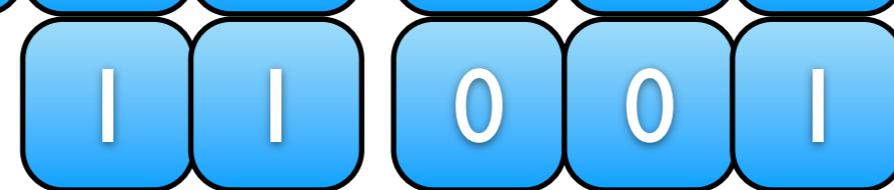
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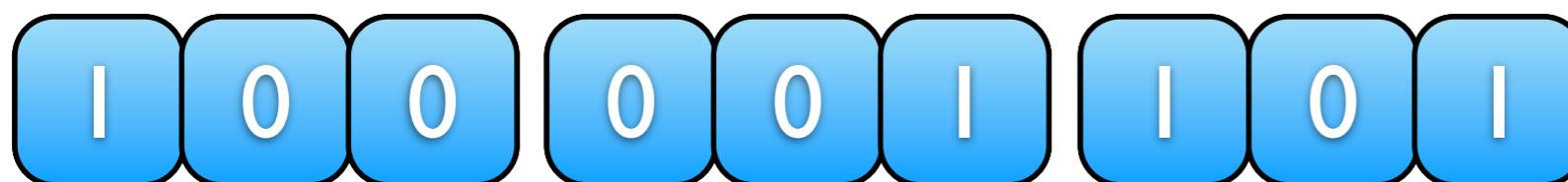


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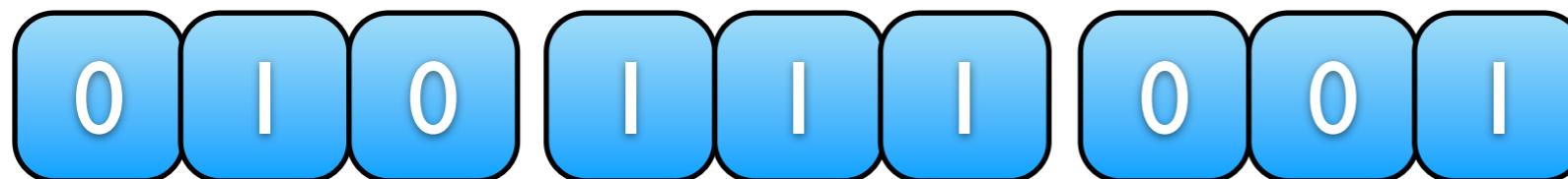




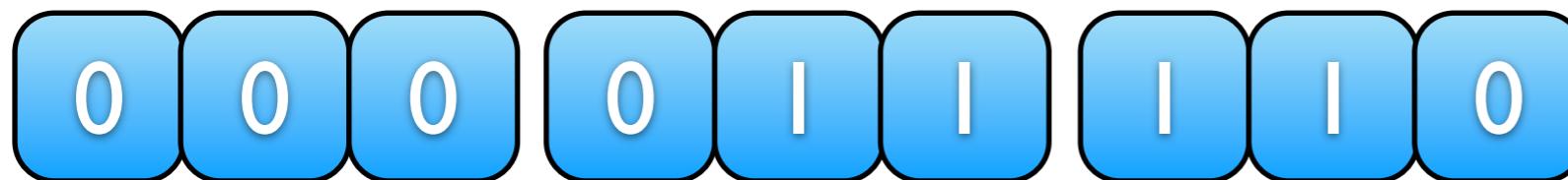
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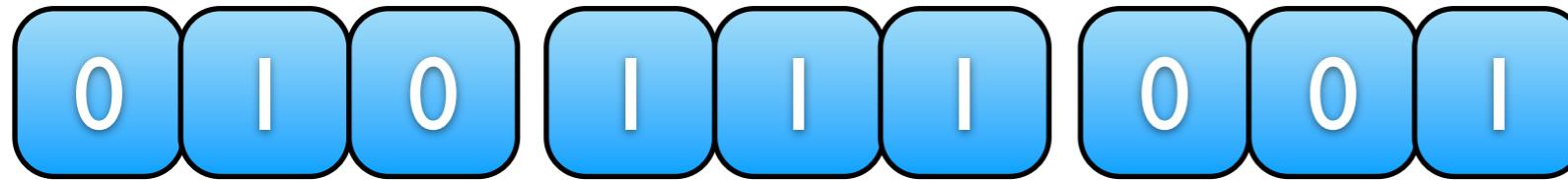
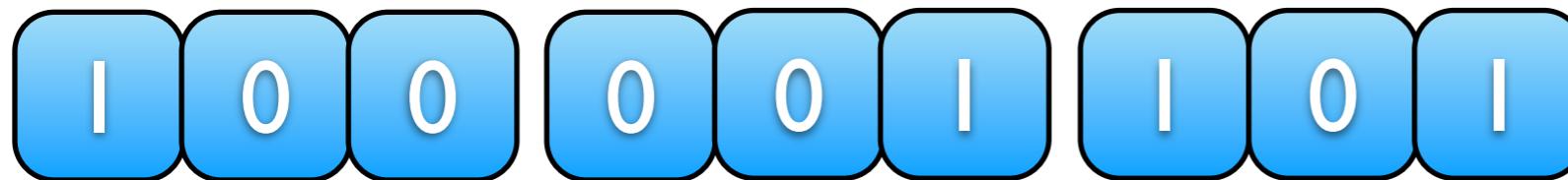
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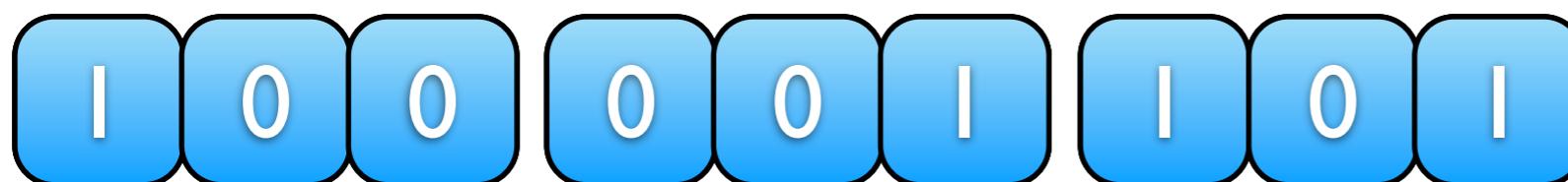


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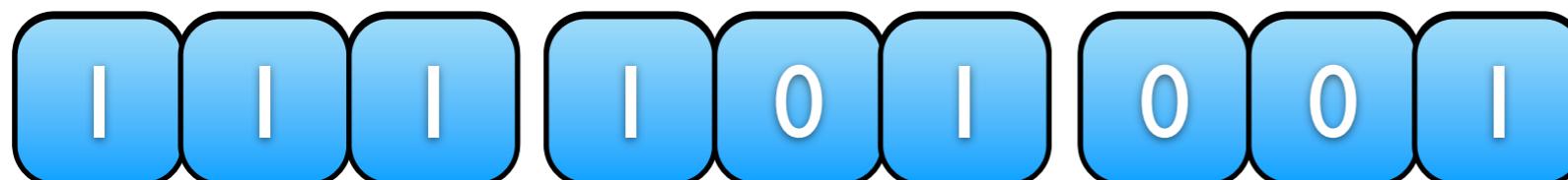


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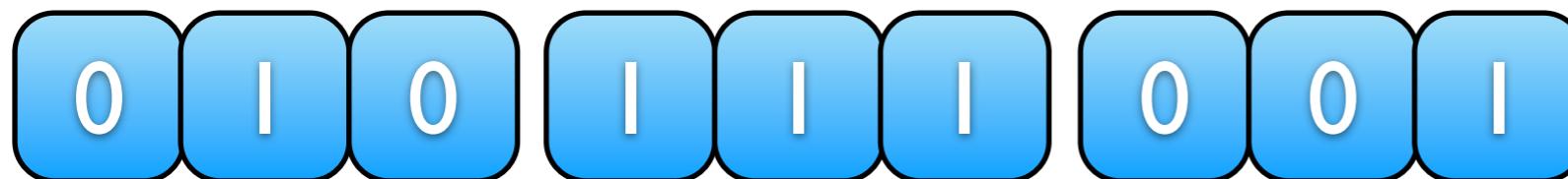




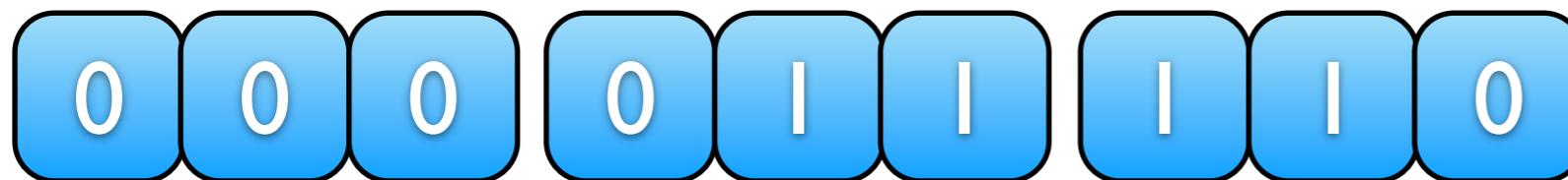
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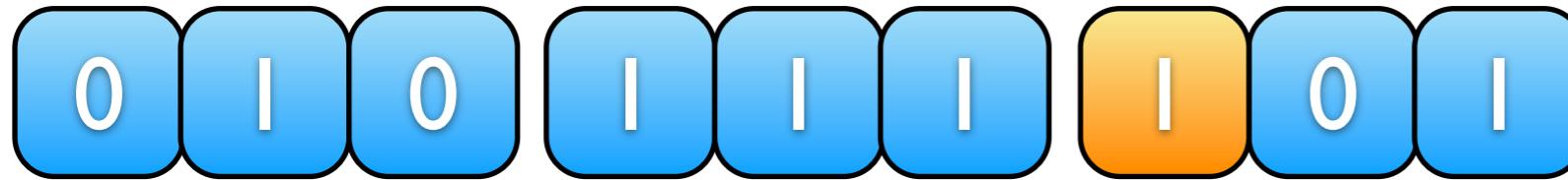
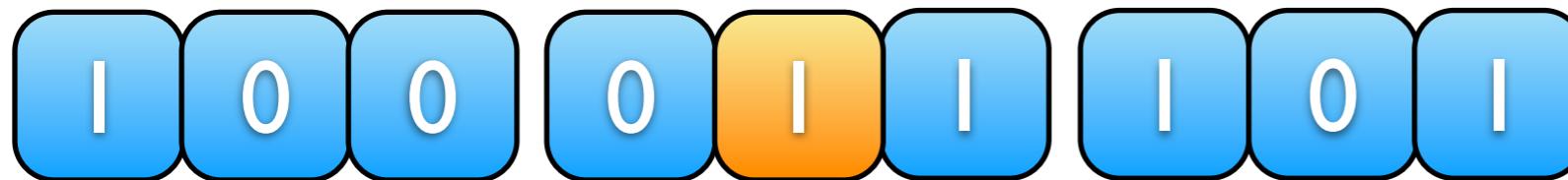
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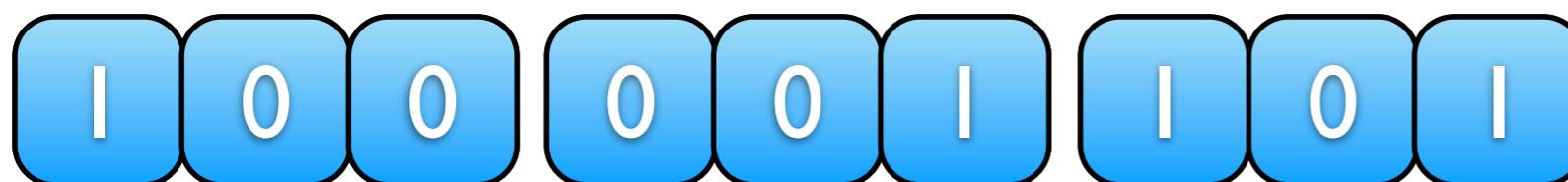


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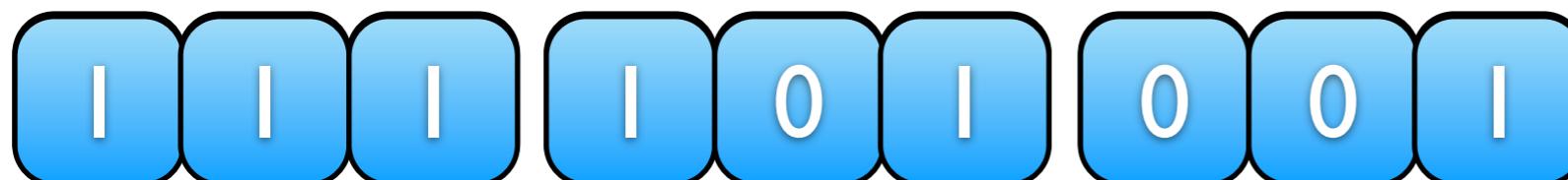


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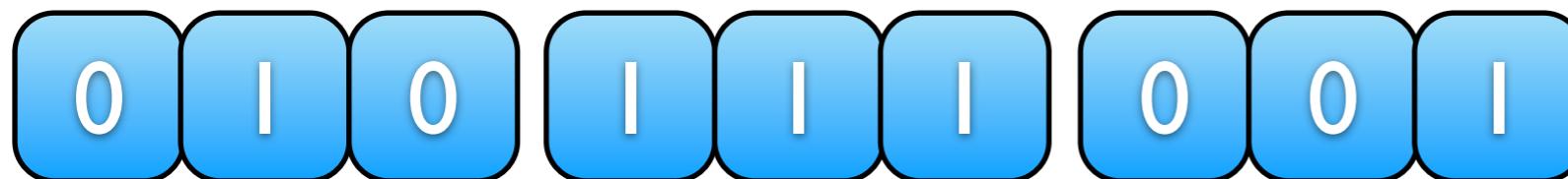




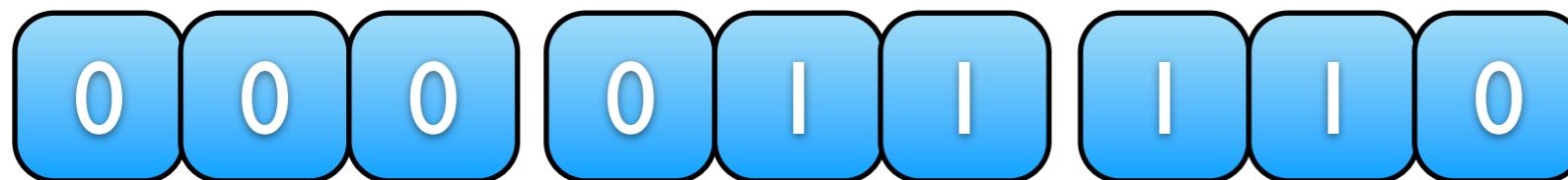
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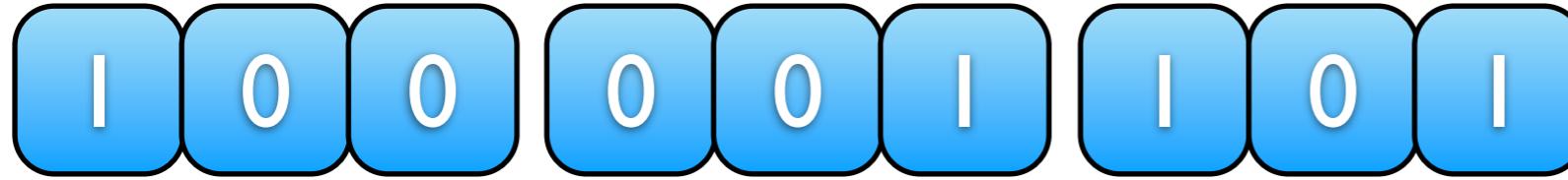
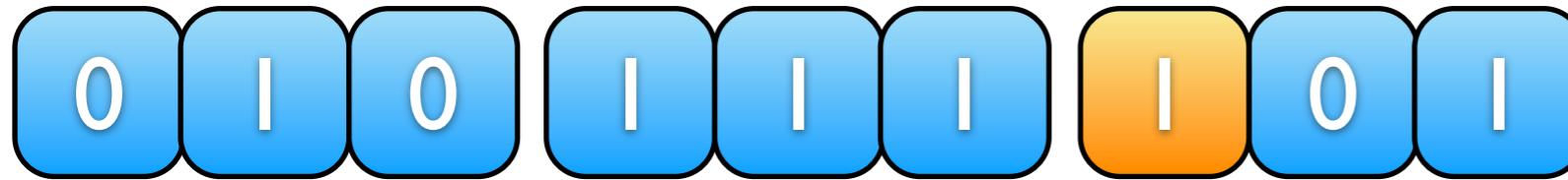
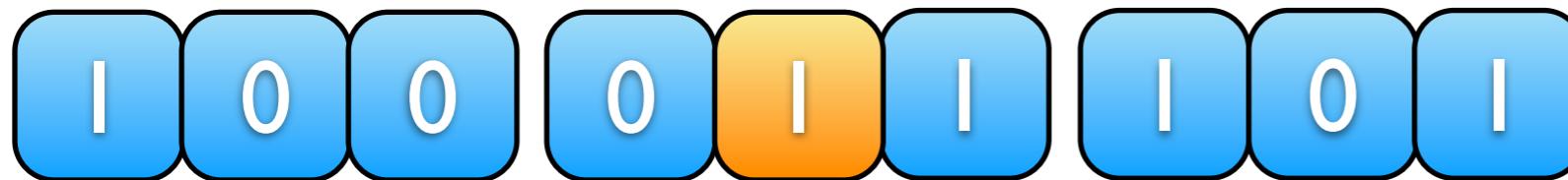
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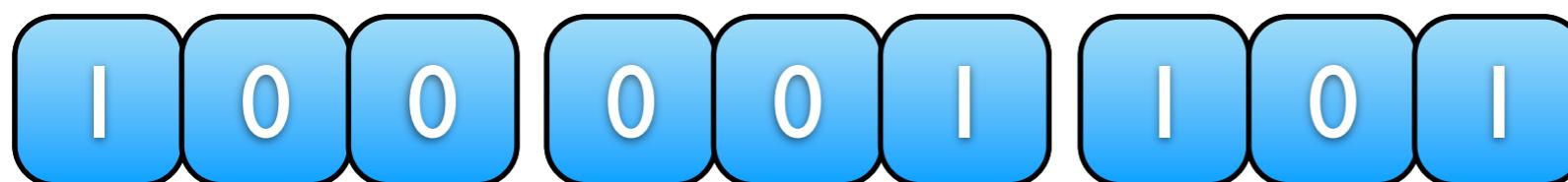


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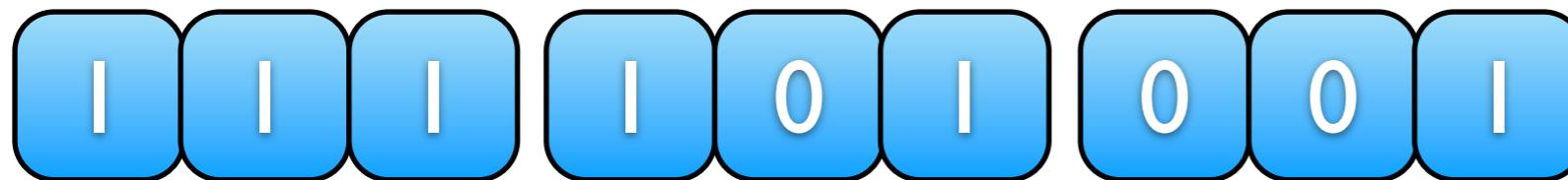


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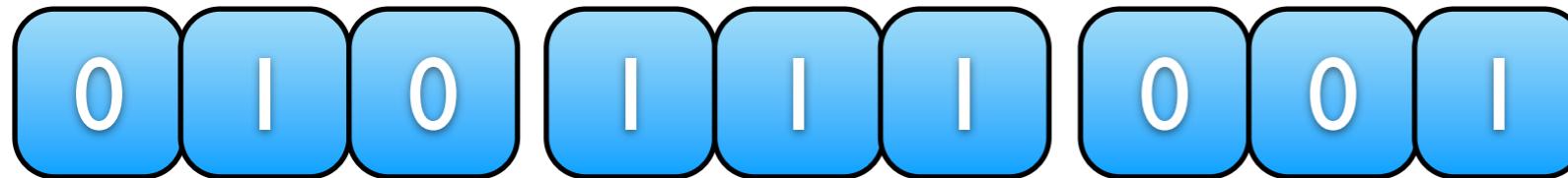




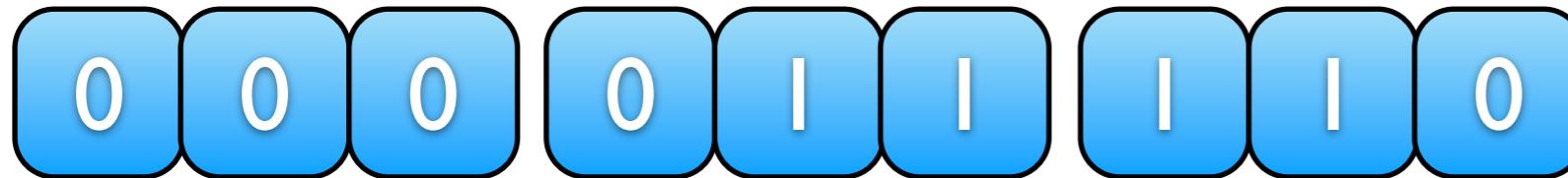
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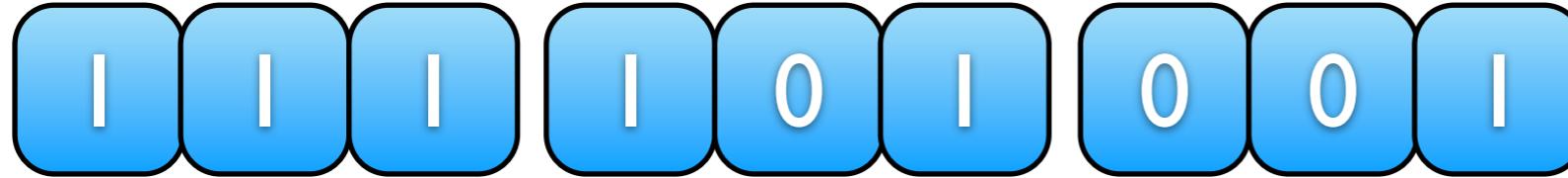
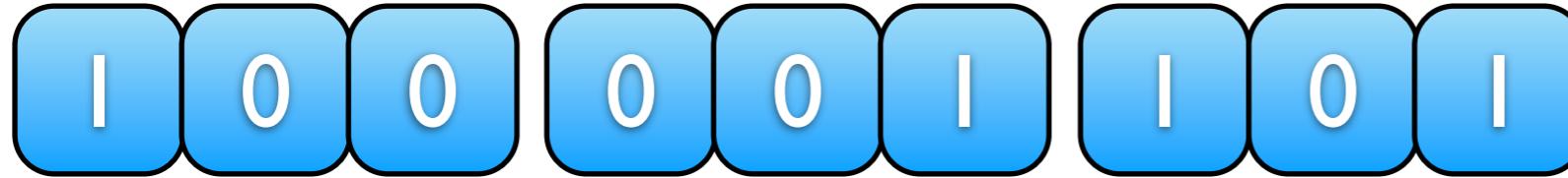
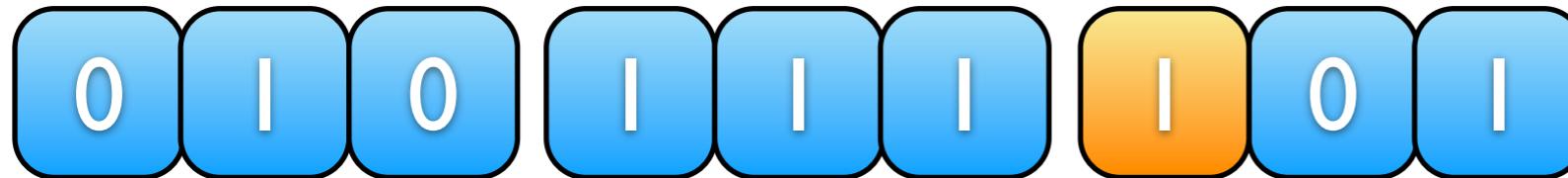
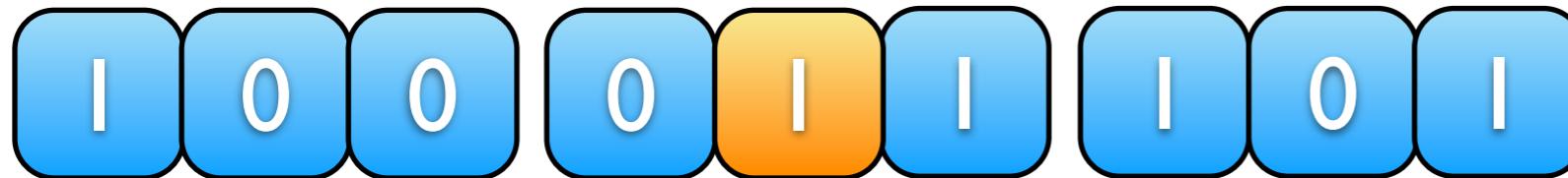
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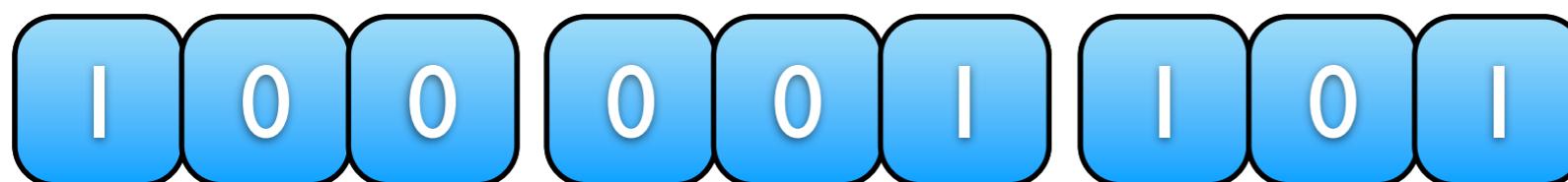


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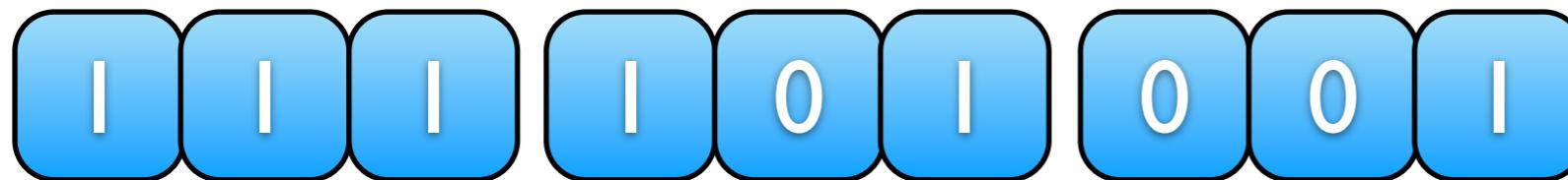


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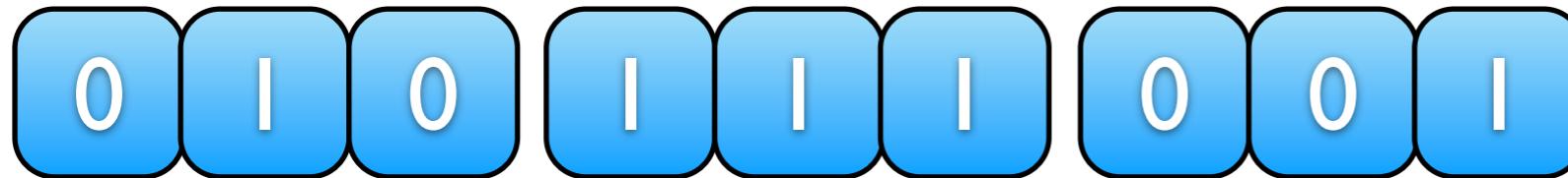




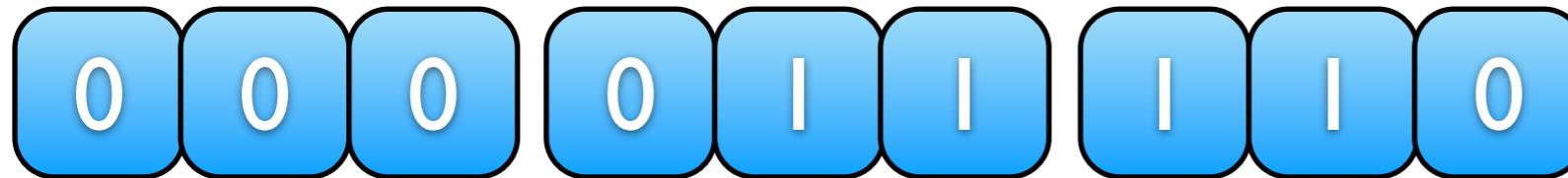
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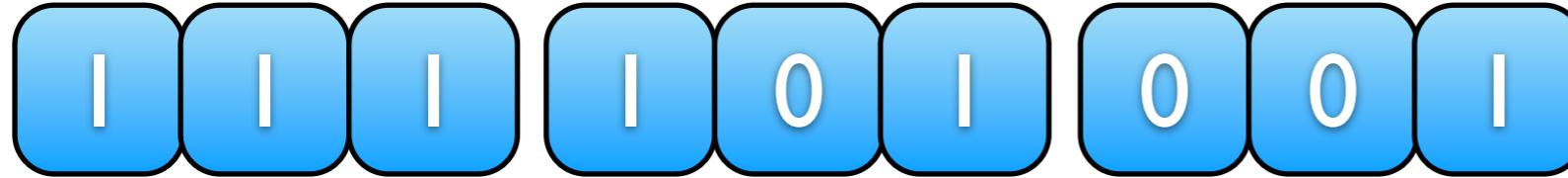
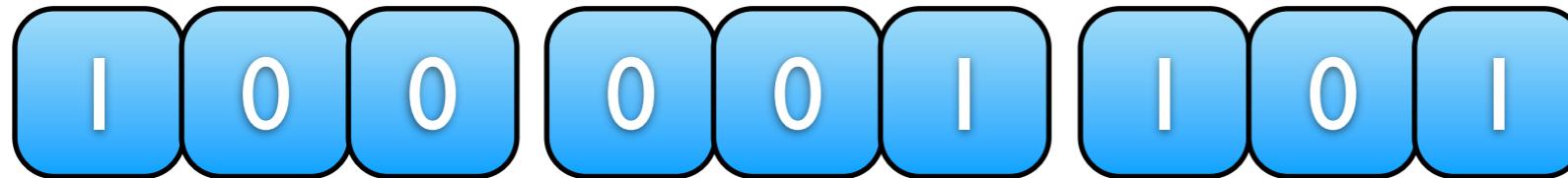
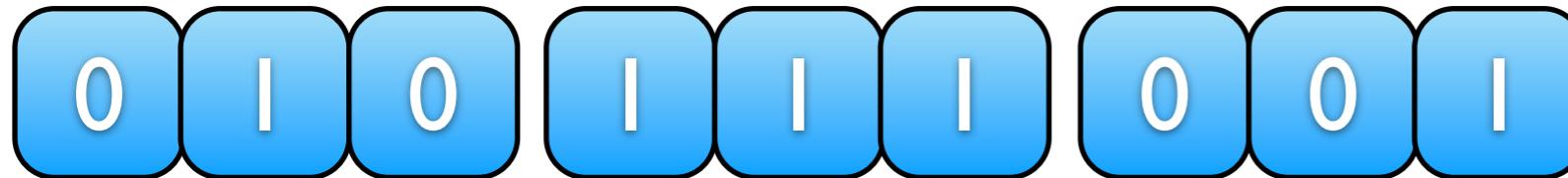
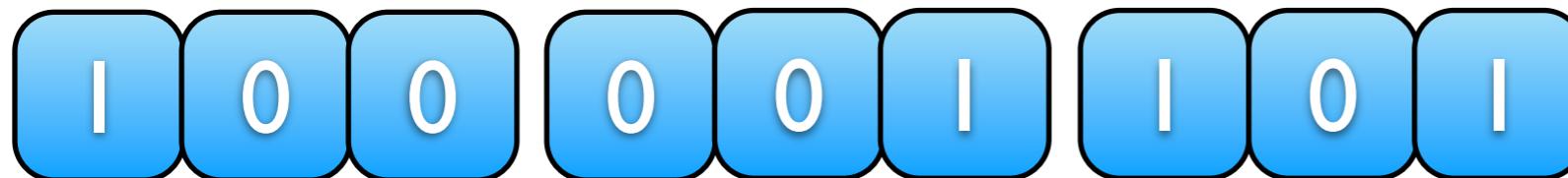
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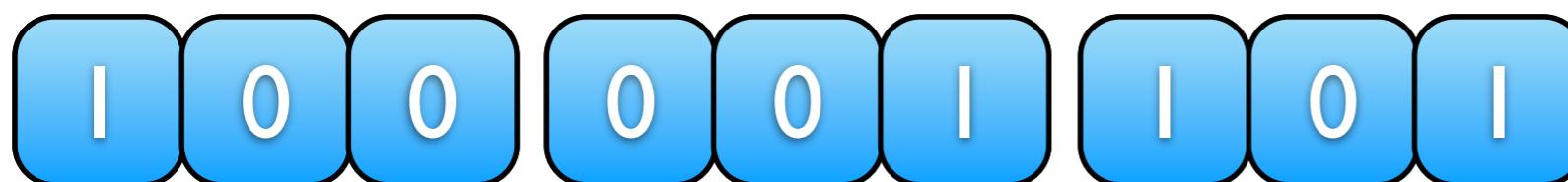


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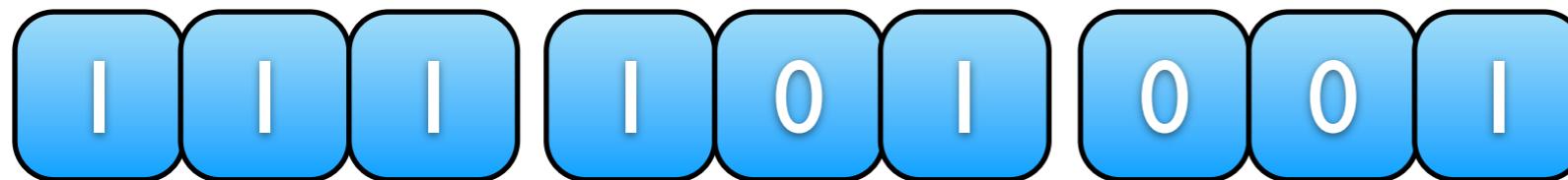


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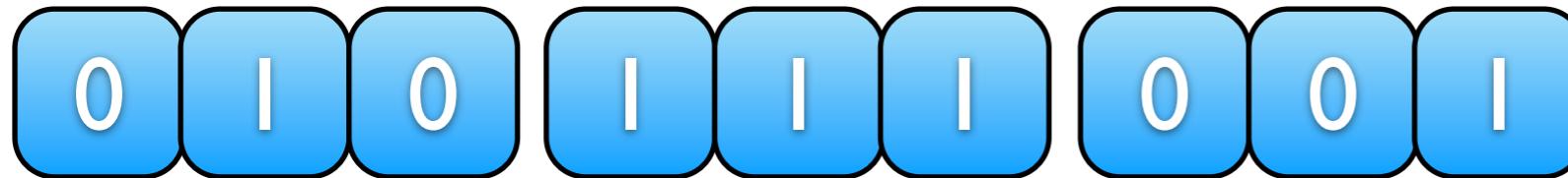




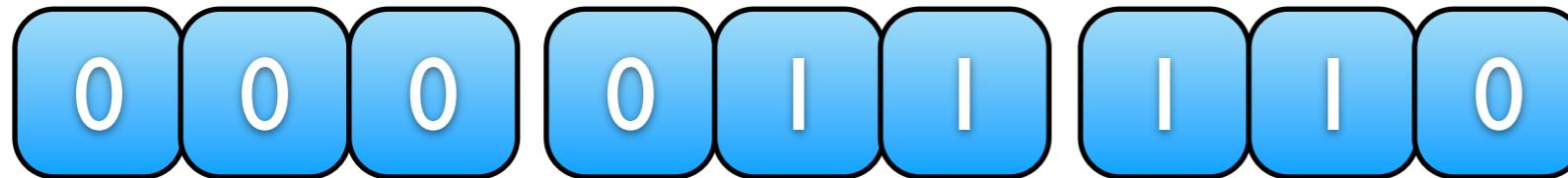
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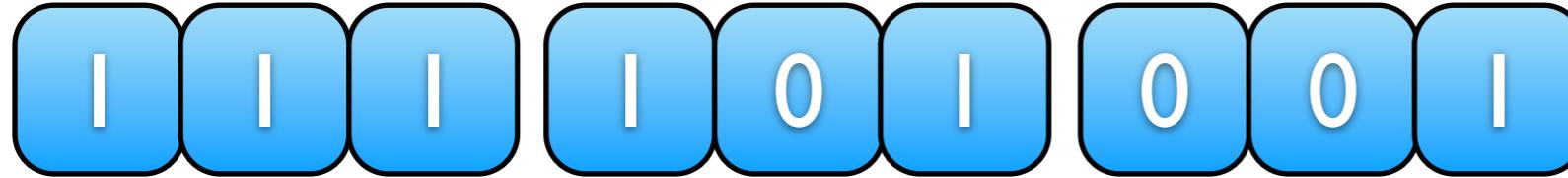
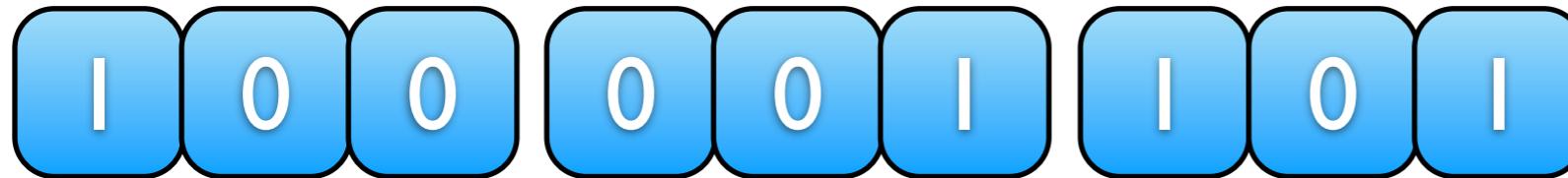
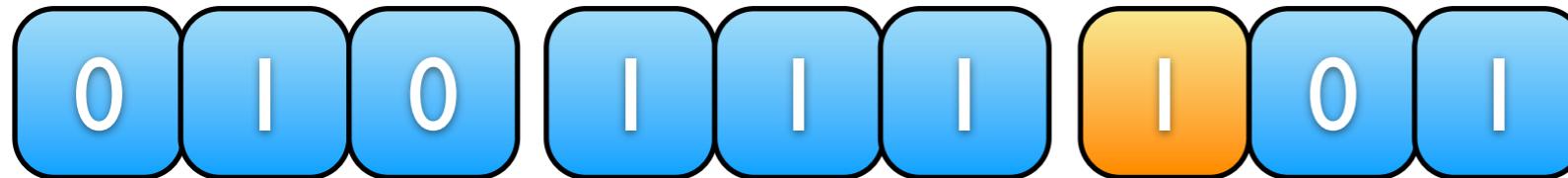
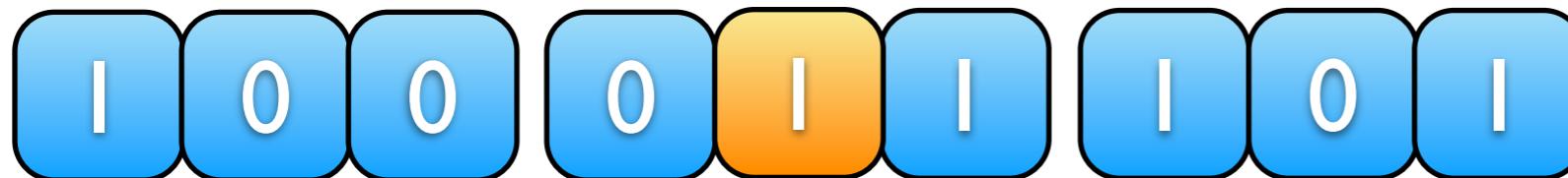
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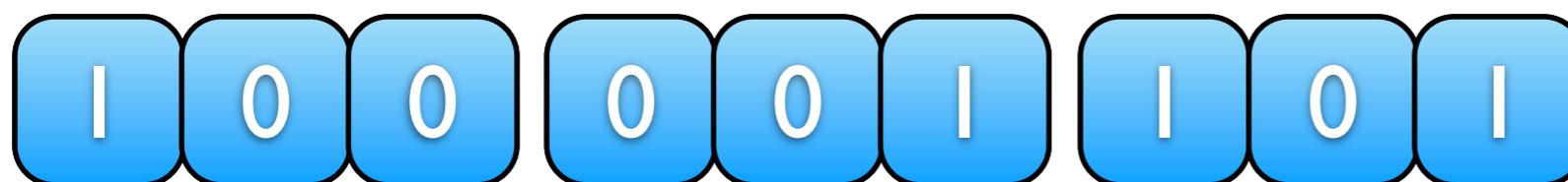


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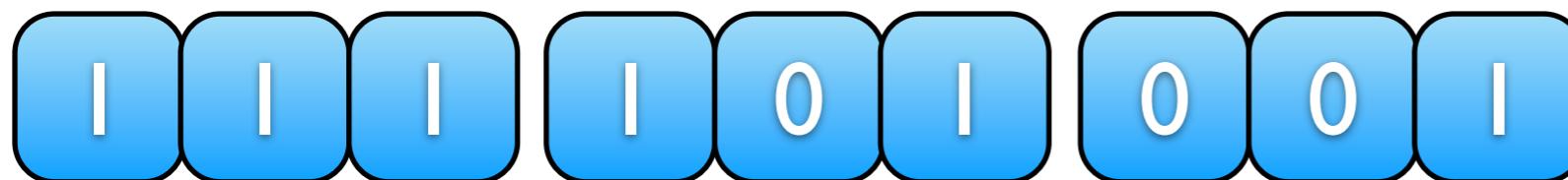


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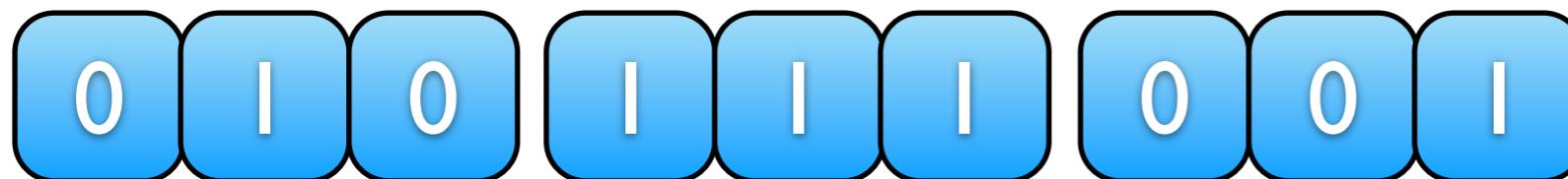




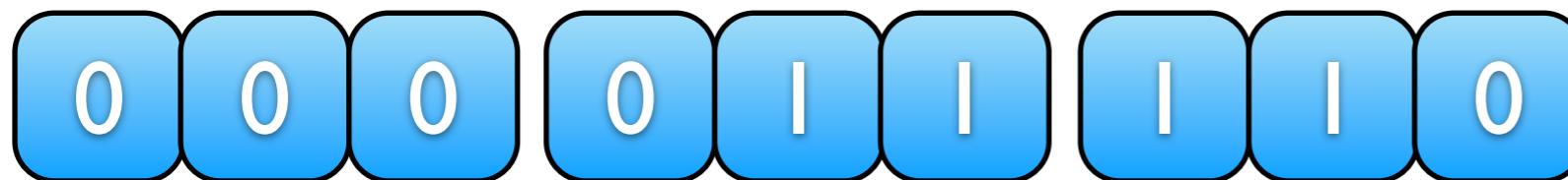
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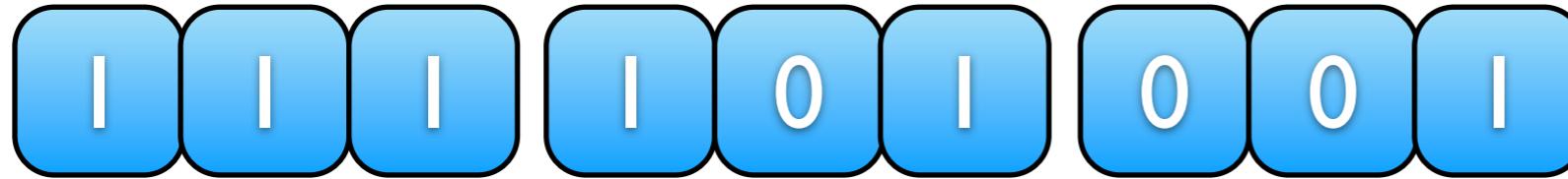
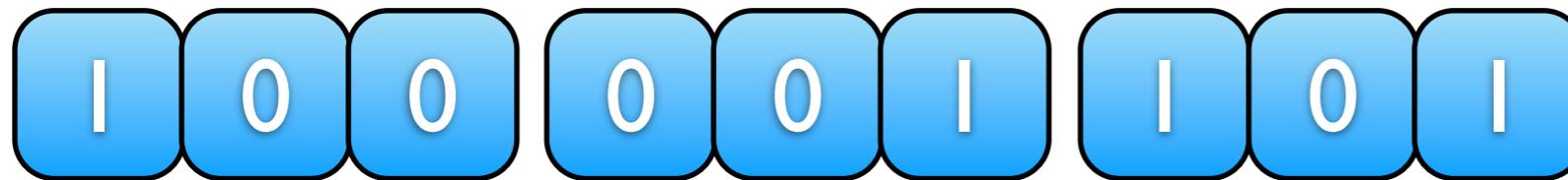
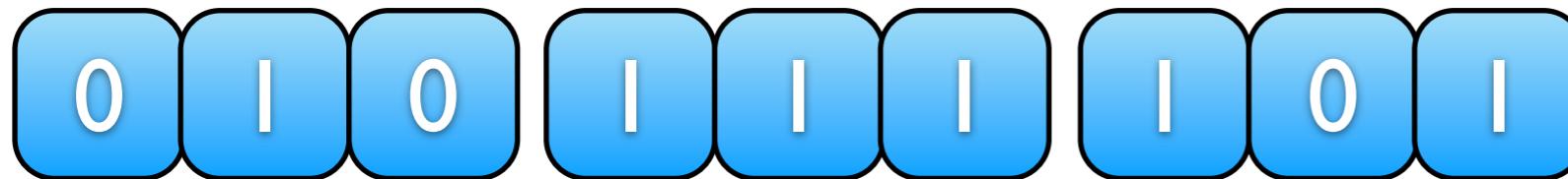
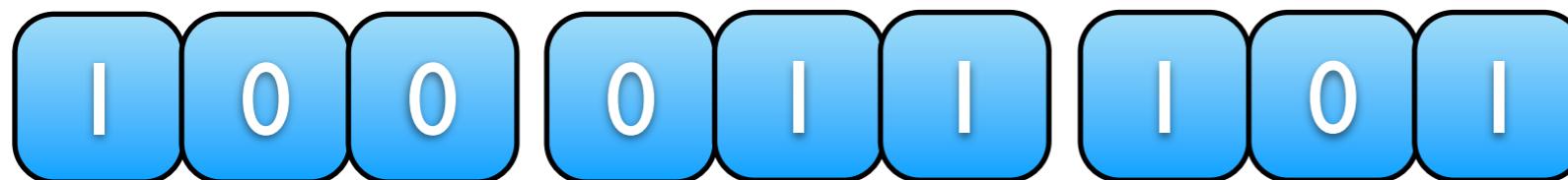
(7,5,1) A: 3, B: 2



(2,7,1) A: 3, B: 5



(0,3,6) A: 4, B: 1



I	0	0	0	I	I	I	0	I
0	I	0	I	I	I	I	0	I
I	0	0	0	0	I	I	0	I
I	I	I	I	0	I	0	0	I

I	0	0	0	I	I	I	0	I
0	I	0	I	I	I	I	0	I
I	0	0	0	0	I	I	0	I
I	I	I	I	0	I	0	0	I

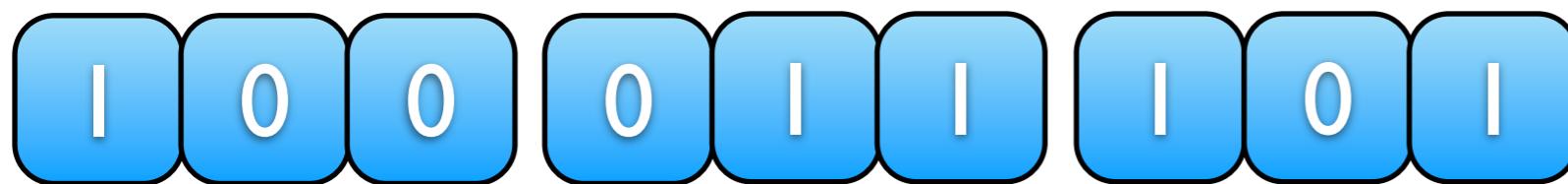
(4,3,5)

I	0	0	0	I	I	I	0	I
0	I	0	I	I	I	I	0	I
I	0	0	0	0	I	I	0	I
I	I	I	I	0	I	0	0	I

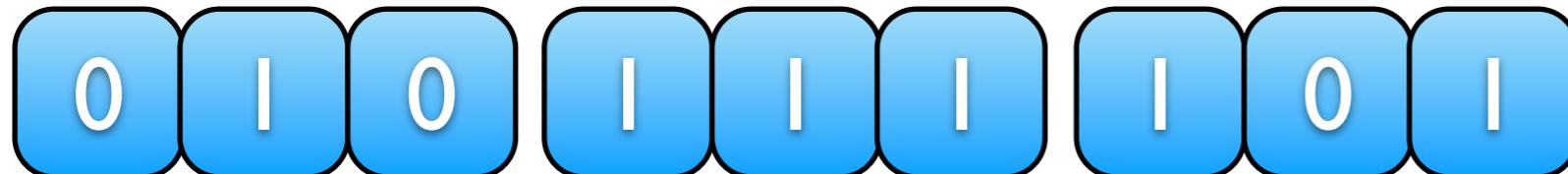
(4,3,5) A: 2, B: 2

I	0	0	0	I	I	I	0	I
0	I	0	I	I	I	I	0	I
I	0	0	0	0	I	I	0	I
I	I	I	I	0	I	0	0	I

(4,3,5) A: 2, B: 2  
(2,7,5)

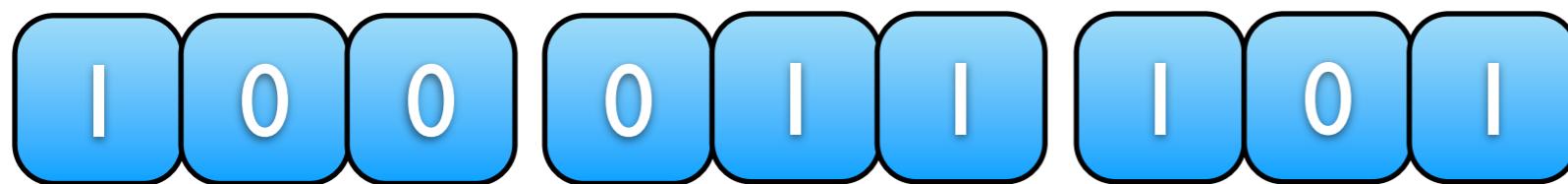


(4,3,5) A: 2, B: 2

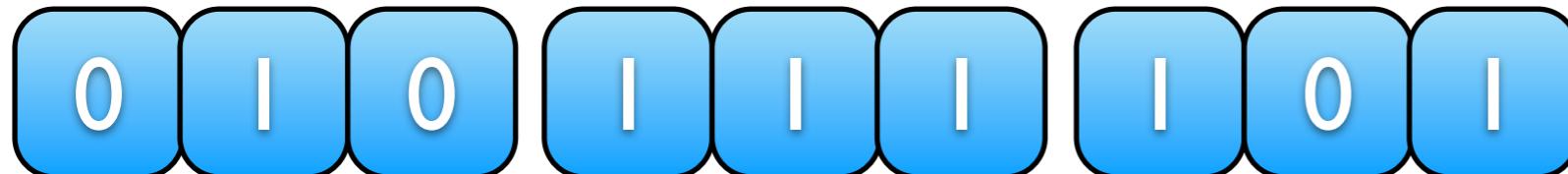


(2,7,5) A: 3, B: 1





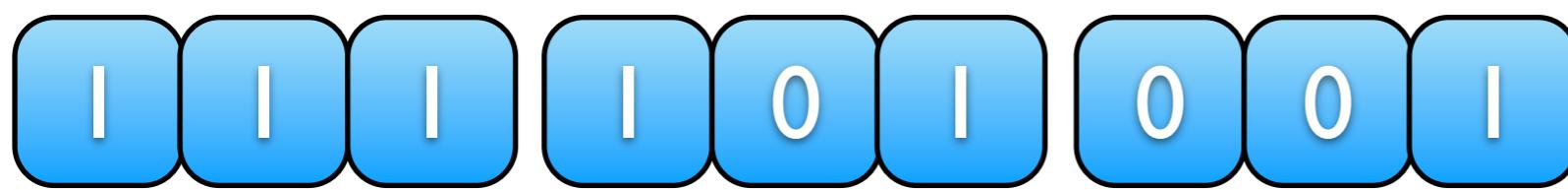
(4,3,5) A: 2, B: 2

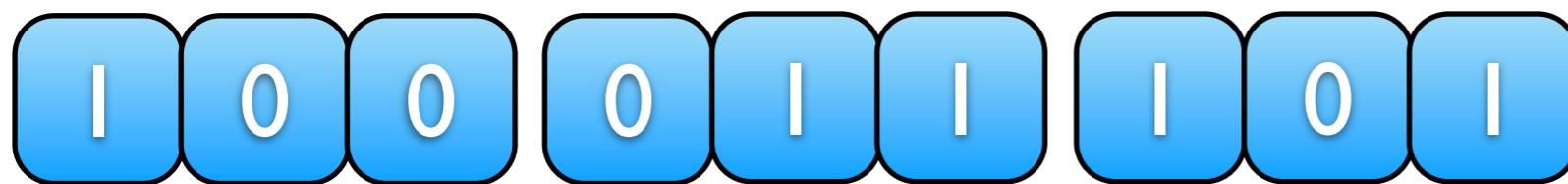


(2,7,5) A: 3, B: 1

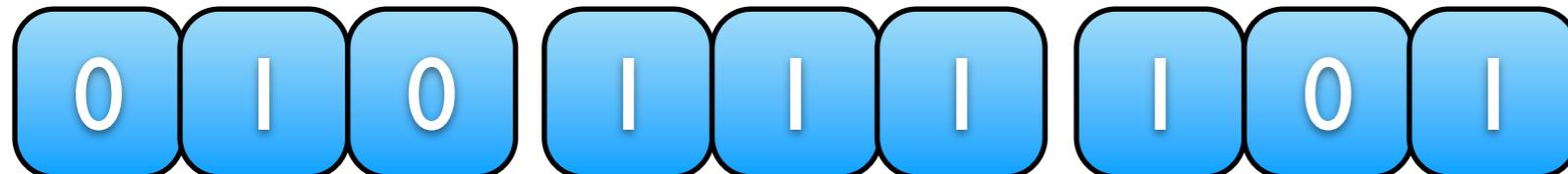


(4,1,5)





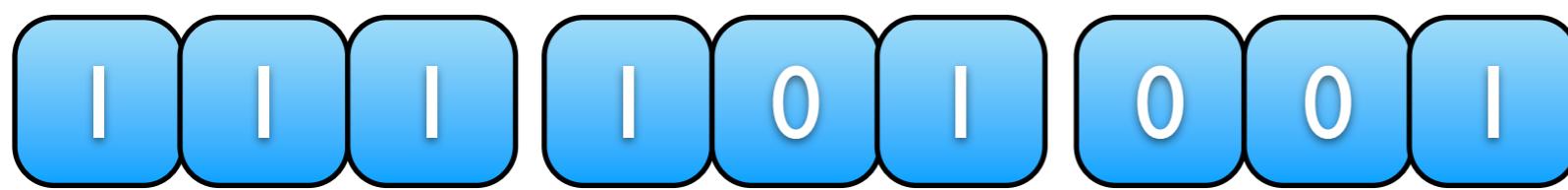
(4,3,5) A: 2, B: 2

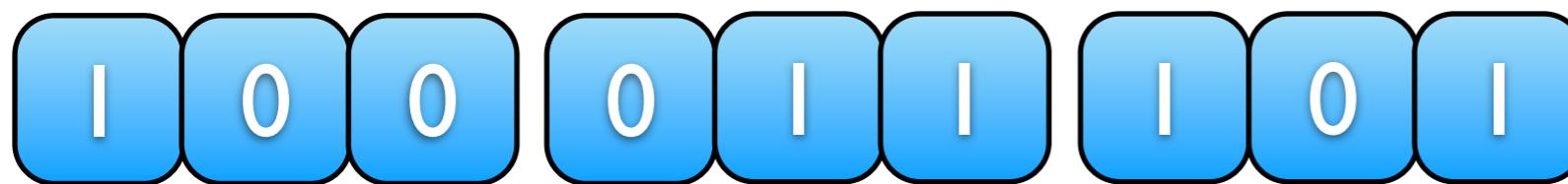


(2,7,5) A: 3, B: 1

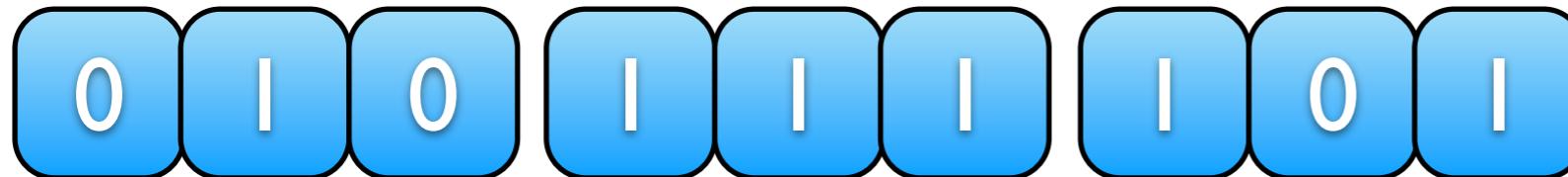


(4,1,5) A: 3, B: 1

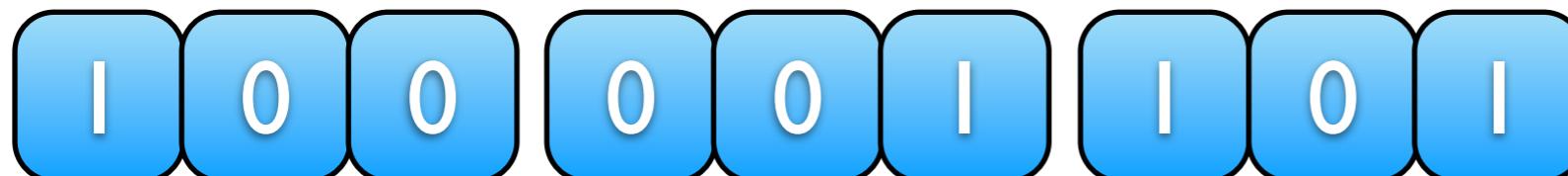




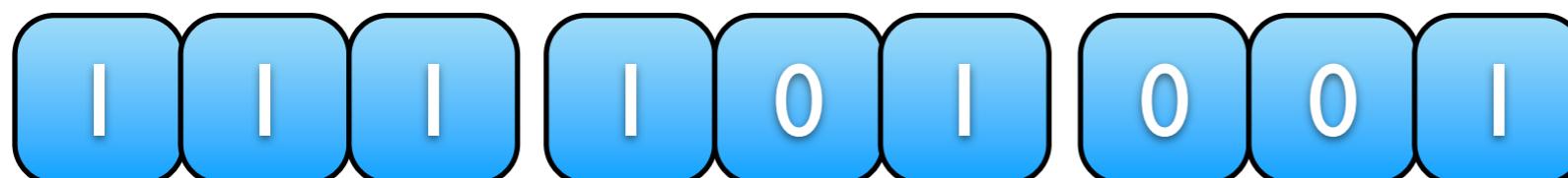
(4,3,5) A: 2, B: 2



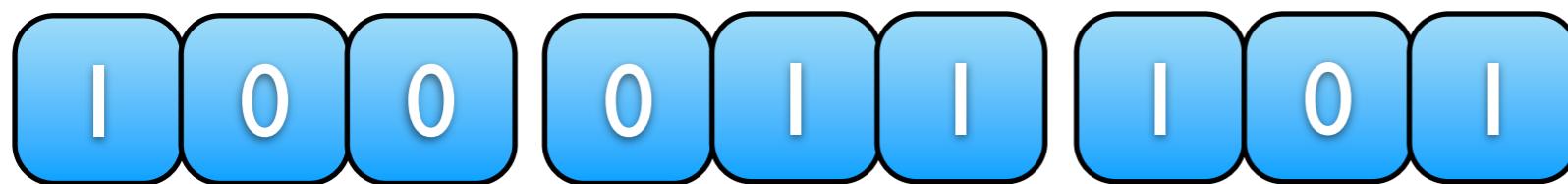
(2,7,5) A: 3, B: 1



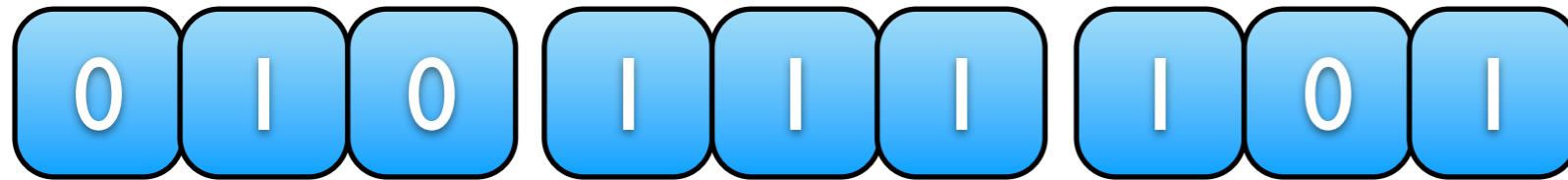
(4,1,5) A: 3, B: 1



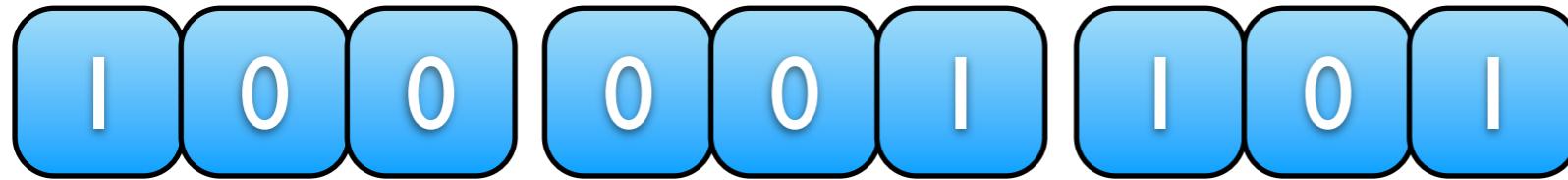
(7,5,1)



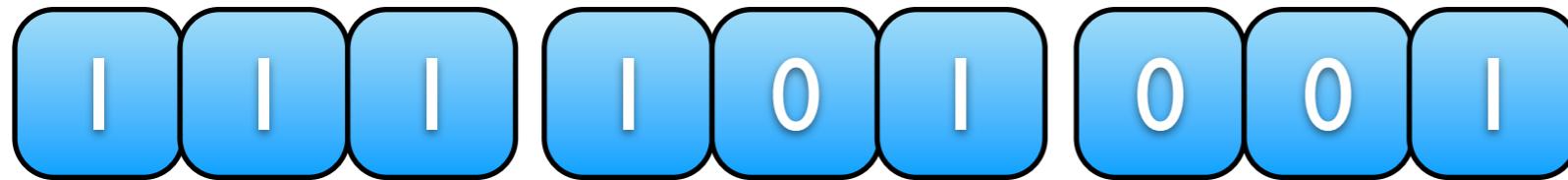
(4,3,5) A: 2, B: 2



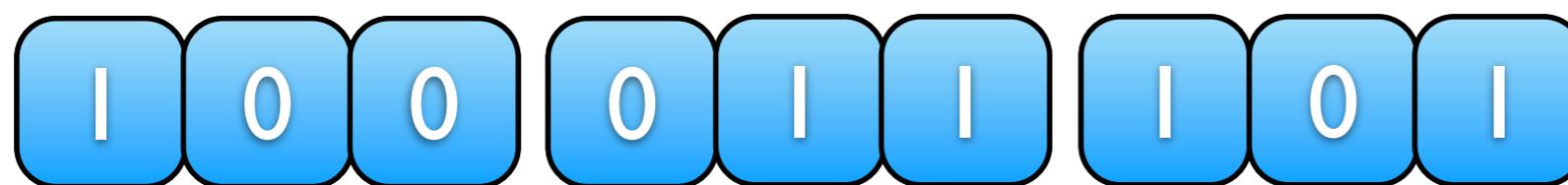
(2,7,5) A: 3, B: 1



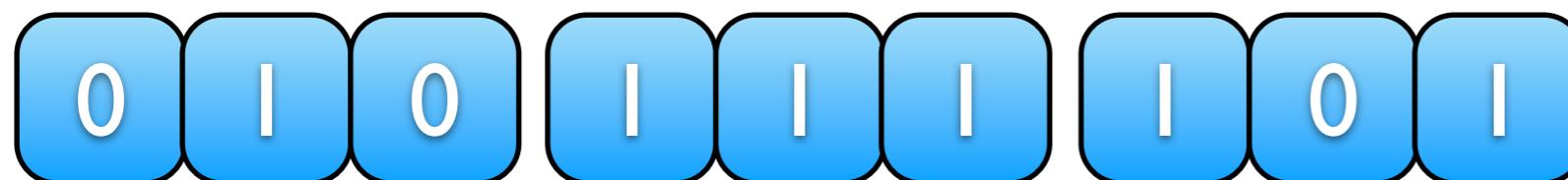
(4,1,5) A: 3, B: 1



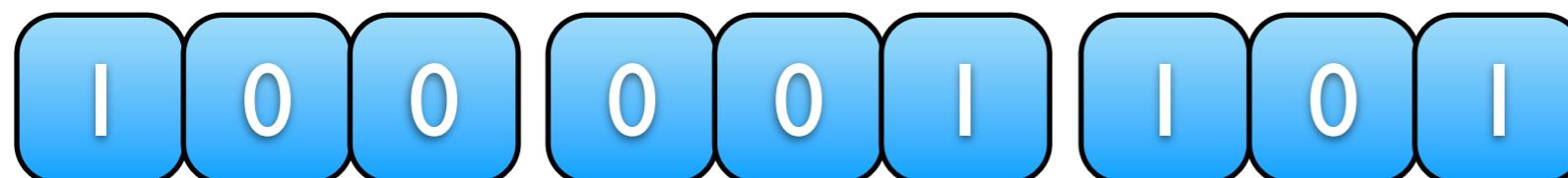
(7,5,1) A: 3, B: 2



(4,3,5) A: 2, B: 2



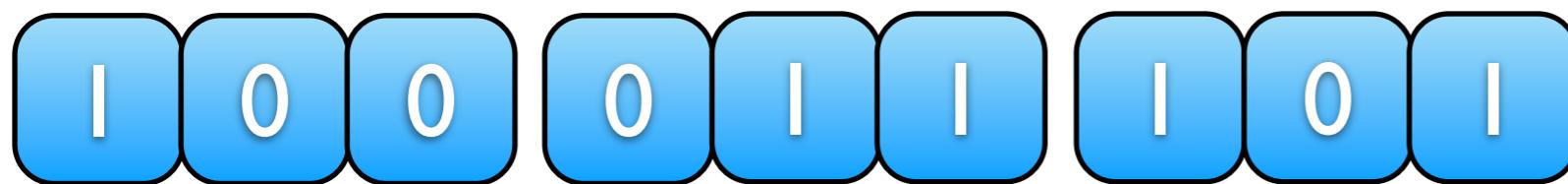
(2,7,5) A: 3, B: 1



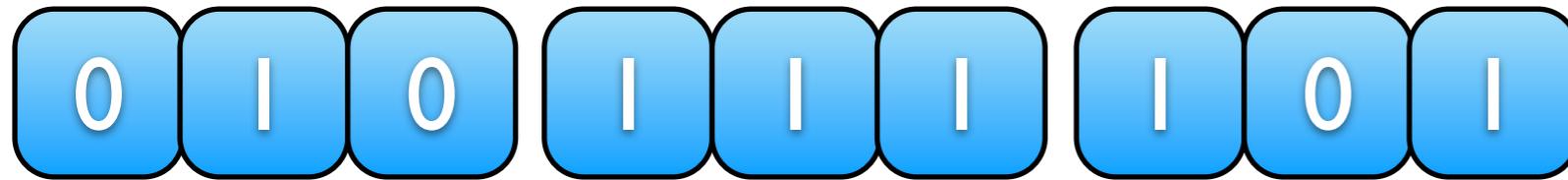
(4,1,5) A: 3, B: 1



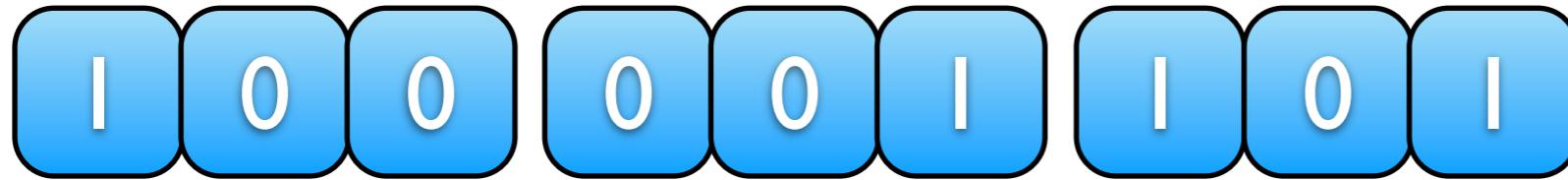
(7,5,1) A: 3, B: 2



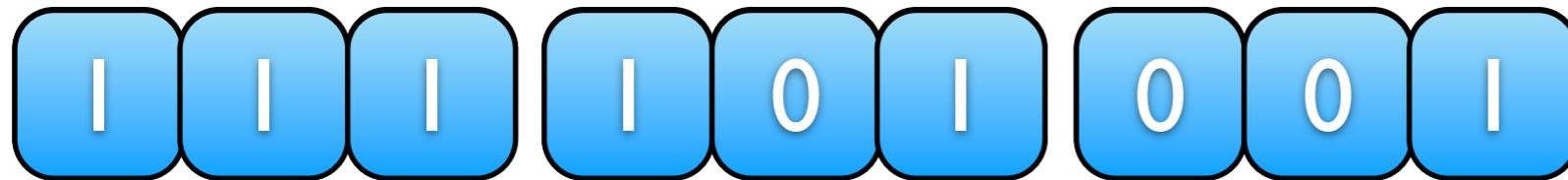
(4,3,5) A: 2, B: 2



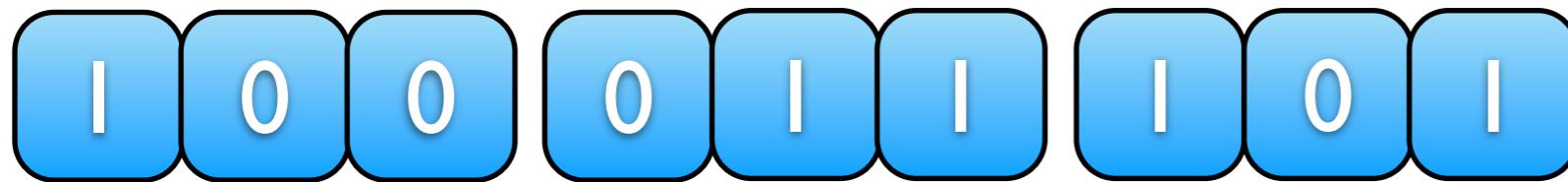
(2,7,5) A: 3, B: 1

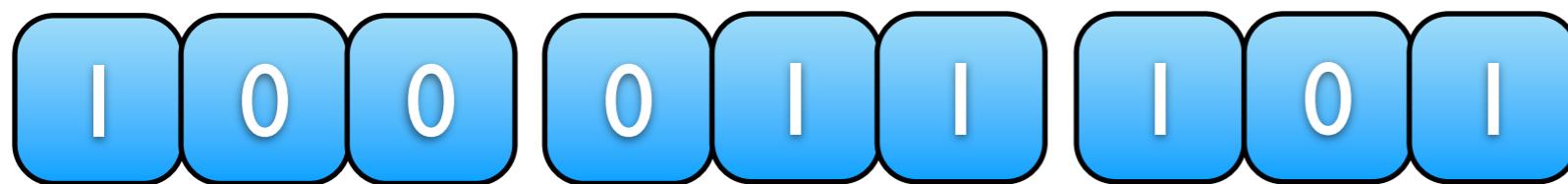


(4,1,5) A: 3, B: 1

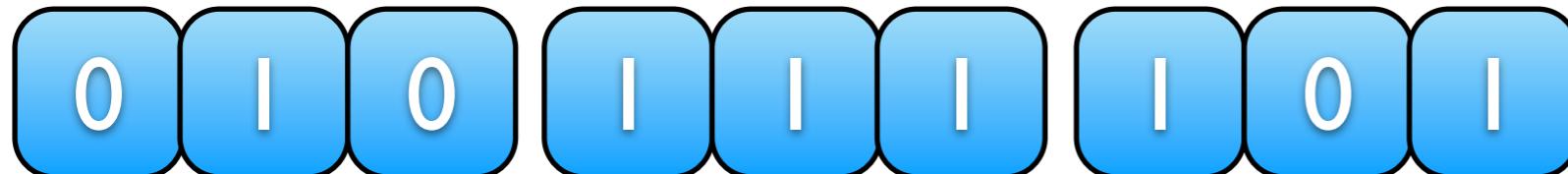


(7,5,1) A: 3, B: 2





(4,3,5) A: 2, B: 2



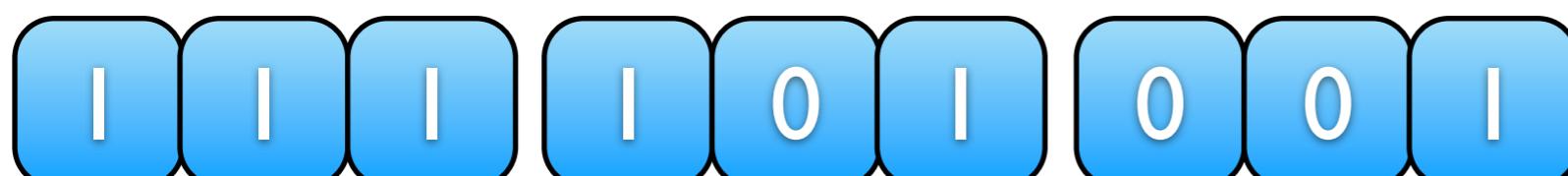
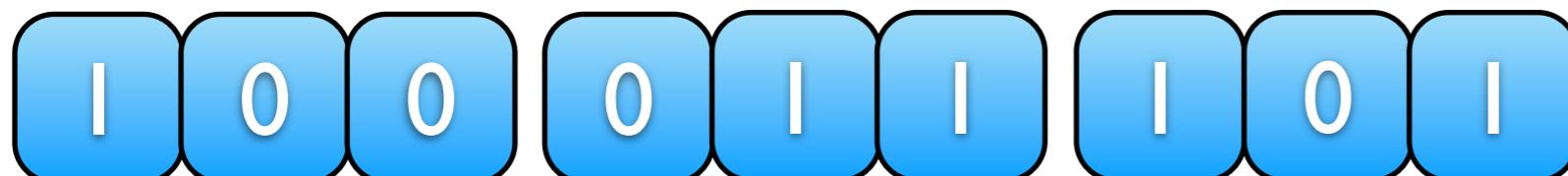
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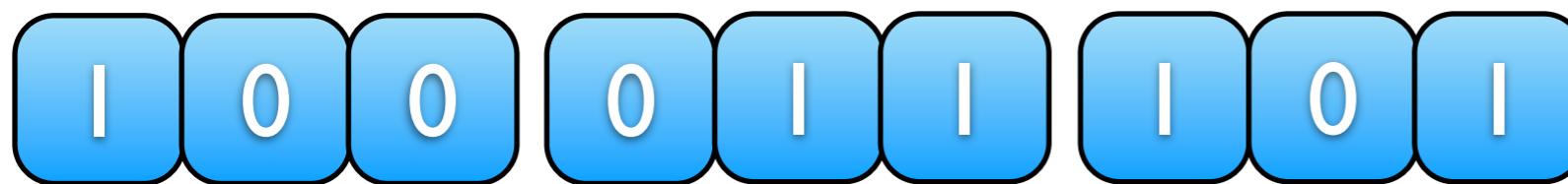


(4,1,5) A: 3, B: 1

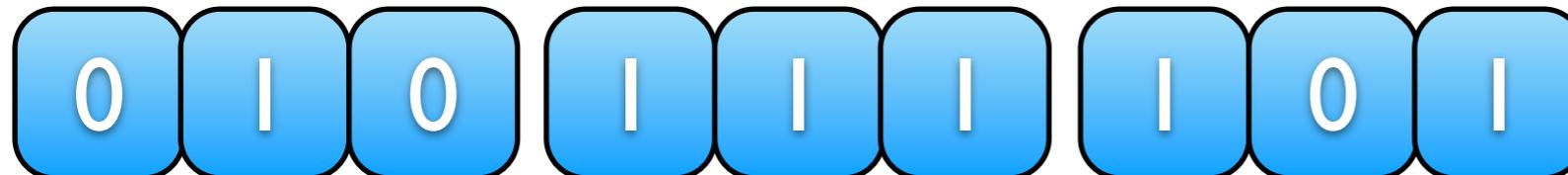


(7,5,1) A: 3, B: 2





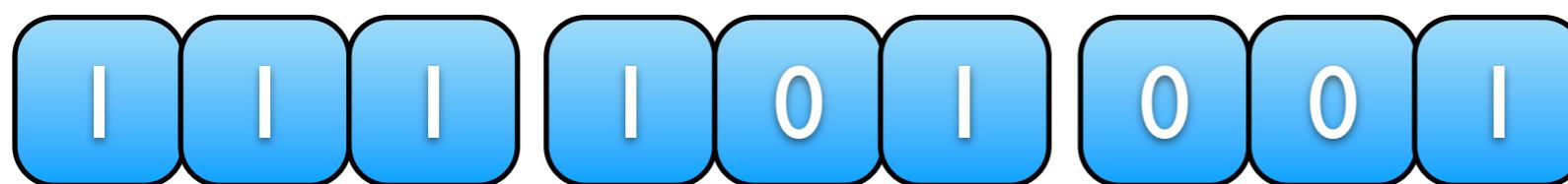
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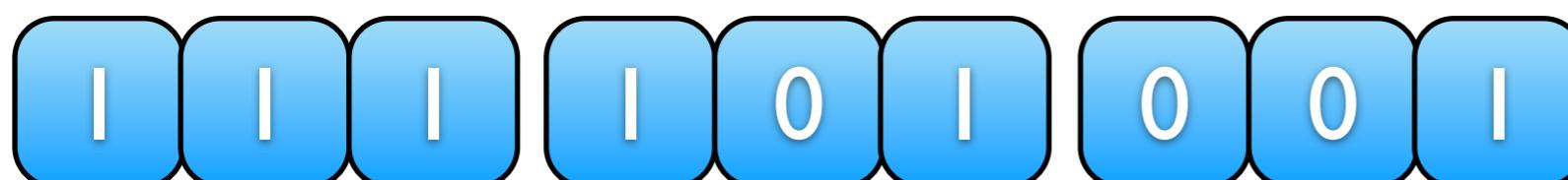
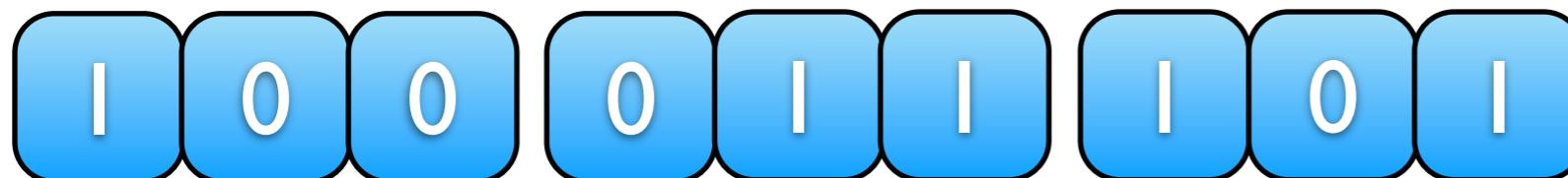
(2,7,5) A: 3, B: 1

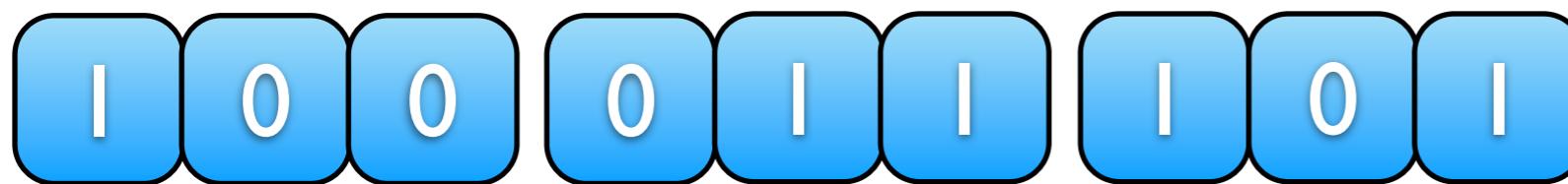


(4,1,5) A: 3, B: 1

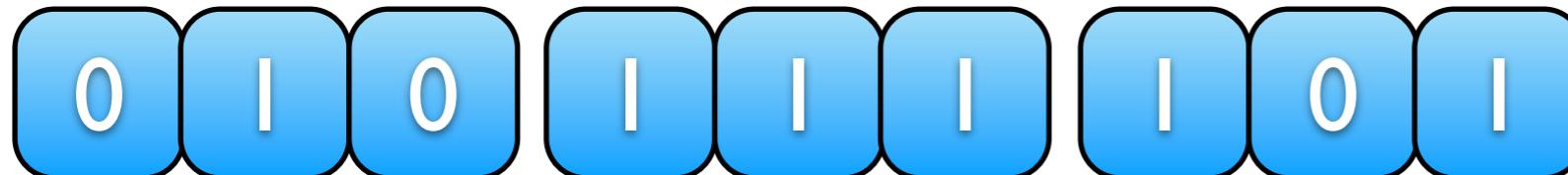


(7,5,1) A: 3, B: 2





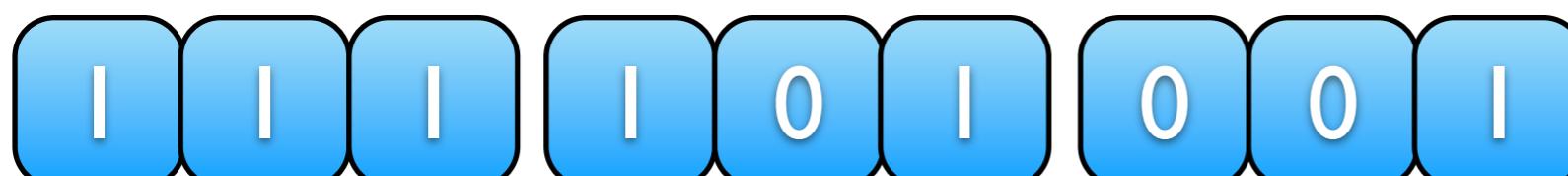
(4,3,5) A: 2, B: 2



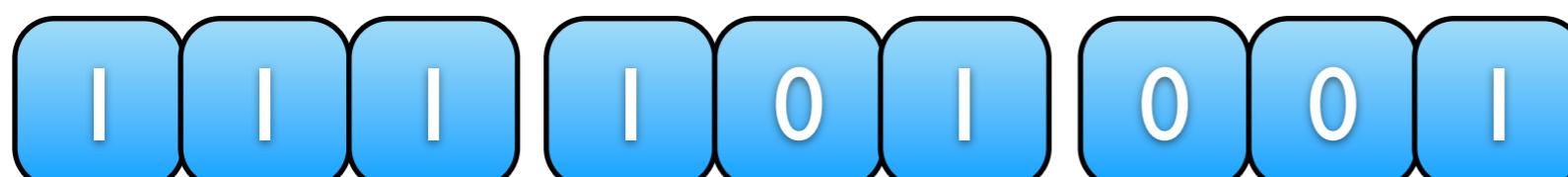
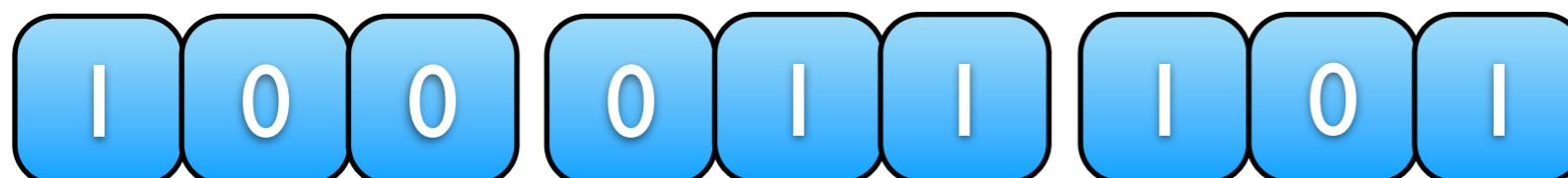
(2,7,5) A: 3, B: 1

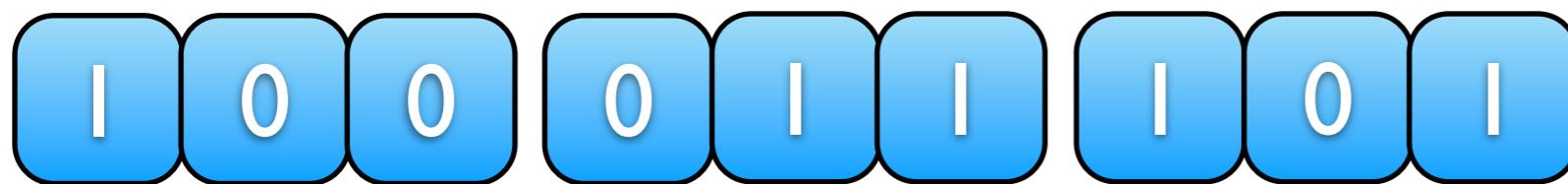


(4,1,5) A: 3, B: 1

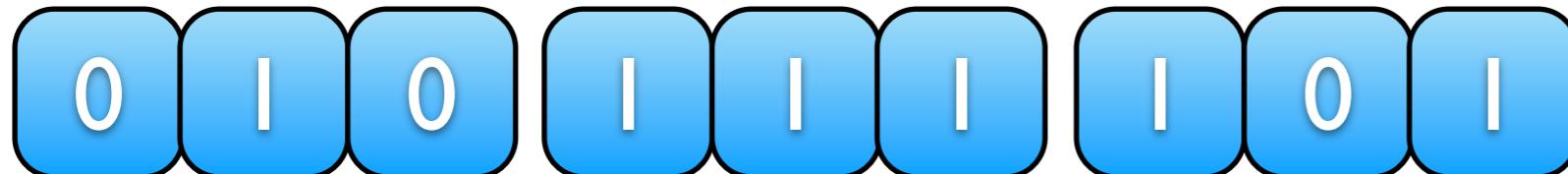


(7,5,1) A: 3, B: 2





(4,3,5) A: 2, B: 2



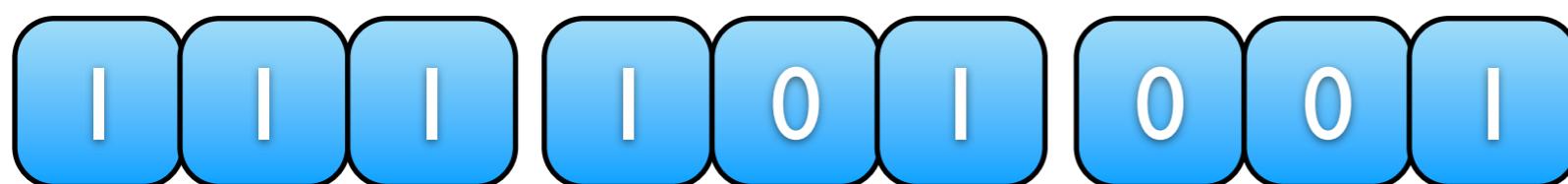
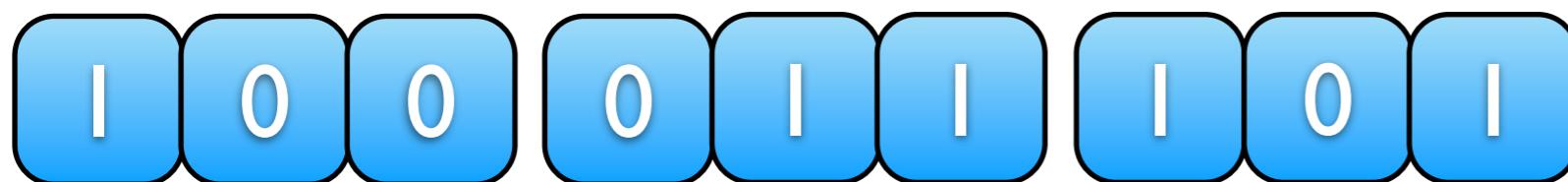
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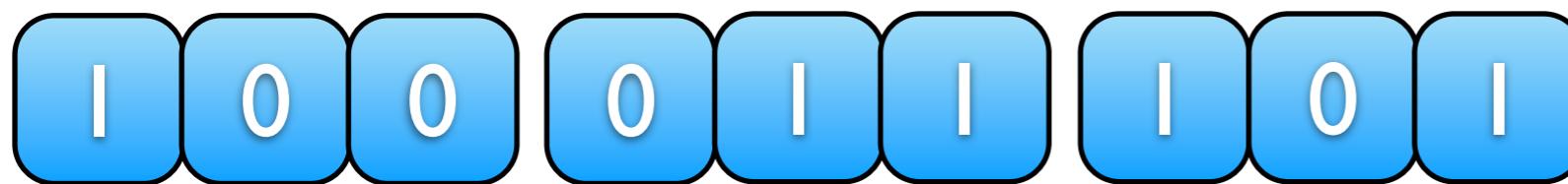


(4,1,5) A: 3, B: 1

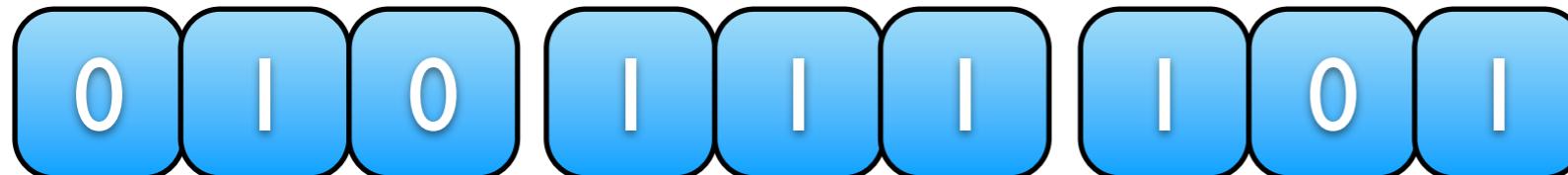


(7,5,1) A: 3, B: 2

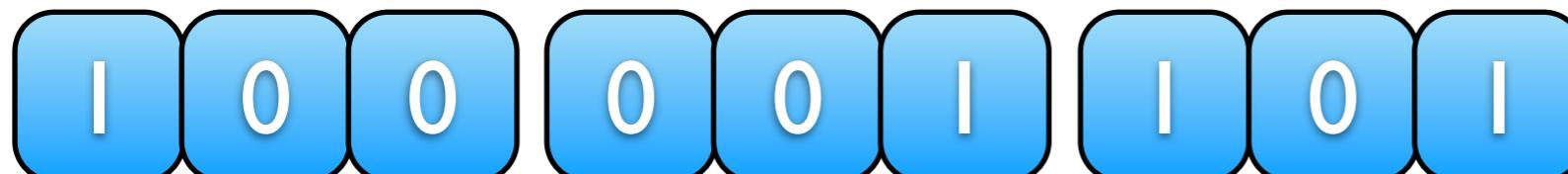




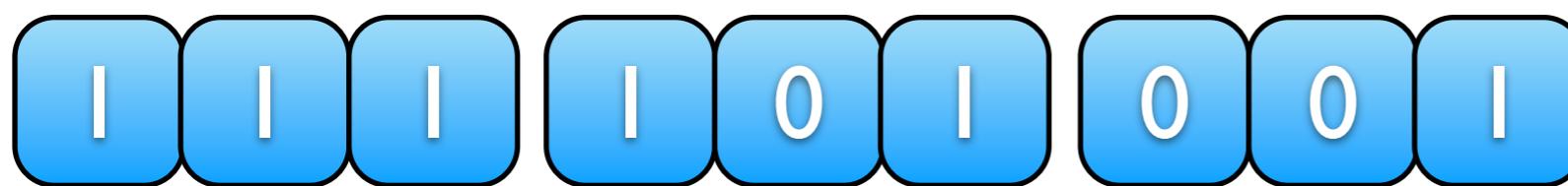
(4,3,5) A: 2, B: 2



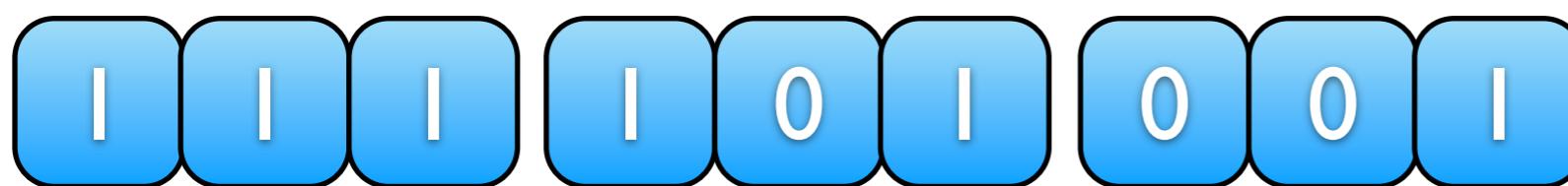
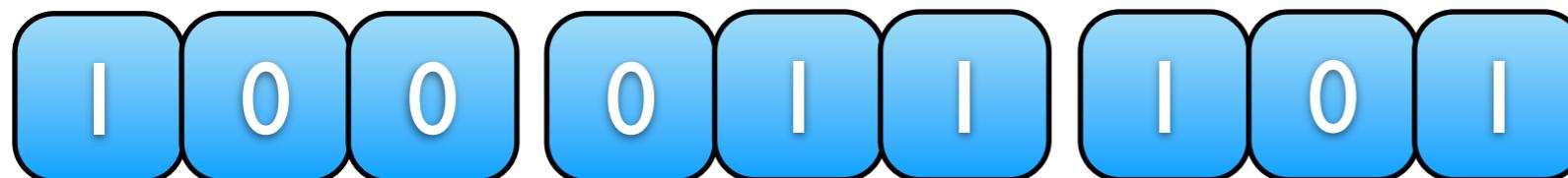
(2,7,5) A: 3, B: 1

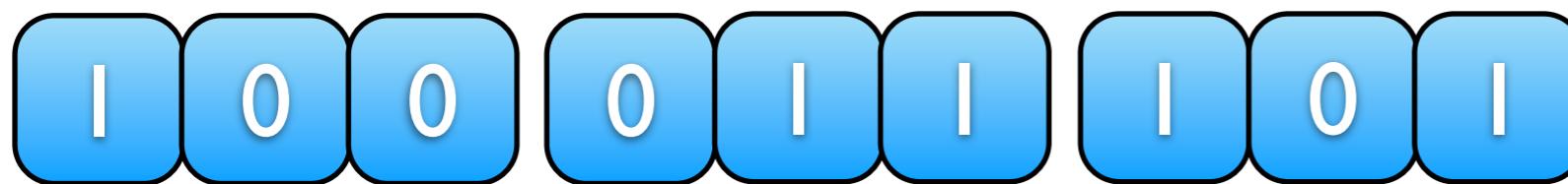


(4,1,5) A: 3, B: 1

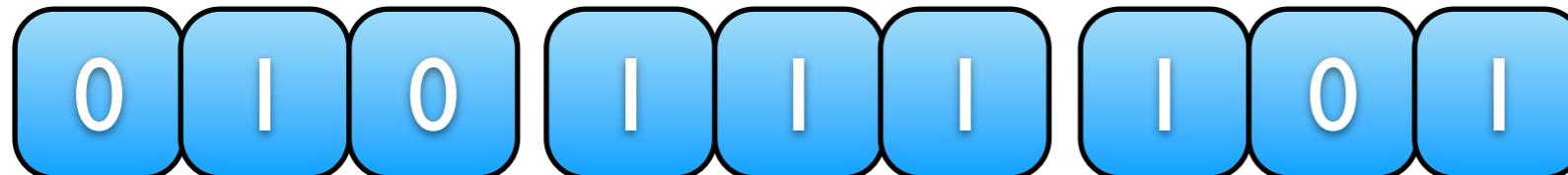


(7,5,1) A: 3, B: 2





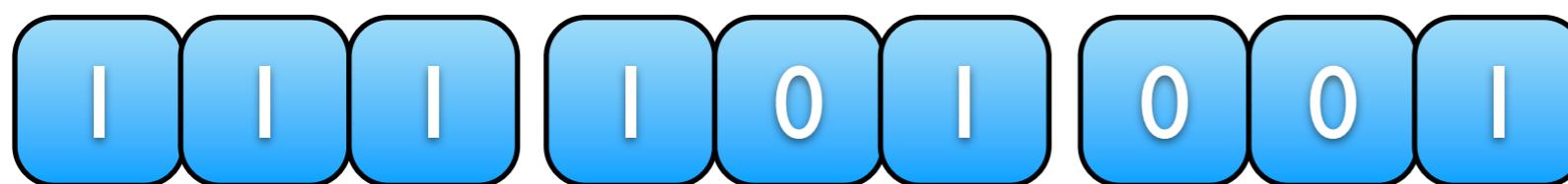
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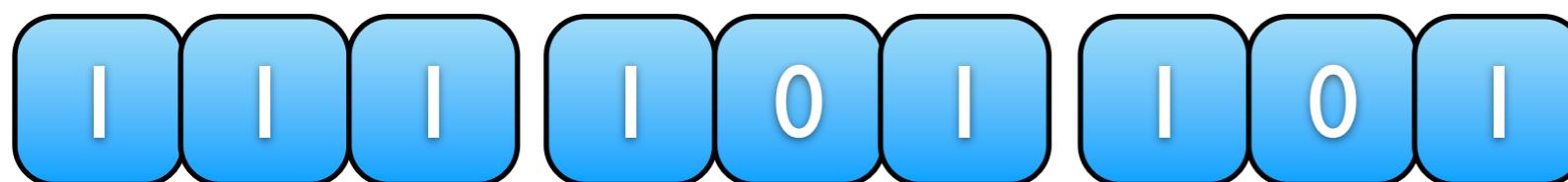
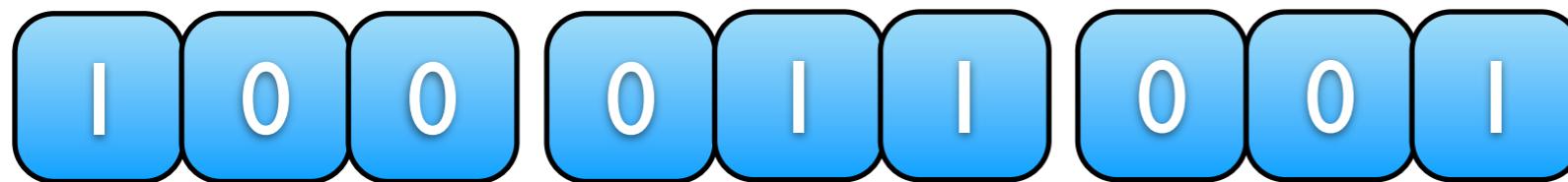
(2,7,5) A: 3, B: 1

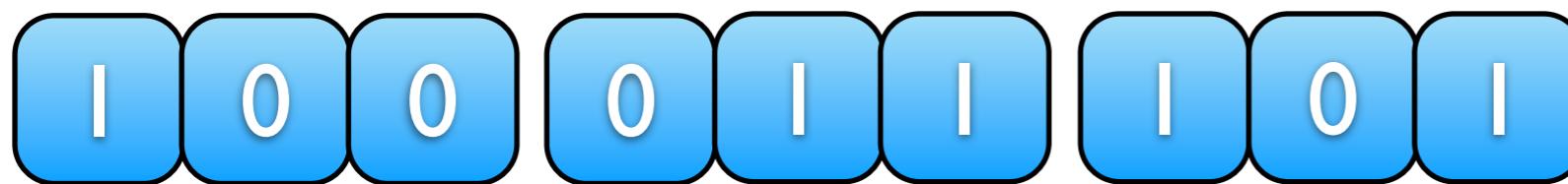


(4,1,5) A: 3, B: 1

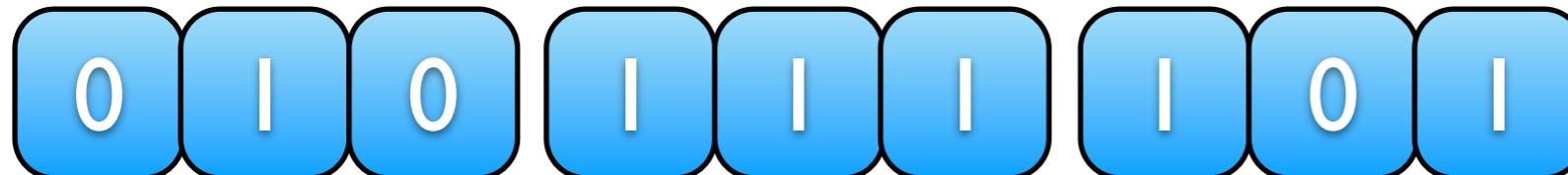


(7,5,1) A: 3, B: 2





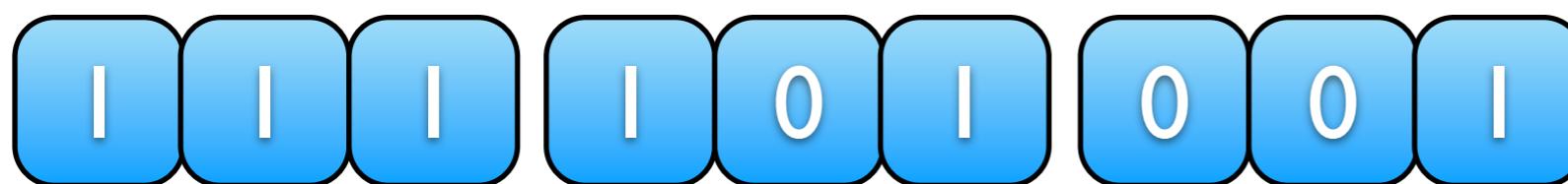
(4,3,5) A: 2, B: 2



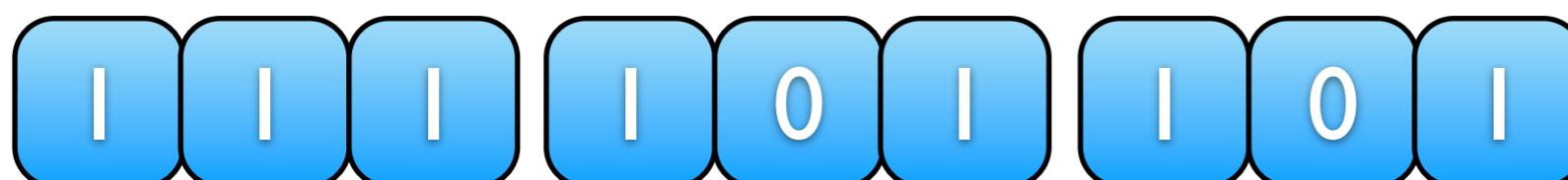
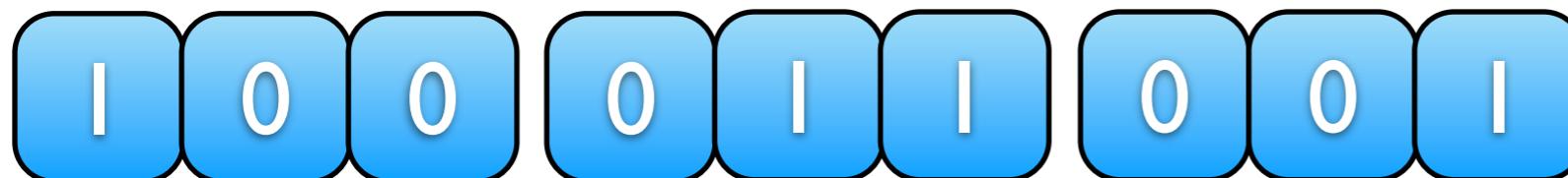
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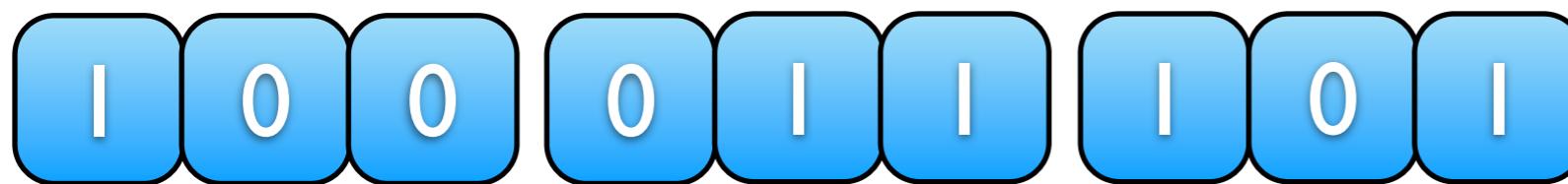


(4,1,5) A: 3, B: 1

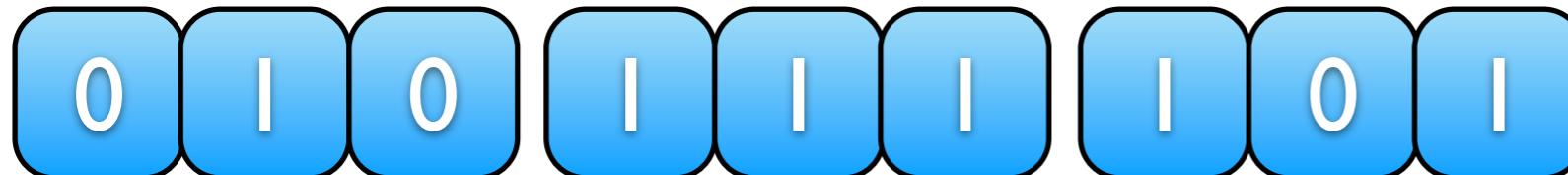


(7,5,1) A: 3, B: 2





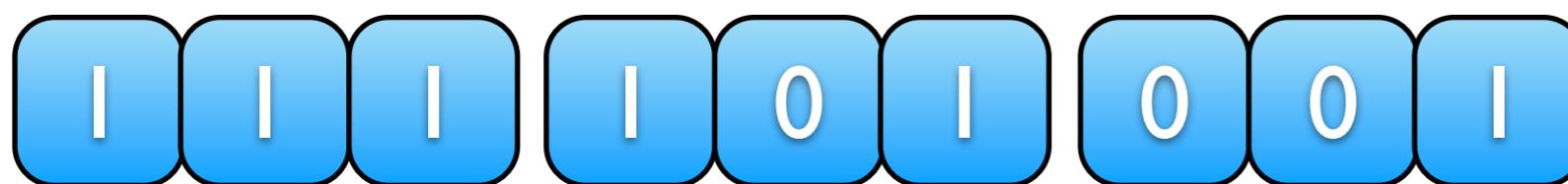
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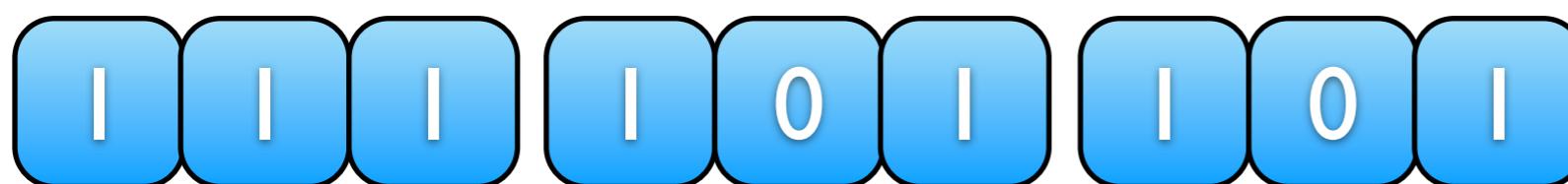
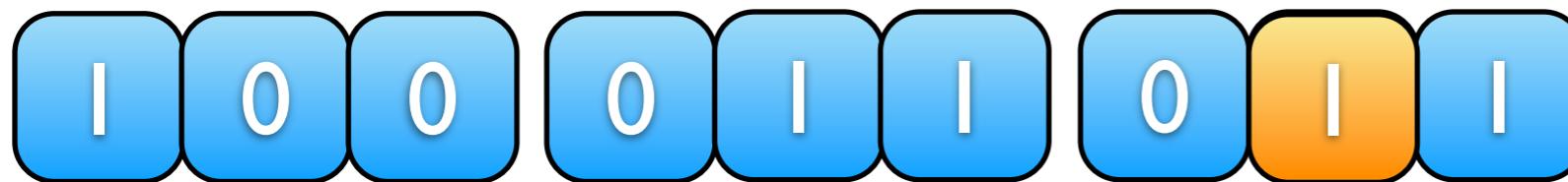
(2,7,5) A: 3, B: 1

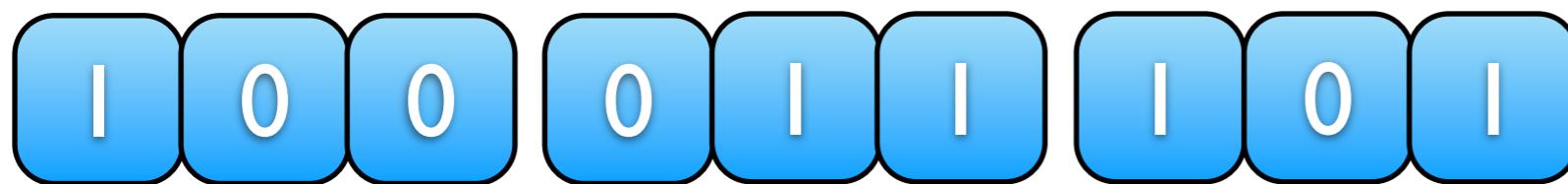


(4,1,5) A: 3, B: 1

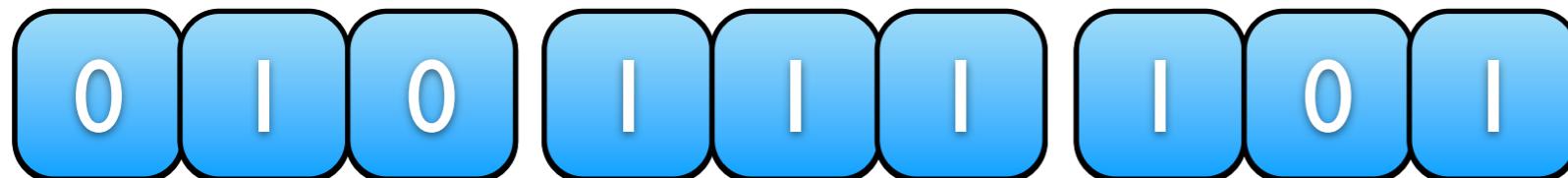


(7,5,1) A: 3, B: 2





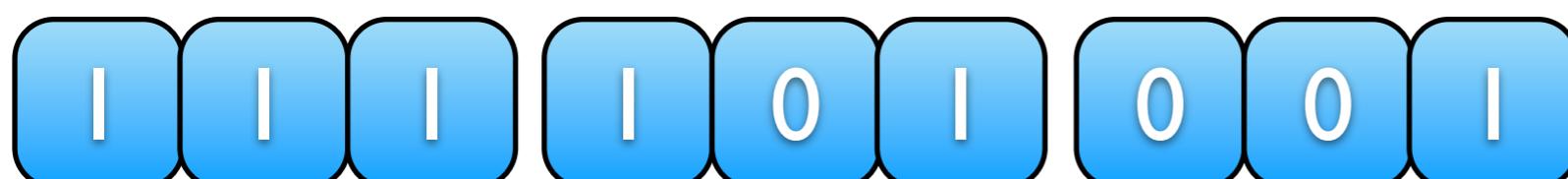
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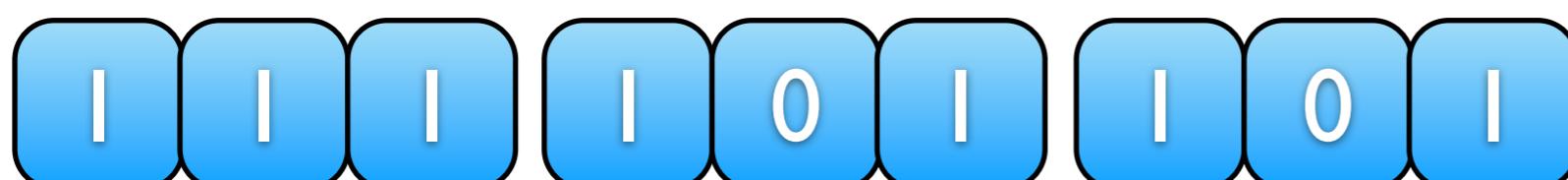
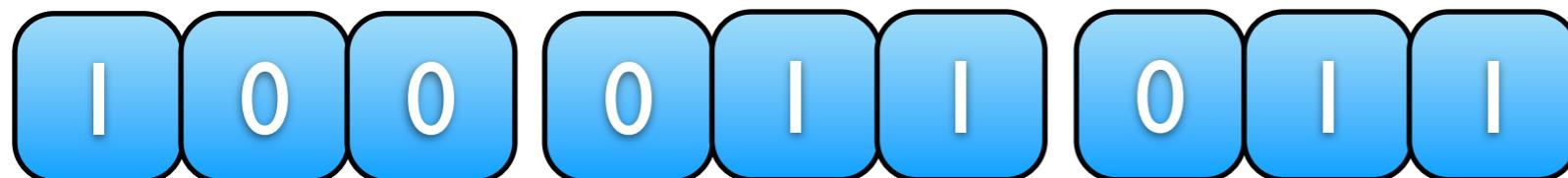
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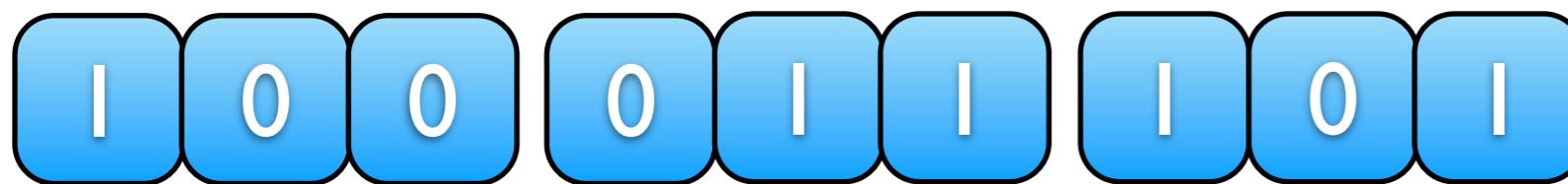


(4,1,5) A: 3, B: 1

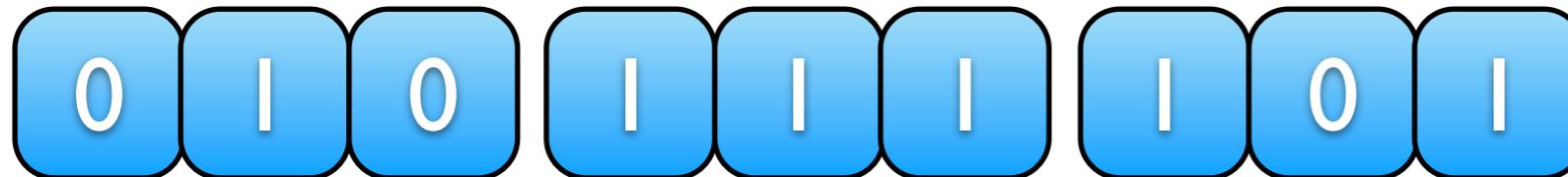


(7,5,1) A: 3, B: 2

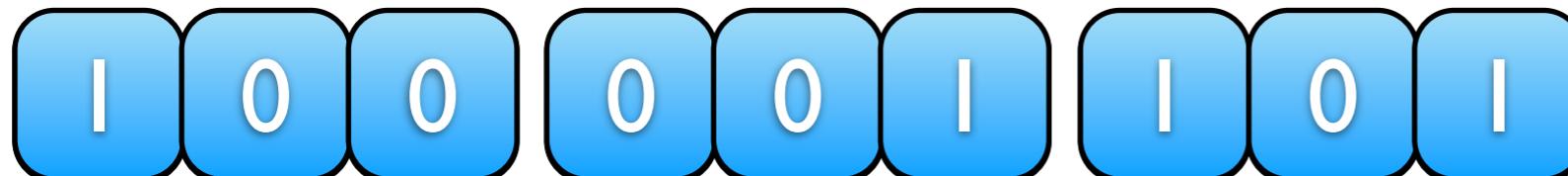




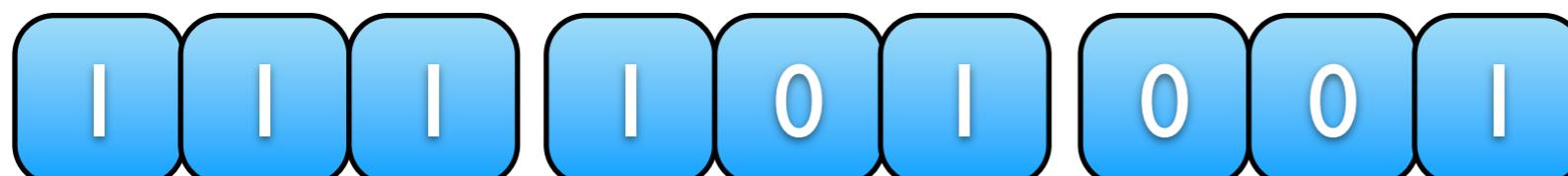
(4,3,5) A: 2, B: 2



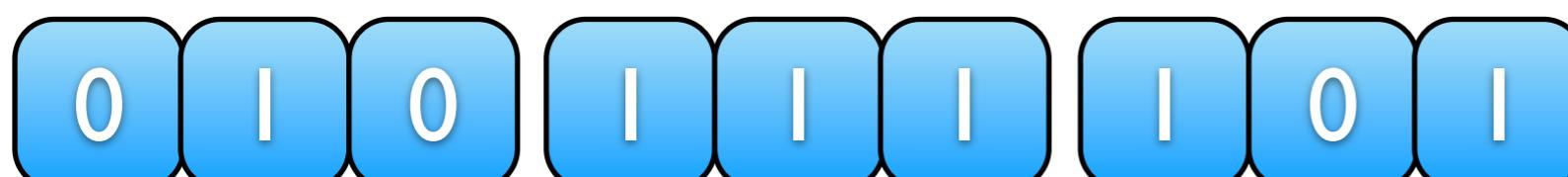
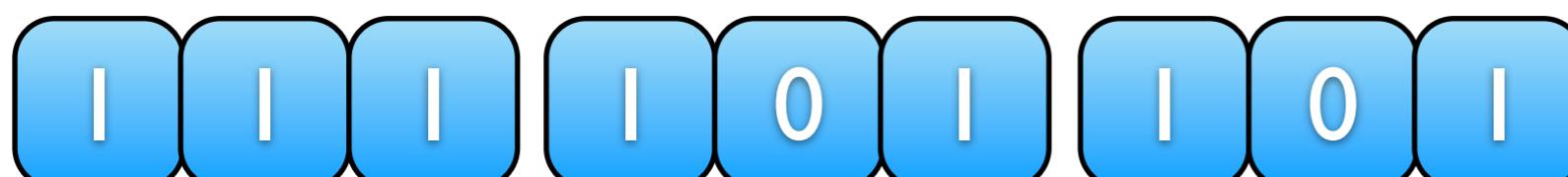
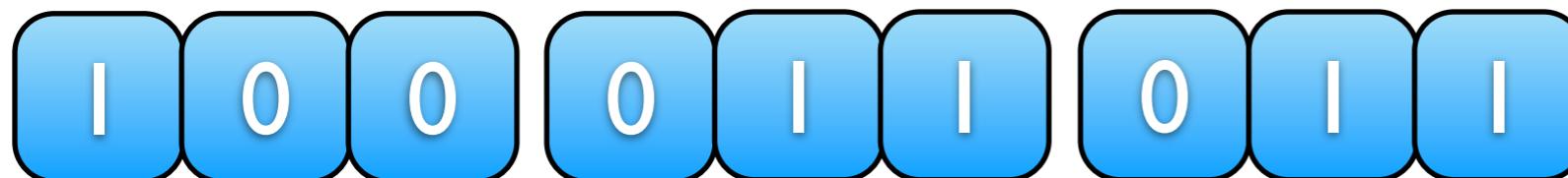
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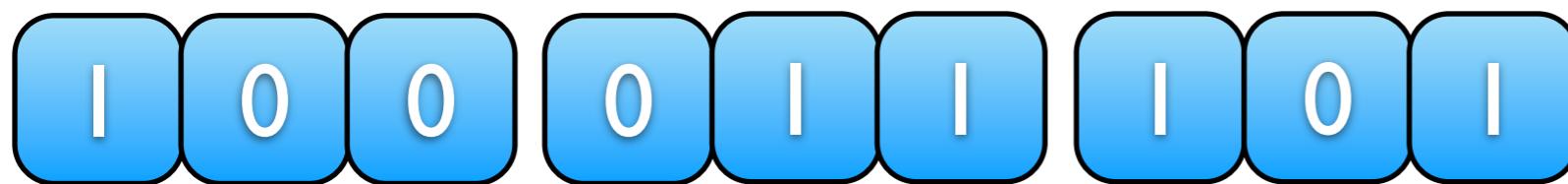


(4,1,5) A: 3, B: 1

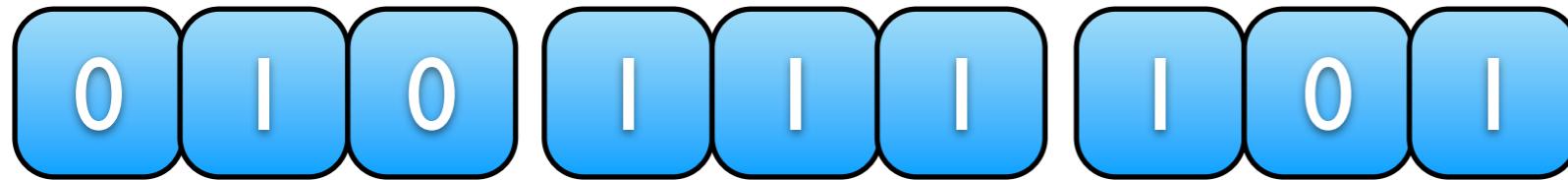


(7,5,1) A: 3, B: 2

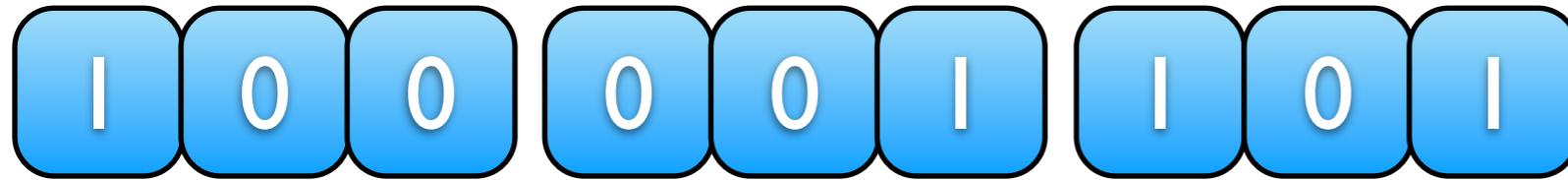




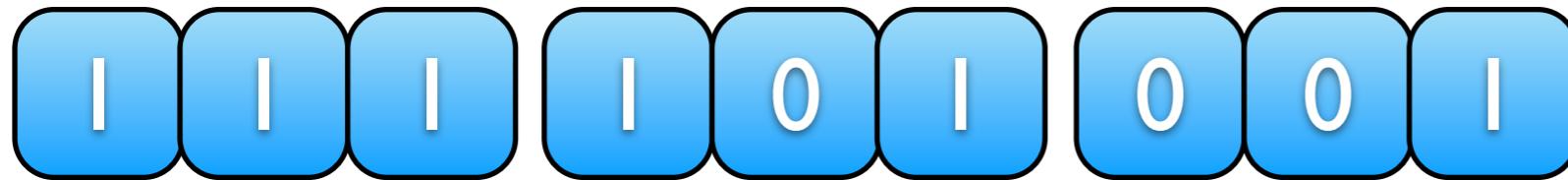
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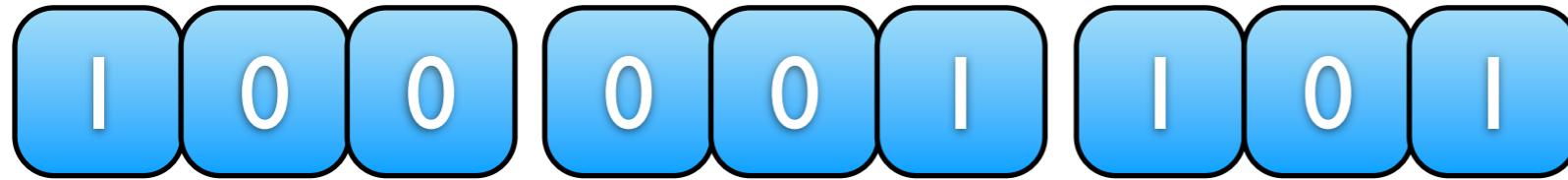
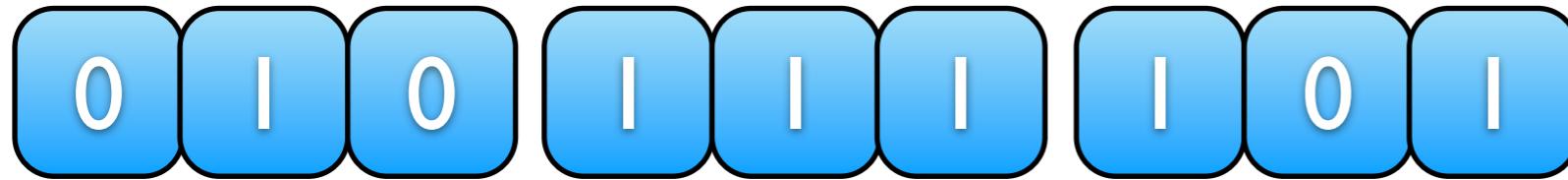
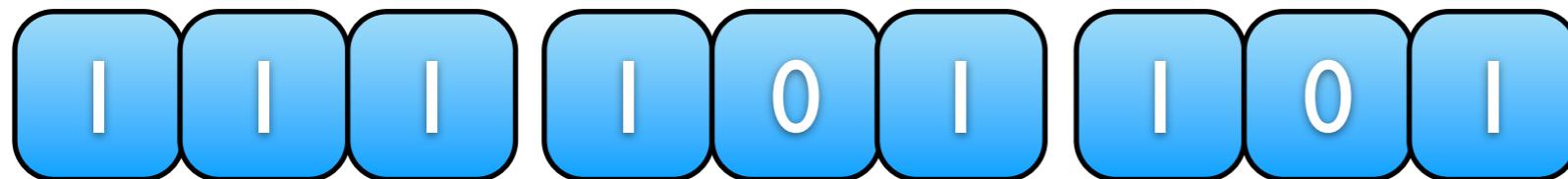
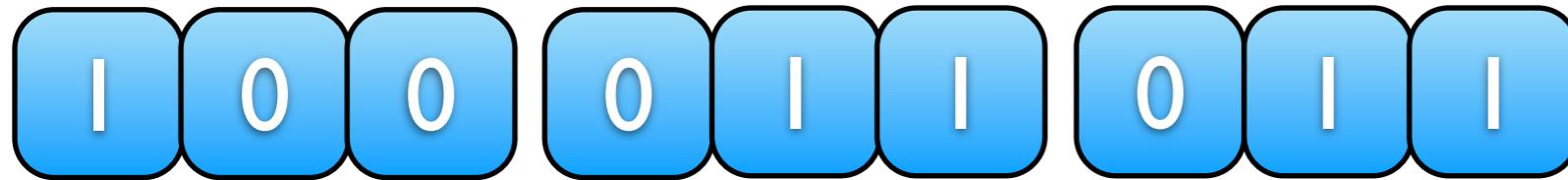
(2,7,5) A: 3, B: 1

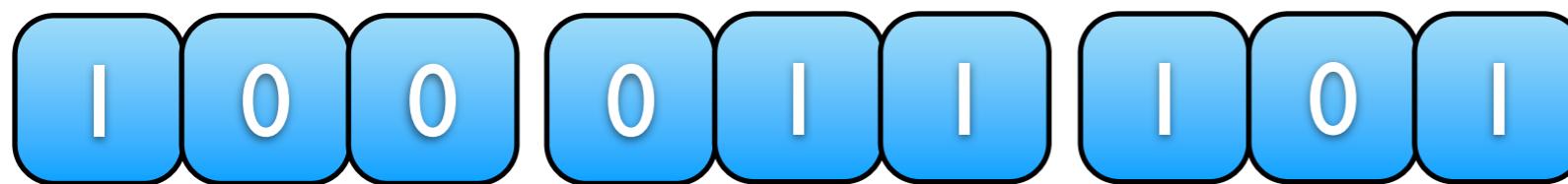


(4,1,5) A: 3, B: 1

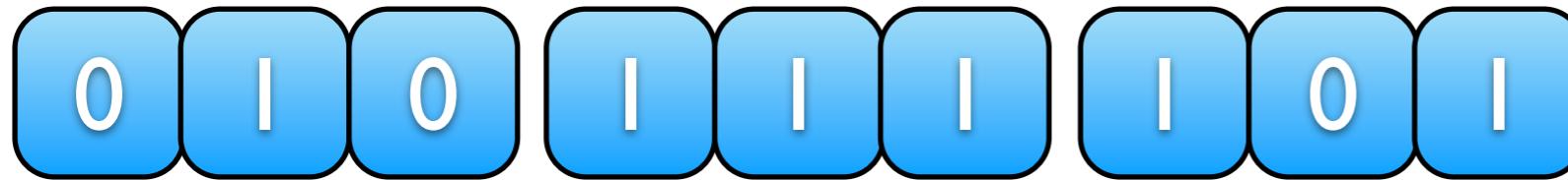


(7,5,1) A: 3, B: 2

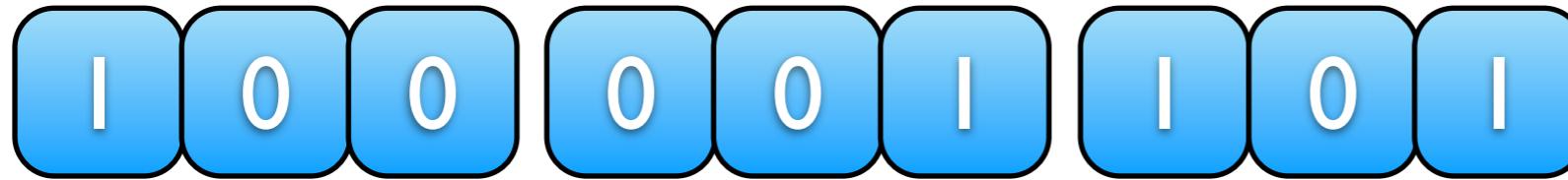




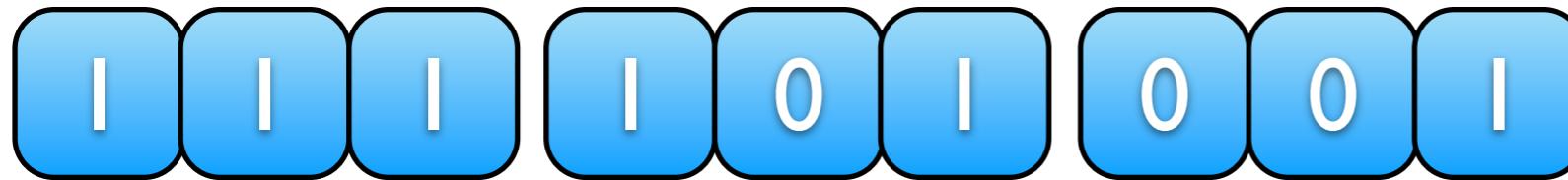
(4,3,5) A: 2, B: 2



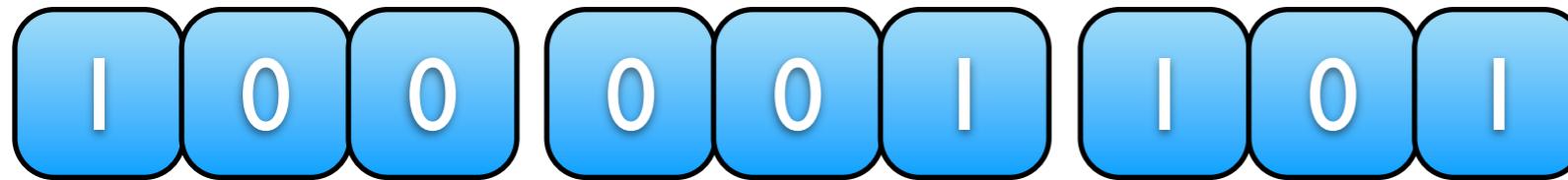
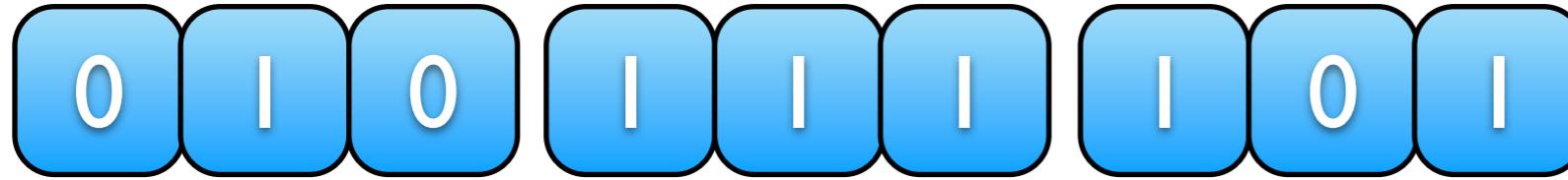
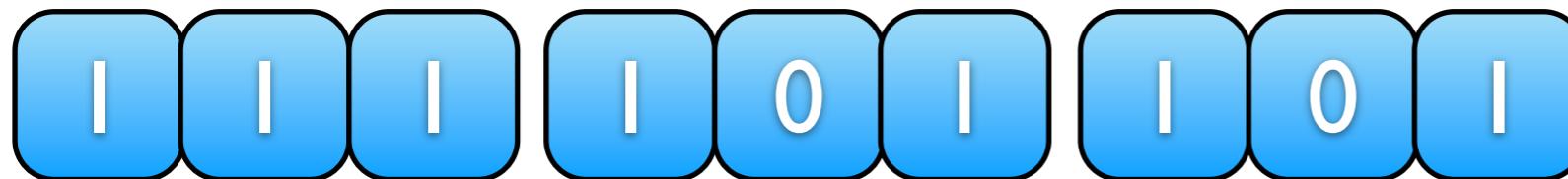
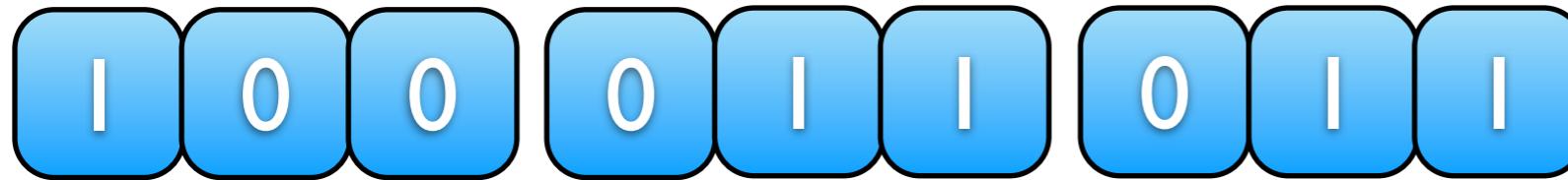
(2,7,5) A: 3, B: 1

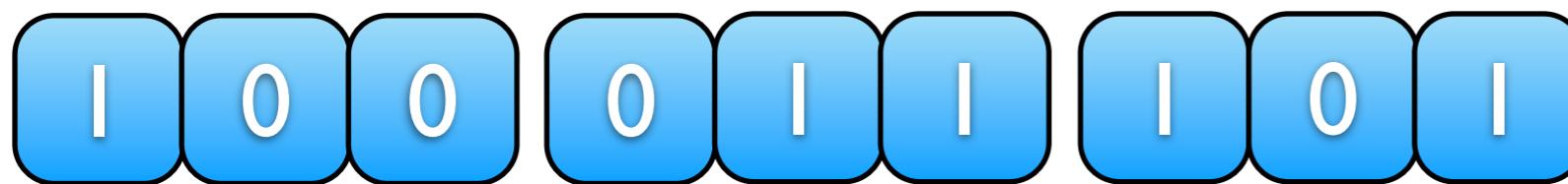


(4,1,5) A: 3, B: 1

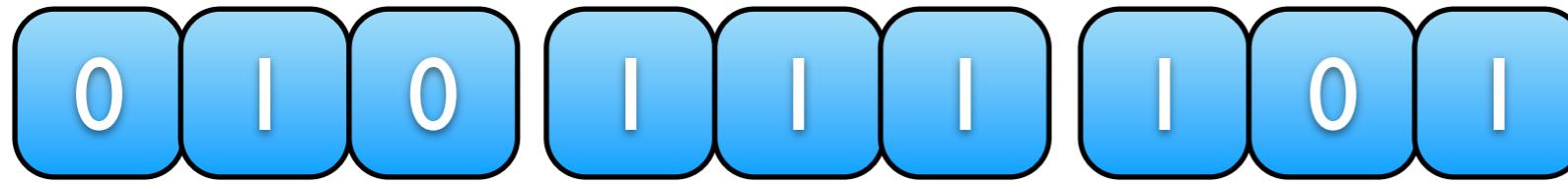


(7,5,1) A: 3, B: 2

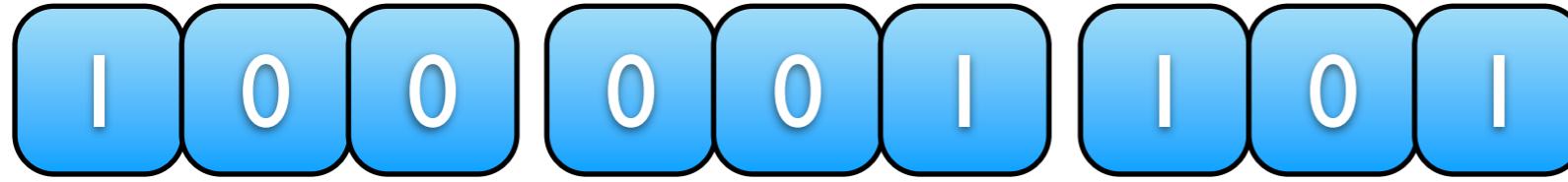




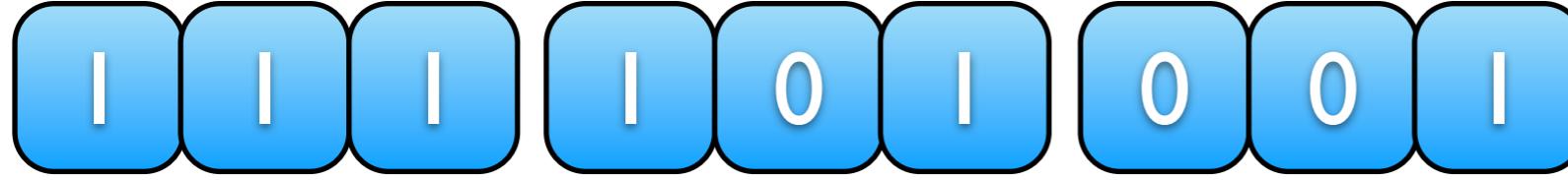
(4,3,5) A: 2, B: 2



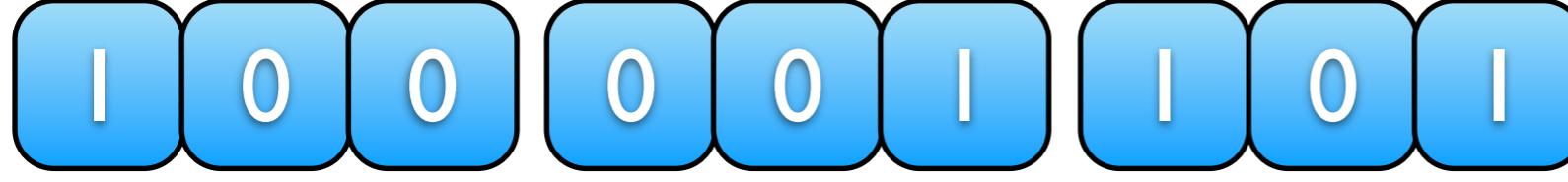
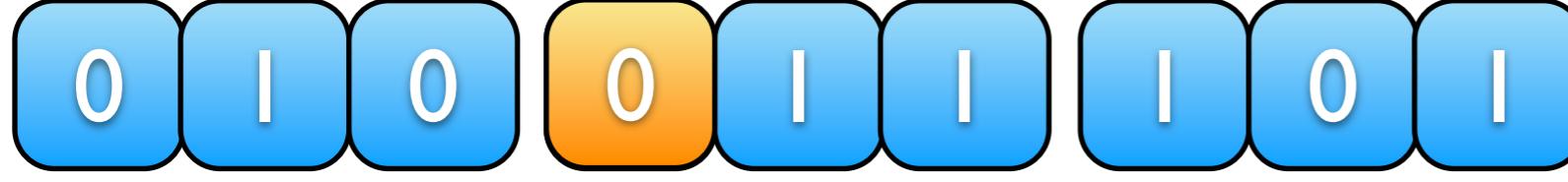
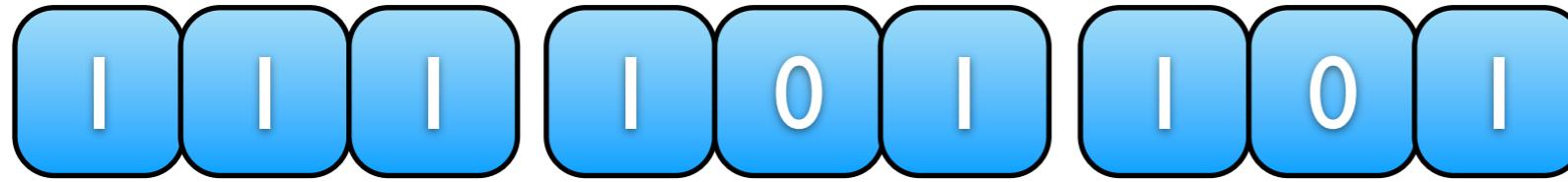
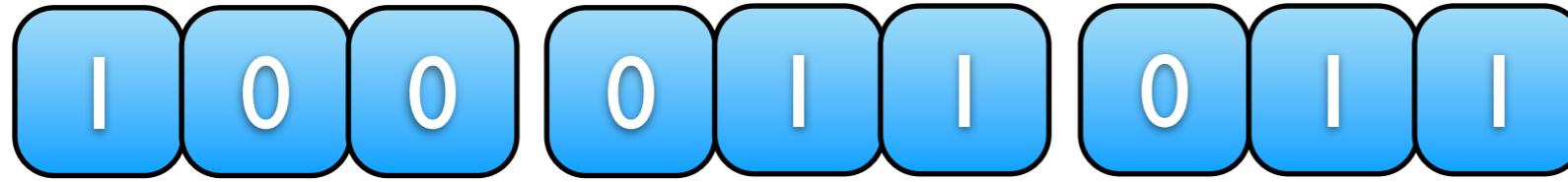
(2,7,5) A: 3, B: 1

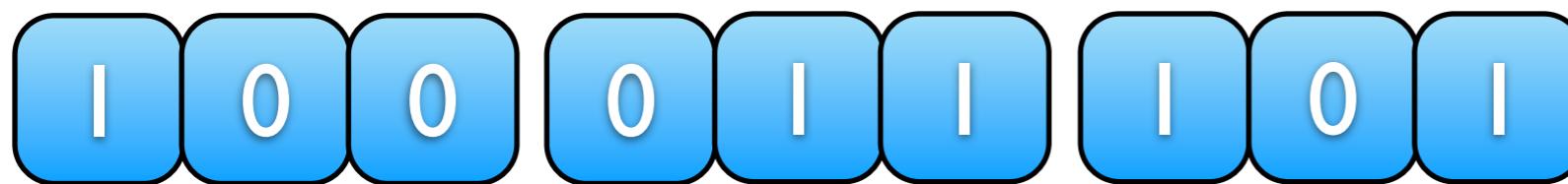


(4,1,5) A: 3, B: 1

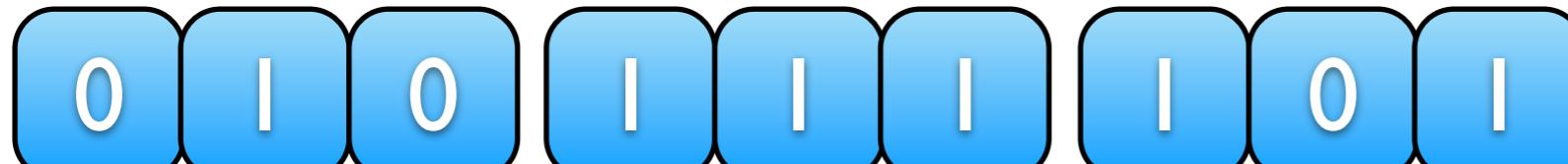


(7,5,1) A: 3, B: 2

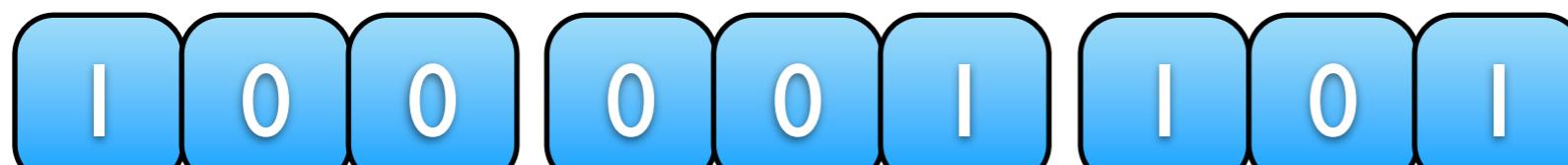




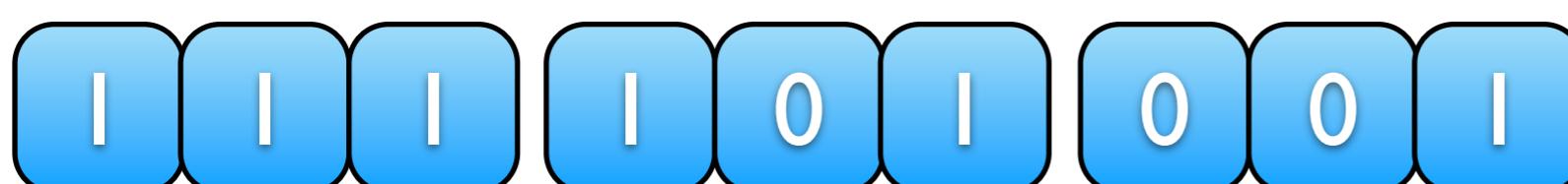
(4,3,5) A: 2, B: 2



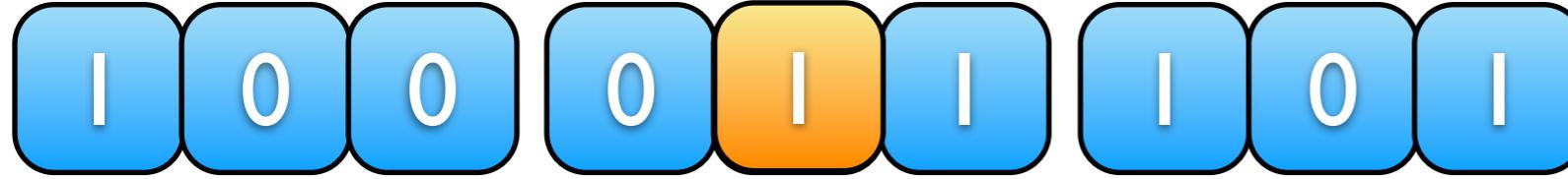
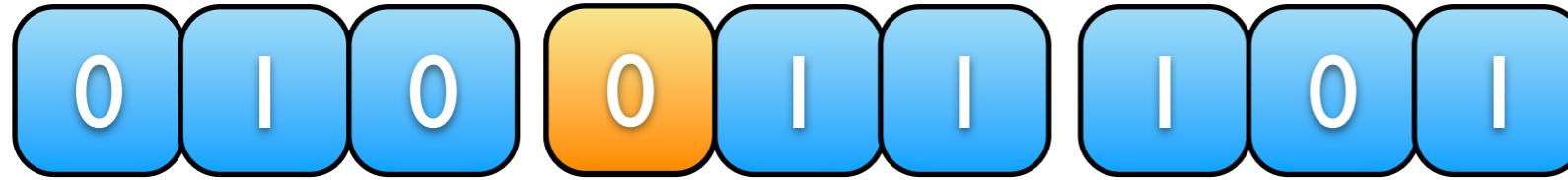
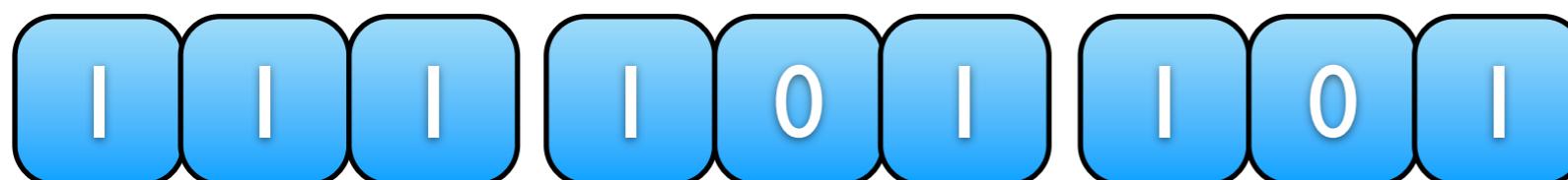
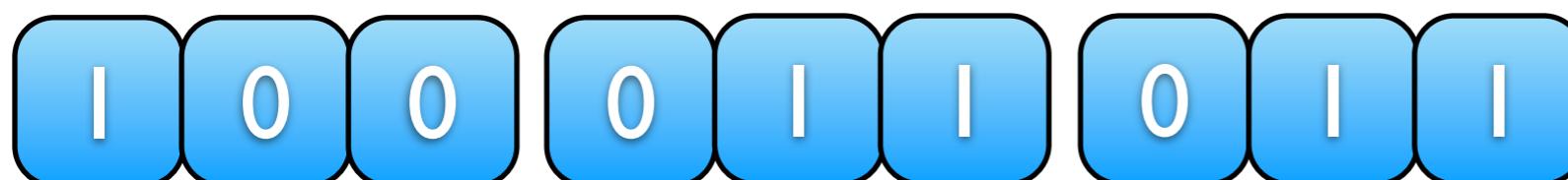
(2,7,5) A: 3, B: 1

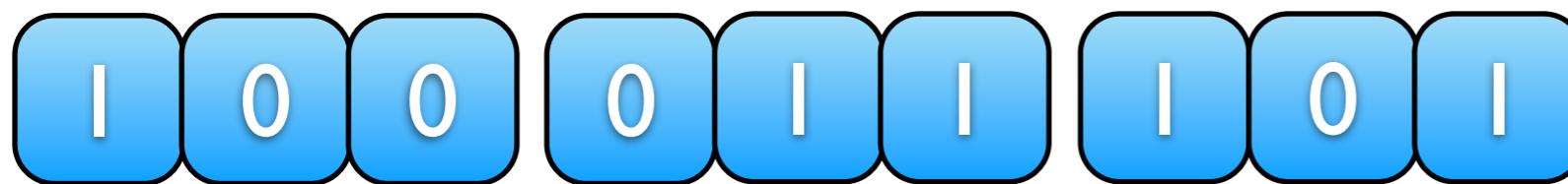


(4,1,5) A: 3, B: 1

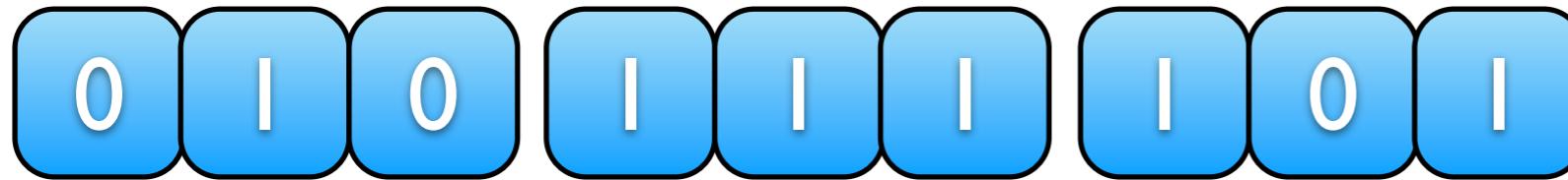


(7,5,1) A: 3, B: 2

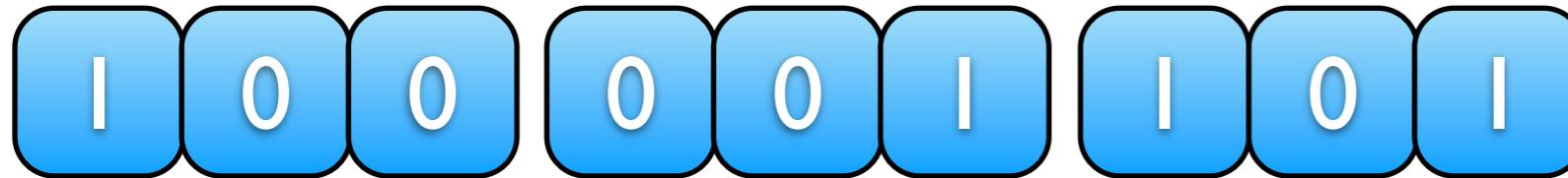




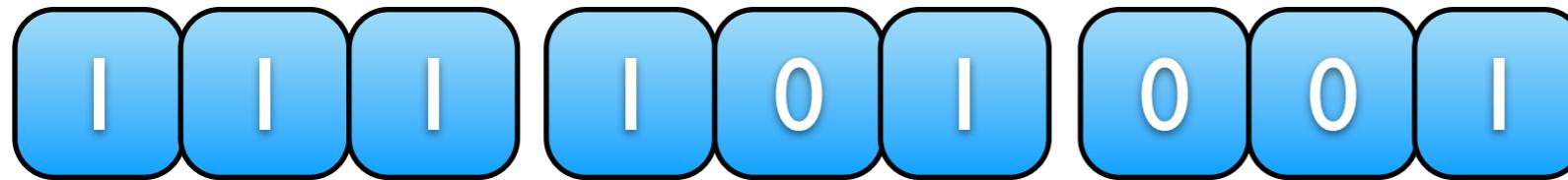
(4,3,5) A: 2, B: 2



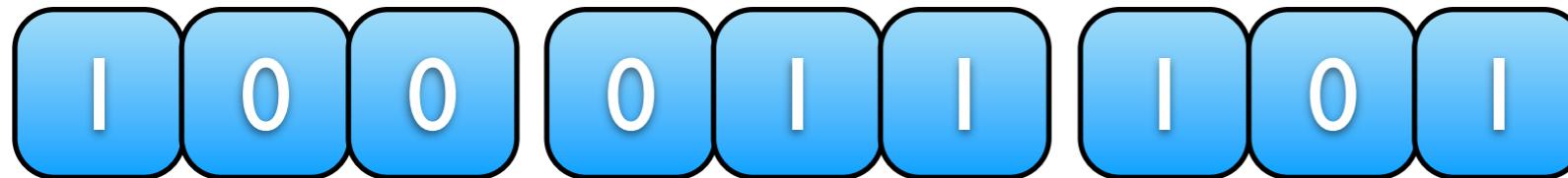
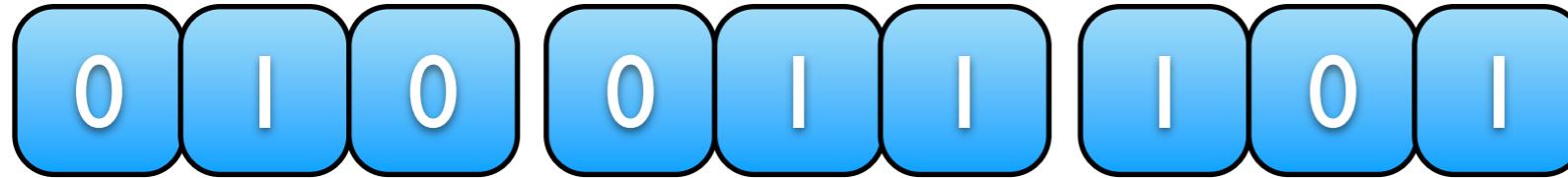
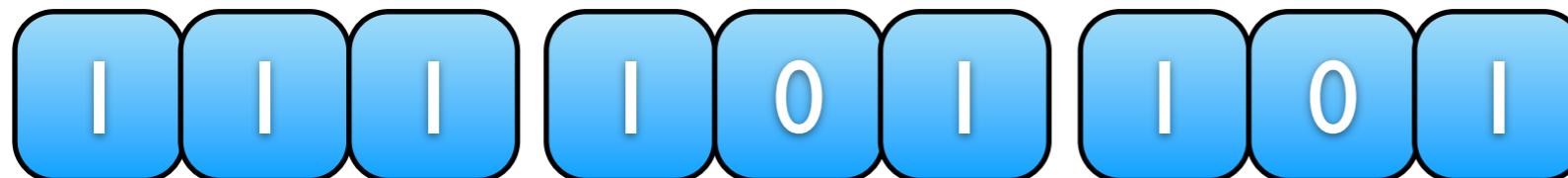
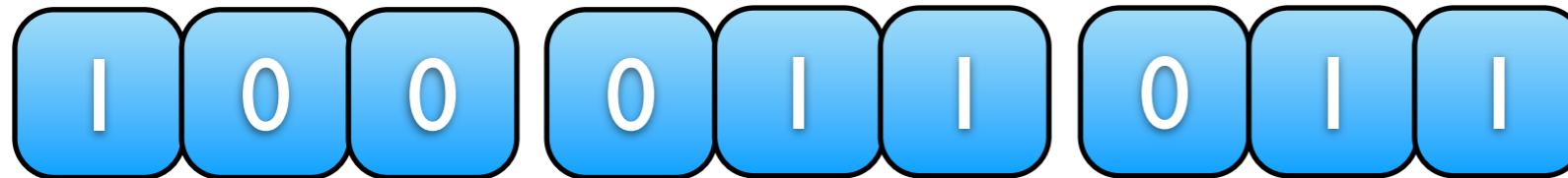
(2,7,5) A: 3, B: 1

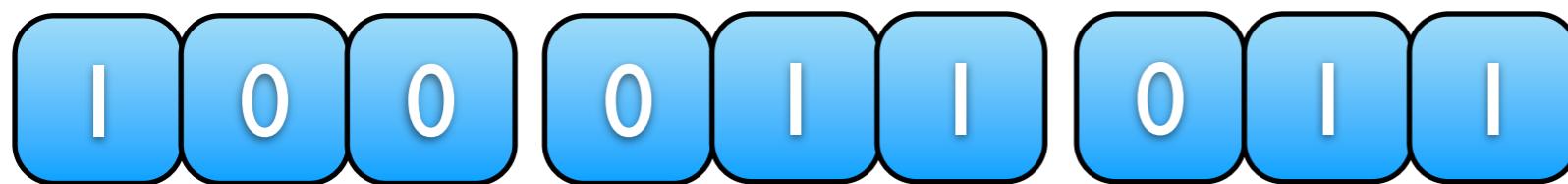


(4,1,5) A: 3, B: 1

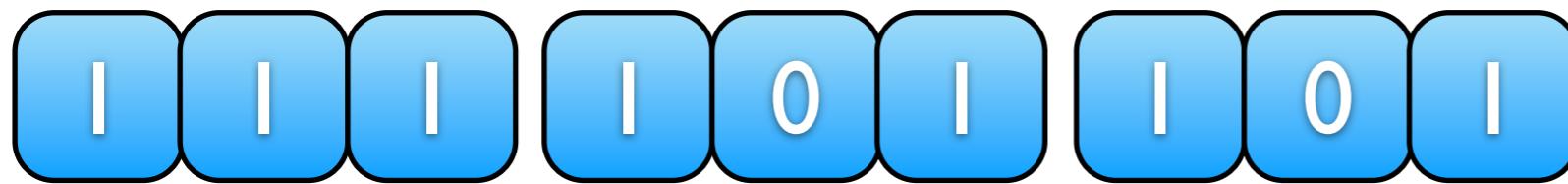


(7,5,1) A: 3, B: 2

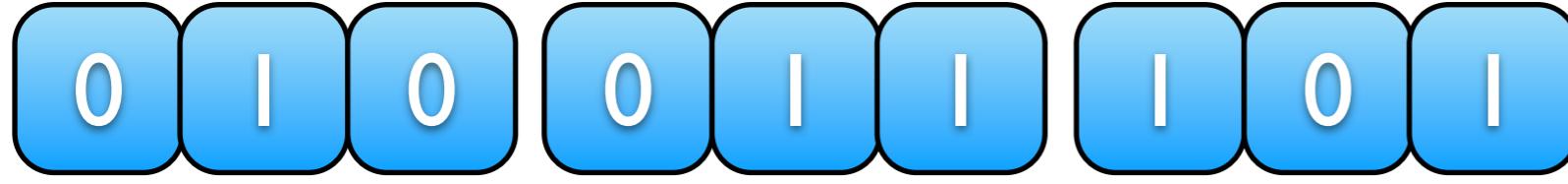




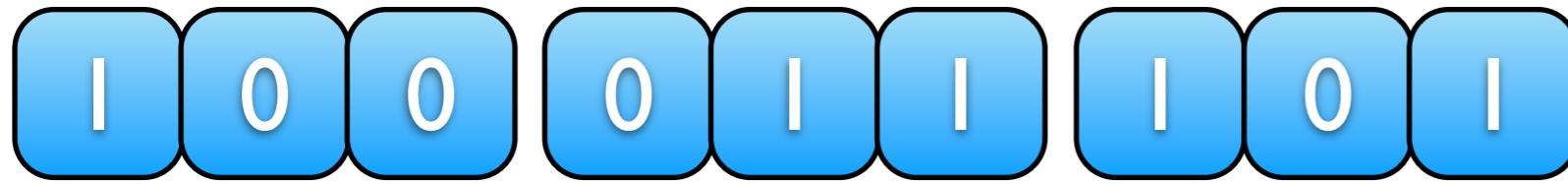
(4,2,2) A: 3, B: I



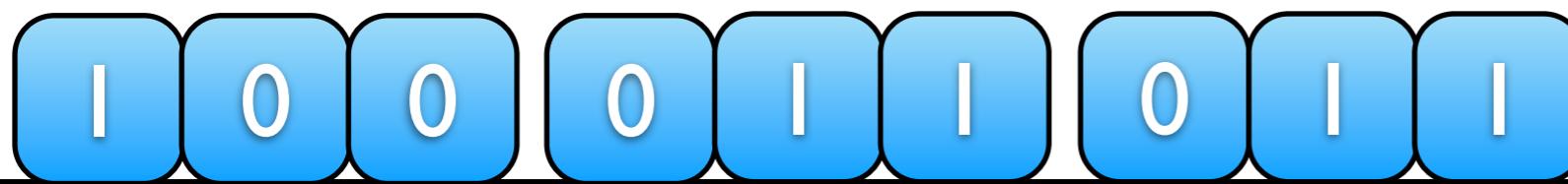
(7,5,5) A: 0, B: 0



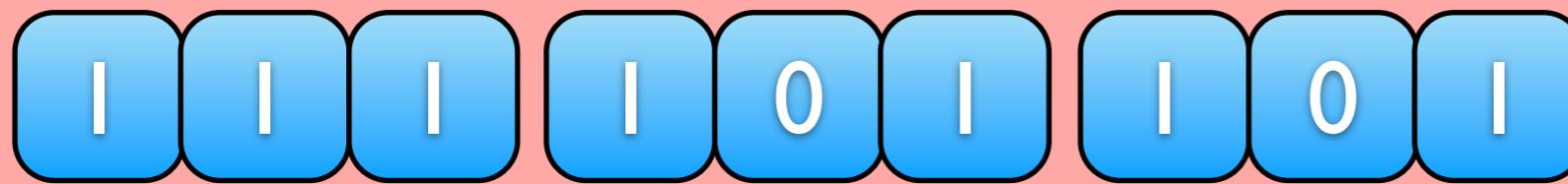
(2,3,5) A: 3, B: I



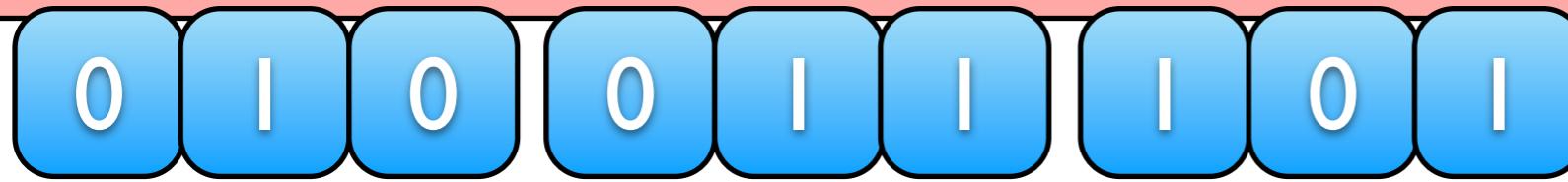
(4,3,5) A: I, B: 2



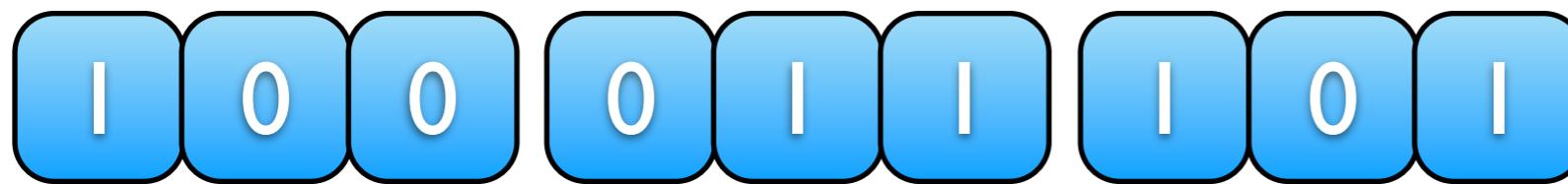
(4,2,2) A: 3, B: I



(7,5,5) A: 0, B: 0



(2,3,5) A: 3, B: I



(4,3,5) A: I, B: 2

# Test Data Generation

Given a function and a location we want to reach, how do we derive inputs to the function that lead the control flow to the desired statement?



## Biological Evolution

- Gene  
Unit of information • passed from one generation to another
- Natural Selection
- Survival of The Fittest
- Origin of New Species

